Sequential supersymmetry breaking in $\mathcal{N} = 2$ SUSY

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Review of partial/sequential SUSY breaking

No-Go Theorem

Suppose $\mathcal{Q}^1 \left|0\right\rangle = \bar{\mathcal{Q}}_1 \left|0\right\rangle = 0$. According to : $\mathrm{tr}\left\{\mathcal{Q}^A, \bar{\mathcal{Q}}_B\right\} = 4H\delta^A{}_B$,

$$\left\langle 0\right| H \left|0\right\rangle = 0 \Leftrightarrow \left\|\mathcal{Q}^2\left|0\right\rangle\right\|^2 + \left\|\bar{\mathcal{Q}}_2\left|0\right\rangle\right\|^2 = 0 \Leftrightarrow \mathcal{Q}^2\left|0\right\rangle = \bar{\mathcal{Q}}_2\left|0\right\rangle = 0.$$

Evading the No-Go theorem [Hughes-Polchinski '86] **:**

- $\nexists\ \mathcal{Q}^{\mathbf{A}}, \bar{\mathcal{Q}}_{\mathbf{A}} \longrightarrow$ **Modification of the SUSY current algebra**

$$\delta^A \lambda^I Z_{IJ}(\phi)\,\delta_B \bar{\lambda}^J = \delta^A{}_B V_S + C^A{}_B$$

- **Non-positive-definiteness of the Hilbert space**

$$\delta^A \lambda^I Z_{IJ}(\phi)\,\delta_B \bar{\lambda}^J - 3M_{Pl}^2 M^{*AC} M_{CB} = \delta^A{}_B V_S$$

("T-identity" [Cecotti-Girardello-Porrati '84])

Motivations

- Only one mechanism is known in $\mathcal{N} = 2$ global SUSY, based on electric **AND** magnetic Fayet-Iliopoulos terms;
- Led to the discovery of new results in differential geometry: Kähler quotients of quaternionic-Kähler manifolds `[Cortés-Louis-Smyth-Triendl '11]`;
- Could be a step towards a reconciliation between extended ($\mathcal{N} = 2$) SUSY and chirality ?
  - Is it possible to break the two SUSYs at two widely separated scales?
  - Is the effective $\mathcal{N} = 1$ theory chiral?

Problem addressed in this talk:

What are the minimal ingredients for seq. SUSY breaking in $\mathcal{N} = 2$ models with hyper and vector multiplets ?

Minimal $\mathcal{N}=2$ model : Gauge sector

$\mathcal{N}=2$ superspace formalism naturally describes partial/sequential SUSY breaking:

- General $\mathcal{N}=2$ chiral superfield $\mathcal{Z}(y,\theta,\tilde{\theta})$, $\bar{D}\mathcal{Z}=\bar{\tilde{D}}\mathcal{Z}=0$:

$$\mathcal{Z}(y,\theta,\tilde{\theta})=Z(y,\theta)+\sqrt{2}\tilde{\theta}\omega(y,\theta)+\tilde{\theta}^2\left(\Phi_Z(y,\theta)-\frac{1}{4}\bar{D}^2\bar{Z}(y,\theta)\right)$$

- Contraint defining a Maxwell multiplet: $D\widetilde{D}\mathcal{Z}+\overline{D}\overline{\widetilde{D}}\bar{\mathcal{Z}}=0$

$$\begin{cases} Z\,(y,\theta)=X\,(y,\theta) & \\ \omega\,(y,\theta)=iW\,(y,\theta) & DW=\bar{D}\bar{W} \\ \Phi_Z\,(y,\theta)=-\frac{iu}{2\kappa} & |u|=1, \kappa\in\mathbb{R} \end{cases}$$

$\Longrightarrow$ Non-linear realization of the SUSY algebra : $\tilde{\delta}\lambda=\sqrt{2}\tilde{\epsilon}\bar{F}^{\bar{x}}+\frac{1}{\sqrt{2}\kappa}\tilde{\epsilon}+\dots$

- $\mathcal{L}_{Max}=-\frac{i}{2}\int d^2\theta d^2\tilde{\theta}\mathcal{F}(\mathcal{W}^a-\frac{iu}{2\kappa_a}\tilde{\theta}^2)+$ h.c.

Minimal $\mathcal{N}=2$ model : F.I. terms

- Auxiliary fields F^{x^a} and D^a embedded in triplets Y^{Ia} of $SU(2)_R$:

$$Y^{Ia} \equiv \left(-2\text{Im}F^{x^a}, 2\text{Re}F^{x^a}, \sqrt{2}D^a\right)$$

$\Longrightarrow$ Triplet of electric F.I. terms with F.I. coefficients

$$E^I_a \equiv \left(-\text{Im}e_a, -\text{Re}e_a, \sqrt{2}\xi_a\right)$$

$$\mathcal{L}_{gauge} = \int d^4\theta \left[\text{Im}\left(\mathcal{F}_a(X)\bar{X}^a\right) + \xi_a V^a\right]$$
$$+ \int d^2\theta \left[-\frac{i}{4}\mathcal{F}_{ab}(X)W^aW^b - \frac{e_a}{4}X^a - \frac{u}{4\kappa_a}\mathcal{F}_a(X)\right] + \text{ h.c.}$$

- $(X^a, \mathcal{F}_a)$: symplectic section under electric-magnetic duality

$\Longrightarrow$ $-\frac{u}{4\kappa_a}\mathcal{F}_a(X)$ magnetic F.I. term.

$$M^{I\ a} \equiv \left(\text{Im}\frac{u}{\kappa_a}, \text{Re}\frac{u}{\kappa_a}, 0\right)$$

Minimal $\mathcal{N}=2$ model : Matter sector

- Matter multiplet with $U(1)$ isometry : off-shell realization in $M^{4|8}$:
  - $\mathcal{N}=1$ superfield content of the single-tensor multiplet: $L = D\chi + \bar{D}\bar{\chi} \mid \Phi$;
  - Gauge invariance of the linear multiplet: $b_{\mu\nu} \xrightarrow{g} b_{\mu\nu} + 2\partial_{[\mu}\Lambda_{\nu]}$;

$$\begin{cases} \mathcal{Y}(y,\theta,\tilde{\theta}) = Y(y,\theta) + \sqrt{2}\tilde{\theta}\chi(y,\theta) + \tilde{\theta}^2\left(-\frac{i}{2}\Phi(y,\theta) - \frac{1}{4}\bar{D}^2\bar{Y}(y,\theta)\right) \\ \delta_g\mathcal{Y} = -\hat{\mathcal{W}} \end{cases}$$

  - Y can be eliminated, but is necessary for the closure of the algebra [Ambrosetti-Antoniadis-Derendinger-Tziveloglou '09].
- Kinetic terms and self-interactions [Lindström-Rocek '83]:

$$\mathcal{L}_{ST} = \int d^4\theta H\left(L,\Phi,\bar{\Phi}\right) \quad , \quad H_{LL} + 2H_{\Phi\bar{\Phi}} = 0$$

- Interaction Lagrangian : $\mathcal{N}=2$ version of the Chern-Simons term $b\wedge F$

$$\mathcal{L}_{CS} = -2ig_a \int d^2\theta d^2\tilde{\theta}\,\mathcal{Y}\left(\mathcal{W}^a - \frac{iu}{2\kappa_a}\tilde{\theta}^2\right) + \text{ h.c.}$$

Minimal $\mathcal{N}=2$ model : Minimal gauge group

- $\mathcal{L}_{CS} = 2g_a \int d^4\theta L V^a - g_a \int d^2\theta \left(\Phi X^a + \frac{i}{\kappa_a} Y \right) + \text{ h.c.}$
  - CS interaction related to the canonical gauge interaction by the scalar-tensor duality [Lindström-Rocek '83]
- K obtained as Legendre transform of H :
  - Canonical K dual to $H\left(L,\Phi,\bar{\Phi}\right) = \sqrt{L^2 + 2\Phi\bar{\Phi}} - L \ln\left(L + \sqrt{L^2 + 2\Phi\bar{\Phi}}\right)$
- No kinetic term for Y in both formulations of the model with n_v abelian $\mathcal{N}=2$ vector multiplets and 1 single-tensor/hyper multiplet :

  $\Longrightarrow$ Constraint on the value of n_v [Itoyama-Maruyoshi-Sakaguchi '07]:

$$\sum_{a=1}^{n_v} \frac{g_a}{\kappa_a} = 0 \longrightarrow n_v \geq 2$$

Minimal gauge group for partial/sequential SUSY breaking with charged matter: $\mathbf{U(1) \times U(1)}$.

Goldstini

- Determination of the number of broken SUSY's with the Goldstini:

$$\eta_1 = \frac{1}{N_1}\left(g_{u\bar{v}}\bar{F}^{\bar{q}^{\bar{v}}}\chi^u + h_{ab}\bar{F}^{\bar{x}^a}\psi^{x^b} - \frac{i}{\sqrt{2}}h_{ab}D^a\lambda^b\right),$$

$$\eta_2 = \frac{1}{N_2}\left(g_{u\bar{v}}\bar{\tilde{F}}^{\bar{q}^{\bar{v}}}\chi^u - \frac{i}{\sqrt{2}}h_{ab}D^a\psi^{x^b} + h_{ab}\left(F^{x^a} + \frac{1}{2\kappa_a}\right)\lambda^b\right)$$

- Normalization (valid for $\mathcal{N}=0$ vacua):

$$N_1 = \sqrt{g_{u\bar{v}}F^{q^u}\bar{F}^{\bar{q}^{\bar{v}}} + h_{ab}\left[F^{x^a}\bar{F}^{\bar{x}^b} + \frac{1}{2}D^aD^b\right]} = \sqrt{\langle V_S\rangle}\,,$$

$$N_2 = \sqrt{N_1^2 + E} \quad , \quad E = \frac{\mathrm{Re}\, e_a}{4\kappa_a}$$

  - $\mathcal{N}=1$ vacuum: η_1, η_2 linearly dependent or η_1 or η_2 ill-defined;
  - $\mathcal{N}=0$ vacuum: η_1, η_2 linearly independent.

SUSY-breaking scales

- $\mathcal{N}=1$ SUSY-breaking scale: $\Lambda_{SUSY} \sim \langle V_S \rangle^{1/4}$;

  Goldstino SUSY variation: $\delta\eta_1 = \sqrt{2\langle V_S \rangle}$;

- $\mathcal{N}=2$ SUSY: $\left(\Lambda^{1,2}_{SUSY}\right)^2$ are the eigenvalues of the 2×2 matrix $\delta_i\eta_j$:

$$\left(\Lambda^{1,2}_{SUSY}\right)^2 = \frac{N_1+\sqrt{N_1^2+E}}{\sqrt{2}} \pm \frac{1}{2}\sqrt{2\left(N_1-\sqrt{N_1^2+E}\right)^2 + \frac{\left(\frac{h_{ab}D^b}{\kappa_a}\right)^2}{N_1\sqrt{N_1^2+E}}}$$

- Two sources of degeneracy lifting: $E \sim \frac{\mathrm{Re}e_a}{\kappa_a}$, $\frac{h_{ab}D^b}{\kappa_a} \sim \frac{\xi_a}{\kappa_a}$:
  - $\begin{cases} E_a^I \equiv \left(-\mathrm{Im}e_a, -\mathrm{Re}e_a, \sqrt{2}\xi_a\right) \\ M^{I\ a} \equiv \left(\frac{1}{\kappa_a}, 0, 0\right) \end{cases} \Longrightarrow$ Two scales if $\vec{E}_a \times \vec{M}^a \neq 0$;
  - $C^A{}_B \sim \vec{\sigma}^A{}_B \cdot \left(\vec{E}_a \times \vec{M}^a\right)$ [Ferrara-Girardello-Porrati '96].

Conclusion and Outlook

- Minimal model with charged matter of sequential (global) SUSY breaking:
  - Two $U(1)$ Maxwell multiplets;
  - 1 Single-tensor/Hyper multiplet;
- Necessary ingredients for sequential SUSY breaking:

  $\longrightarrow$ Identical to those for partial SUSY breaking:
  - $\vec{E}_a$, $\vec{M}^a$ such that $\vec{E}_a \times \vec{M}^a \neq 0$;
  - Non-renormalizable prepotential $\mathcal{F}(X^a)$;
  - Fully depends on F.I. terms;
- Could the spectrum in the effective $\mathcal{N}=1$ theory at a scale Λ_{int}, $\Lambda^1_{SUSY} \ll \Lambda_{int} \ll \Lambda^2_{SUSY}$, be chiral ?
- How do F.I. terms arise from a local supersymmetric theory in general ?
  - Extension of the sequential breaking to $\mathcal{N}=2$ SUGRA;
  - Rigid limit of this type of theories.

Backup slides

SUSY-breaking scales

- Separate study of the two contributions:
  - D-flatness: $\left(\Lambda^1_{SUSY}\right)^2 = \sqrt{2}N_1$, $\left(\Lambda^2_{SUSY}\right)^2 = \sqrt{2\left(N_1^2 + E\right)}$;
  - $\mathrm{Re}e_a = 0$:

$$\left(\Lambda^{1,2}_{SUSY}\right)^2 = \frac{\sqrt{2}}{N_1}\left[g_{u\bar{v}} F^{q^u} \bar{F}^{\bar{q}^{\bar{v}}} + h_{ab} \mathrm{Im} F^{x^a} \mathrm{Im} F^{x^b} + \ h_{ab}\left(\frac{1}{4\kappa_a} \pm \frac{D^a}{\sqrt{2}}\right)\left(\frac{1}{4\kappa_b} \pm \frac{D^b}{\sqrt{2}}\right)\right]$$

- Partial SUSY Breaking $\Longleftrightarrow$ one scale vanishes:
  - Conditions for PSB ($\mathrm{Re}e_a = 0$):

$$\langle F^{x^a} \rangle = -\frac{1}{4\kappa_a} \ , \langle D^a \rangle = \mp \frac{1}{2\sqrt{2}\kappa_a} \ , \langle F^{q^u} \rangle = \langle \tilde{F}^{q^u} \rangle = 0$$

$\longrightarrow$ $\eta_2 = \mp i\eta_1 \Longrightarrow \mathcal{Q} \pm i\tilde{\mathcal{Q}}$ is preserved;

$\longrightarrow$ The unbroken SUSY only depends on the orientation of $\vec{E}_a$, $\vec{M}^a$.

Sequential SUSY breaking - Simplest example

- Separable prepotential $\mathcal{F}\left(X^1, X^2\right) = f_1\left(X^1\right) + f_2\left(X^2\right)$;

- $\left(\Lambda^1_{SUSY}\right)^2 = \frac{g_1\xi_2}{2Ng_2\kappa_1}$, $\left(\Lambda^2_{SUSY}\right)^2 = \frac{g_2\xi_1}{g_1\xi_2}\left(\Lambda^1_{SUSY}\right)^2$:

  $\longrightarrow$ The ratio $\Lambda^1_{SUSY}/\Lambda^2_{SUSY}$ can be arbitrarily large/small, by tuning the F.I. coefficients;

- $\mathcal{N}=1$ spectrum at a scale Λ_{int}, $\Lambda^1_{SUSY} \ll \Lambda_{int} \ll \Lambda^2_{SUSY}$:
  - $\eta_1 = \frac{1}{2\sqrt{2}N}\left[-\xi_1\left(\psi^{x^1} - i\lambda^1\right) + \xi_2\left(\psi^{x^2} + i\lambda^2\right)\right]$,

    $\eta_2 = \frac{1}{2\sqrt{2}N}\left[i\xi_1\left(\psi^{x^1} - i\lambda^1\right) + i\xi_2\left(\psi^{x^2} + i\lambda^2\right)\right]$;
  - Non-renormalizable prepotential for $\mathcal{N}=1$ (instead of $\mathcal{N}=2$) spectrum at this scale.