Local Supersymmetry Without SUSY Partners

Pedro Álvarez, Pablo Pais, Jorge Zanelli

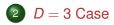
Centro de Estudios Científicos (CECs) & Universidad Andrés Bello (UNAB)

> Trieste, Italy August 2013

Pedro Álvarez, Pablo Pais, Jorge Zanelli Local Supersymmetry Without SUSY Partners

イロト イポト イヨト イヨト









 $\begin{array}{l} \mbox{Motivation}\\ D=3\ \mbox{Case}\\ D=4\ \mbox{Case}\\ \mbox{Future Work}\\ \mbox{Bonus Track} \end{array}$









イロト 不得 とくほと くほとう

∃ <\0,0</p>

 The idea of unification has proven a strong guide for progress in Physics

イロト 不得 とくほ とくほとう

- The idea of unification has proven a strong guide for progress in Physics
- Supersymmetry (SUSY) is a nontrivial extension of the Poincaré group that includes internal symmetries unifying fermions and bosons

イロト イポト イヨト イヨト

э

 $\begin{array}{l} \text{Motivation}\\ D=3 \text{ Case}\\ D=4 \text{ Case}\\ \text{Future Work}\\ \text{Bonus Track} \end{array}$

- The idea of unification has proven a strong guide for progress in Physics
- Supersymmetry (SUSY) is a nontrivial extension of the Poincaré group that includes internal symmetries unifying fermions and bosons
- A distinct signal of standard SUSY would be the existence of partners that duplicate the spectrum of observed particles

In an unbroken supersymmetric phase these SUSY partners would have degenerate masses

イロト イポト イヨト イヨト

 $\begin{array}{l} \text{Motivation} \\ D = 3 \text{ Case} \\ D = 4 \text{ Case} \\ \text{Future Work} \\ \text{Bonus Track} \end{array}$

- The idea of unification has proven a strong guide for progress in Physics
- Supersymmetry (SUSY) is a nontrivial extension of the Poincaré group that includes internal symmetries unifying fermions and bosons
- A distinct signal of standard SUSY would be the existence of partners that duplicate the spectrum of observed particles

In an unbroken supersymmetric phase these SUSY partners would have degenerate masses \implies spontaneously broken at current experimental energies

イロト 不同 とくほ とくほ とう

 SUSY constrains the field contents of a theory: restricts the interaction patterns, and relates different coupling constants and particle masses

イロト イポト イヨト イヨト

- SUSY constrains the field contents of a theory: restricts the interaction patterns, and relates different coupling constants and particle masses
- The detailed form of those constraints depends on the particular form in which SUSY is presented

イロト イポト イヨト イヨト

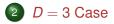
-

- SUSY constrains the field contents of a theory: restricts the interaction patterns, and relates different coupling constants and particle masses
- The detailed form of those constraints depends on the particular form in which SUSY is presented
- Can those fields transform in some particular way obtaining a SUSY without SUSY partners?

イロト イポト イヨト イヨト

Motivation			
D		3	Case
D			Case
Fι	itu		Work
Bo	งทเ	IS	Track









Pedro Álvarez, Pablo Pais, Jorge Zanelli Local Supersymmetry Without SUSY Partners

◆ロト ◆御 ト ◆臣 ト ◆臣 ト ○臣 … 釣んで

• Affirmative answer in D = 3 with U(1) internal group¹

 ¹ P.D.Alvarez, M.Valenzuela and J.Zanelli, Supersymmetry of a Different

 Kind, JHEP 1204, 058 (2012)

Pedro Álvarez, Pablo Pais, Jorge Zanelli Local Supersymmetry Without SUSY Partners

- Affirmative answer in D = 3 with U(1) internal group¹
- Curious similarity:



 ¹ P.D.Alvarez, M.Valenzuela and J.Zanelli, Supersymmetry of a Different

 Kind, JHEP 1204, 058 (2012)

Pedro Álvarez, Pablo Pais, Jorge Zanelli Local Supersymmetry Without SUSY Partners

Take the 3 × 3 matrix constructed with $A = \gamma_{\mu} A^{\mu}$, extended with the column ψ and row $\overline{\psi}$

$$\mathbb{A} = \begin{pmatrix} \mathbf{A}^{\alpha}{}_{\beta} & \psi^{\alpha} \\ \overline{\psi}_{\beta} & \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \psi^{\alpha} \\ \overline{\psi}_{\beta} & \mathbf{0} \end{pmatrix}$$

イロト 不得 とくほ とくほとう

For the U(1) group, we know that under gauge transformations:

イロト 不得 とくほ とくほとう

For the U(1) group, we know that under gauge transformations:

$$\begin{array}{lll} \mathbf{A}'_{\mu} &=& \mathbf{A}_{\mu} + \partial_{\mu} \alpha \\ \psi' &=& \mathbf{e}^{i\alpha} \psi \\ \overline{\psi}' &=& \mathbf{e}^{-i\alpha} \overline{\psi} \end{array}$$

The gauge transformations for $A_{\mu}, \psi, \overline{\psi}$ can be obtained as

$$\mathbb{A}' = g^{-1}\mathbb{A}g + g^{-1}\#g,$$

being $g = exp[i\mathbb{K}]$ with $i\mathbb{K} = diag(1, 1, 2)$ and $d = diag(\gamma^{\mu}\partial_{\mu}, 0)$

イロト 不同下 不良下 不良下 一度

$$\mathbb{A}_{\mu} = \mathbf{A}_{\mu}\mathbb{K} + \overline{\mathbb{Q}}_{\beta}(\gamma_{\mathbf{a}})^{\beta}{}_{\alpha}\mathbf{e}^{\mathbf{a}}{}_{\mu}\psi^{\alpha} + \overline{\psi}_{\beta}(\gamma_{\mathbf{a}})^{\beta}{}_{\alpha}\mathbf{e}^{\mathbf{a}}{}_{\mu}\mathbb{Q}^{\alpha} + \omega_{\mu}^{\mathbf{a}}\mathbb{J}_{\mathbf{a}}$$

$$\mathbb{A}_{\mu} = \mathbf{A}_{\mu}\mathbb{K} + \overline{\mathbb{Q}}_{\beta}(\gamma_{\mathbf{a}})^{\beta}{}_{\alpha}\mathbf{e}^{\mathbf{a}}{}_{\mu}\psi^{\alpha} + \overline{\psi}_{\beta}(\gamma_{\mathbf{a}})^{\beta}{}_{\alpha}\mathbf{e}^{\mathbf{a}}{}_{\mu}\mathbb{Q}^{\alpha} + \omega_{\mu}^{\mathbf{a}}\mathbb{J}_{\mathbf{a}}$$

with the graded Lie algebra

$$\begin{split} [\mathbf{J}_{a},\mathbf{J}_{b}] &= \epsilon^{c}_{ab}\mathbf{J}_{c} \qquad \{\overline{\mathbb{Q}}^{\alpha},\mathbb{Q}_{\beta}\} = -\mathbf{J}_{a}(\gamma^{a})^{\alpha}_{\beta} - \frac{i}{2}\delta^{\alpha}_{\beta}\mathbb{K}, \\ [\mathbf{J}_{a},\mathbb{Q}^{\alpha}] &= \frac{1}{2}(\gamma^{a})^{\alpha}_{\beta}\mathbb{Q}^{\beta} \qquad [\mathbf{J}_{a},\mathbb{Q}_{\alpha}] = -\frac{1}{2}(\gamma^{a})^{\beta}_{\alpha}\mathbb{Q}_{\beta}, \\ [\mathbb{K},\mathbb{Q}^{\alpha}] &= i\mathbb{Q}^{\alpha} \qquad [\mathbb{K},\mathbb{Q}_{\alpha}] = -i\mathbb{Q}_{\alpha} \end{split}$$

;

イロト 不得 トイヨト イヨト



• The Chern-Simons (CS) form provides a Lagrangian for the connection A without additional ingredients

$$L = \left\langle \mathbb{A} d\mathbb{A} + \frac{2}{3} \mathbb{A}^3 \right\rangle$$

イロト 不得 とくほ とくほとう

• The Chern-Simons (CS) form provides a Lagrangian for the connection A without additional ingredients

$$L = \left\langle \mathbb{A}d\mathbb{A} + \frac{2}{3}\mathbb{A}^3 \right\rangle$$

Using the standard conventions, the Lagrangian reads

$$L = 2AdA + \frac{1}{4} [\omega^{a}{}_{b}d\omega^{b}{}_{a} + \frac{2}{3} \omega^{a}{}_{b}\omega^{b}{}_{c}\omega^{c}{}_{a}] - 2\overline{\psi}\psi e^{a}T_{a} + 2\overline{\psi}(\overleftarrow{\partial} - \overrightarrow{\partial} + 2iA + \frac{1}{2}\gamma^{a}\psi_{ab}\gamma^{b})\psi|e|d^{3}x,$$

where $|e| = \det[e^a_{\mu}] = \sqrt{-g}$, and $T^a = de^a + \omega^a{}_b e^b$ is the torsion 2-form.

The supersymmetric transformations are deduced by $\delta \mathbb{A} = \textit{d} \Lambda + [\mathbb{A}, \Lambda] \text{, with}$

$$\Lambda = \alpha \mathbb{K} + \overline{\mathbb{Q}}\epsilon - \overline{\epsilon}\mathbb{Q} + \lambda^{a}\mathbb{J}_{a}$$

イロン 不得 とくほ とくほ とうほ

The supersymmetric transformations are deduced by $\delta \mathbb{A} = \textit{d} \Lambda + [\mathbb{A}, \Lambda] \text{, with}$

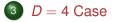
$$\Lambda = \alpha \mathbb{K} + \overline{\mathbb{Q}}\epsilon - \overline{\epsilon} \mathbb{Q} + \lambda^{a} \mathbb{J}_{a}$$

given the result

$$\begin{split} \delta \mathbf{A} &= \mathbf{d}\alpha - \frac{i}{2} \left(\overline{\epsilon} \mathbf{\phi} \psi + \overline{\psi} \mathbf{\phi} \epsilon \right) \\ \delta(\mathbf{\phi}\psi) &= \overrightarrow{\nabla} \epsilon + i\alpha \mathbf{\phi}\psi + \frac{1}{2} \lambda^a \gamma_a \mathbf{\phi}\psi \\ \delta(\overline{\psi}\mathbf{\phi}) &= \epsilon \overleftarrow{\nabla} - i\alpha \overline{\psi}\mathbf{\phi} - \frac{1}{2} \lambda^a \overline{\psi}\mathbf{\phi}\gamma_a \\ \delta\omega^a &= \mathbf{d}\lambda^a + \epsilon^a_{bc} \omega^b \lambda^c - \left(\overline{\epsilon}\gamma^a \mathbf{\phi}\psi + \overline{\psi}\mathbf{\phi}\gamma^a_a \epsilon \right)_{\text{constraints}} \end{split}$$









イロト 不得 とくほと くほとう

∃ <\0,0</p>

Connection

$$\mathbb{A} = A\mathbb{K} + \overline{\mathbb{Q}} \not e \psi + \overline{\psi} \not e \mathbb{Q} + f^{a} \mathbb{J}_{a} + \frac{1}{2} \omega^{ab} \mathbb{J}_{ab},$$

for the supercharges

$$\{\overline{\mathbb{Q}}_{\alpha}, \mathbb{Q}^{\beta}\} = -\frac{i}{s} (\Gamma^{a})_{\alpha}^{\beta} \mathbb{J}_{a} + \frac{i}{2} (\Gamma^{ab})_{\alpha}^{\beta} \mathbb{J}_{ab} - \delta_{\alpha}^{\beta} \mathbb{K} ,$$

イロト イポト イヨト イヨト



In D = 4 we do not have the Chern Simons form as in D = 3

(ロ) (同) (ヨ) (ヨ) (ヨ) (コ) (の)

- In D = 4 we do not have the Chern Simons form as in D = 3
- The curvature $\mathbb{F} = d\mathbb{A} + \mathbb{A}\mathbb{A}$ takes the form

$$\mathbb{F} = F_0 \mathbb{K} + \overline{\mathbb{Q}}_{\alpha} \mathcal{F}^{\alpha} + \overline{\mathcal{F}}_{\alpha} \mathbb{Q}^{\alpha} + F^a \mathbb{J}_a + \frac{1}{2} F^{ab} \mathbb{J}_{ab}$$

◆ロト ◆御 ト ◆臣 ト ◆臣 ト ○臣 … 釣んで

We can try naively with

 $\langle \mathbb{F} \wedge \mathbb{F} \rangle$

イロン イ団ン イヨン イヨン

∃ ∽ へ (~

• We can try naively with

 $\langle \mathbb{F} \wedge \mathbb{F} \rangle$

• But this does not give field equations: it is an invariant so when we perform a variation it vanish identically

イロト 不得 トイヨト イヨト

• We can try naively with

 $\langle \mathbb{F} \wedge \mathbb{F} \rangle$

- But this does not give field equations: it is an invariant so when we perform a variation it vanish identically
- $\bullet\,$ We can break the symmetry, introducing some kind of dual $\circledast \mathbb{F}$

イロト 不得 トイヨト イヨト

$$\circledast \mathbb{F} = *F_0 \mathbb{K} + (\overline{\mathbb{Q}})_{\alpha} (\Gamma_5 \mathcal{F})^{\alpha} + (\overline{\mathcal{F}})_{\alpha} (\Gamma_5 \mathbb{Q})^{\alpha} + \hat{\Gamma} \left[F^a \mathbb{J}_a + \frac{1}{2} F^{ab} \mathbb{J}_{ab} \right],$$

with

With this rules we write the Lagrangian as

$$L = -\frac{1}{4} \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

¹ Towsend Phys. Rev. D 15, 2795 (1977)

ヘロト 人間 とくほとく ほとう

With this rules we write the Lagrangian as

$$L = -rac{1}{4} \langle \mathbb{F} \circledast \mathbb{F}
angle$$

A natural way to avoid parity violation is to impose $f^a \wedge e_a = 0$ which means¹ $f^a = \mu e^a$

with μ a constant of canonical dimension L^{-1}

¹ Towsend Phys. Rev. D 15, 2795 (1977)

・ロン ・ 一 レ ・ 日 と ・ 日 と

э

Doing the computations we obtain

$$L = -\frac{1}{16} \epsilon_{abcd} (R^{ab} - \mu^2 e^a e^b) (R^{cd} - \mu^2 e^c e^d) + \frac{1}{2} \left[\overline{\psi} (\overleftarrow{\nabla} - \overrightarrow{\nabla}) \psi + 4\mu i \overline{\psi} \psi \right] + t^{\mu} \overline{\psi} \Gamma_5 \Gamma_{\mu} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} |e| d^4 x - \frac{1}{3\mu^2} \left[(\overline{\psi} \psi)^2 - (\overline{\psi} \Gamma_5 \psi)^2 \right].$$

with $t^{\mu} \equiv -\frac{1}{3!} \varepsilon^{\mu\nu\rho\tau} e^{a}_{\nu} T_{a\rho\tau} |e|$, and

$$\vec{\nabla} \psi \equiv (\partial - i\mathbf{A} + \frac{1}{2}\phi)\psi$$
$$\vec{\psi} \vec{\nabla} \equiv \vec{\psi} (\overleftarrow{\partial} + i\mathbf{A} - \frac{1}{2}\phi)$$



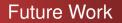






イロト イポト イヨト イヨト

∃ <\0,0</p>



 Can we extend the present analysis in order to include a non-Abelian group like SU(2) or SU(3)?

イロト イポト イヨト イヨト



- Can we extend the present analysis in order to include a non-Abelian group like *SU*(2) or *SU*(3)?
- Can we construct the Lagrangian in *D* = 4 in a more subtle way?

イロト 不得 とくほとくほとう

-



- Can we extend the present analysis in order to include a non-Abelian group like *SU*(2) or *SU*(3)?
- Can we construct the Lagrangian in D = 4 in a more subtle way?
 Starting with a CS form in D = 5?

イロト イポト イヨト イヨト

-

Thanks!

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへで

SUSY algebra D = 4

$$\begin{bmatrix} \mathbf{J}_{a}, \mathbf{J}_{b} \end{bmatrix} = \mathbf{s}^{2} \mathbf{J}_{ab} , \quad \begin{bmatrix} \mathbf{J}_{a}, \mathbf{J}_{bc} \end{bmatrix} = \eta_{ab} \mathbf{J}_{c} - \eta_{ac} \mathbf{J}_{b} ,$$
$$\begin{bmatrix} \mathbf{J}_{ab}, \mathbf{J}_{cd} \end{bmatrix} = \eta_{ad} \mathbf{J}_{bc} - \eta_{ac} \mathbf{J}_{bd} + \eta_{bc} \mathbf{J}_{ad} - \eta_{bd} \mathbf{J}_{ac} ,$$

$$[\mathbb{J}_{a}, \mathbb{Q}^{\alpha}] = -\frac{s}{2} (\Gamma_{a})^{\alpha}{}_{\beta} \mathbb{Q}^{\beta}, \quad [\mathbb{J}_{a}, \overline{\mathbb{Q}}_{\alpha}] = \frac{s}{2} \overline{\mathbb{Q}}_{\beta} (\Gamma_{a})^{\beta}{}_{\alpha},$$

$$[\mathbb{J}_{ab}, \mathbb{Q}^{\alpha}] = -\frac{1}{2} (\Gamma_{ab})^{\alpha}{}_{\beta} \mathbb{Q}^{\beta}, \quad [\mathbb{J}_{ab}, \overline{\mathbb{Q}}_{\alpha}] = \frac{1}{2} \overline{\mathbb{Q}}_{\beta} (\Gamma_{ab})^{\beta}{}_{\alpha}.$$

$$[\mathbb{K}, \mathbb{Q}^{\alpha}] = i\mathbb{Q}^{\alpha}, \qquad [\mathbb{K}, \overline{\mathbb{Q}}_{\alpha}] = -i\overline{\mathbb{Q}}_{\alpha}.$$

$$\{\mathbb{Q}^{\alpha}, \overline{\mathbb{Q}}_{\beta}\} = -\frac{i}{s} (\Gamma^{a})^{\alpha}_{\beta} \mathbb{J}_{a} + \frac{i}{2} (\Gamma^{ab})^{\alpha}_{\beta} \mathbb{J}_{ab} - \delta^{\alpha}_{\beta} \mathbb{K}.$$

Local Supersymmetry Without SUSY Partners

Explicit Form of the Generators D = 4

$$(\mathbb{Q}^{\alpha})^{A}_{B} = -i \begin{bmatrix} 0_{4 \times 4} & 0 & 0 & 0 \\ 0 & 0 & C^{\alpha A} \\ 0 & 0 & 0 \\ \hline \delta^{\alpha}_{B} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = -i (\delta^{A}_{5} \delta^{\alpha}_{B} + C^{\alpha A} \delta^{6}_{B} 0^{1})$$
$$(\overline{\mathbb{Q}}_{\alpha})^{A}_{B} = \begin{bmatrix} 0_{4 \times 4} & 0 & 0 \\ 0_{4 \times 4} & \delta^{A}_{\alpha} & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \delta^{A}_{\alpha} \delta^{5}_{B} + \delta^{A}_{6} C_{\alpha B}, \quad (2)$$

Explicit Form of the Generators D = 4

Pedro Álvarez, Pablo Pais, Jorge Zanelli Local Supersymmetry Without SUSY Partners

SUSY Transformations D = 4

$$(\mathbb{K})^{A}{}_{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0_{4 \times 4} & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -i \end{bmatrix} = i(\delta^{A}_{5}\delta^{5}_{B} - \delta^{A}_{6}\delta^{6}_{B}), \quad (5)$$

イロト 不得 とくほと くほとう

= 990

SUSY Transformations D = 4

The variation of $\mathbb A$ under supersymmetry generated by $\Lambda=\overline{\mathbb Q}\epsilon-\overline{\epsilon}\mathbb Q$ is

$$\delta \mathbb{A} = d\Lambda + [\mathbb{A}, \Lambda].$$

Using the (anti-) commuting relations of the superalgebra, one finds

$$\delta \mathbf{A}_{\mu} = -\left(\overline{\epsilon}\Gamma_{\mu}\psi + \overline{\psi}\Gamma_{\mu}\epsilon\right) \tag{6}$$

$$\delta\omega^{ab} = i\left(\overline{\epsilon}\Gamma^{ab}\phi\psi + \overline{\psi}\phi\Gamma^{ab}\epsilon\right)$$
(8)

(日本) (日本) (日本)

э

$$\delta \left[\Gamma_c e^c \psi \right] = \left[d - iA + \frac{1}{2} s f^a \Gamma_a + \frac{1}{4} \omega^{ab} \Gamma_{ab} \right] \epsilon.$$
 (9)

Relations Between Constants in D = 3 and D = 4

In three dimensions,

Electric chargee = 1Newton's constantG = 1Cosmological constant $\Lambda = -\mu^2$ Fermion mass $m = \mu$

And in four dimensions,

Electric chargee = 1Newton's constant $G = -s^2(4\pi\mu^2)^{-1}$ Cosmological constant $\Lambda = -s^2\mu^2$ Nambu–Jona-Lasinio coupling $g = (3\mu)^{-2}$

イロト イポト イヨト イヨト