Local Supersymmetry Without SUSY Partners

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1. Motivation
2. $D = 3$ Case
3. $D = 4$ Case
4. Future Work

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- Supersymmetry (SUSY) is a nontrivial extension of the Poincaré group that includes internal symmetries unifying fermions and bosons
- A distinct signal of standard SUSY would be the existence of partners that duplicate the spectrum of observed particles
  In an unbroken supersymmetric phase these SUSY partners would have degenerate masses $\Longrightarrow$ spontaneously broken at current experimental energies

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- The detailed form of those constraints depends on the particular form in which SUSY is presented
- Can those fields transform in some particular way obtaining a SUSY without SUSY partners?

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- Affirmative answer in $D = 3$ with $U(1)$ internal group[1]

[1] *P.D.Alvarez, M.Valenzuela and J.Zanelli, Supersymmetry of a Different Kind, JHEP* **1204**, *058 (2012)*

- Affirmative answer in $D = 3$ with $U(1)$ internal group[1]
- Curious similarity:

$$\underbrace{\overline{\psi}\partial\!\!\!/\psi}_{Dirac} \Longleftrightarrow \underbrace{AdA}_{Abelian-Chern-Simons}$$

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Take the 3×3 matrix constructed with $\not{A} = \gamma_\mu A^\mu$, extended with the column ψ and row $\overline{\psi}$

$$\mathbb{A} = \begin{pmatrix} A^\alpha{}_\beta & \psi^\alpha \\ \overline{\psi}_\beta & 0 \end{pmatrix} = \begin{pmatrix} \not{A} & \psi^\alpha \\ \overline{\psi}_\beta & 0 \end{pmatrix}$$

For the $U(1)$ group, we know that under gauge transformations:

$$
\begin{aligned}
A'_\mu &= A_\mu + \partial_\mu \alpha \\
\psi' &= e^{i\alpha}\psi \\
\overline{\psi}' &= e^{-i\alpha}\overline{\psi}
\end{aligned}
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The gauge transformations for $A_\mu, \psi, \overline{\psi}$ can be obtained as

$$\mathbb{A}' = g^{-1}\mathbb{A}g + g^{-1}\not{d}g,$$

being $g = exp[i\mathbb{K}]$ with $i\mathbb{K} = \text{diag}(1, 1, 2)$ and $\not{d} = \text{diag}(\gamma^\mu \partial_\mu, 0)$

$$\mathbb{A}_\mu = A_\mu \mathbb{K} + \overline{\mathbb{Q}}_\beta (\gamma_a)^\beta{}_\alpha e^a{}_\mu \psi^\alpha + \overline{\psi}_\beta (\gamma_a)^\beta{}_\alpha e^a{}_\mu \mathbb{Q}^\alpha + \omega^a_\mu \mathbb{J}_a$$

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with the graded Lie algebra

$$[\mathbb{J}_a, \mathbb{J}_b] = \epsilon^c_{ab} \mathbb{J}_c \qquad \{\overline{\mathbb{Q}}^\alpha, \mathbb{Q}_\beta\} = -\mathbb{J}_a (\gamma^a)^\alpha_\beta - \frac{i}{2} \delta^\alpha_\beta \mathbb{K},$$

$$[\mathbb{J}_a, \mathbb{Q}^\alpha] = \frac{1}{2} (\gamma^a)^\alpha_\beta \mathbb{Q}^\beta \qquad [\mathbb{J}_a, \mathbb{Q}_\alpha] = -\frac{1}{2} (\gamma^a)^\beta_\alpha \mathbb{Q}_\beta,$$

$$[\mathbb{K}, \mathbb{Q}^\alpha] = i \mathbb{Q}^\alpha \qquad [\mathbb{K}, \mathbb{Q}_\alpha] = -i \mathbb{Q}_\alpha$$

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- Using the standard conventions, the Lagrangian reads

$$\begin{aligned} L \;\; = \;\; & 2AdA + \frac{1}{4}[\omega^a{}_b d\omega^b{}_a + \frac{2}{3}\omega^a{}_b\omega^b{}_c\omega^c{}_a] - 2\overline{\psi}\psi e^a T_a \\ & + 2\overline{\psi}(\overleftarrow{\partial\!\!\!/} - \overrightarrow{\partial\!\!\!/} + 2iA\!\!\!/ + \frac{1}{2}\gamma^a \omega\!\!\!/_{ab}\gamma^b)\psi|e|d^3x, \end{aligned}$$

where $|e| = \det[e^a{}_\mu] = \sqrt{-g}$, and $T^a = de^a + \omega^a{}_b e^b$ is the torsion 2-form.

The supersymmetric transformations are deduced by $\delta\mathbb{A} = d\Lambda + [\mathbb{A}, \Lambda]$, with

$$\Lambda = \alpha\mathbb{K} + \overline{\mathbb{Q}}\epsilon - \bar{\epsilon}\mathbb{Q} + \lambda^a\mathbb{J}_a$$

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given the result

$$\delta A = d\alpha - \frac{i}{2}\left(\bar{\epsilon}\not{e}\psi + \overline{\psi}\not{e}\epsilon\right)$$

$$\delta(\not{e}\psi) = \overrightarrow{\nabla}\epsilon + i\alpha\not{e}\psi + \frac{1}{2}\lambda^a\gamma_a\not{e}\psi$$

$$\delta(\overline{\psi}\not{e}) = \epsilon\overleftarrow{\nabla} - i\alpha\overline{\psi}\not{e} - \frac{1}{2}\lambda^a\overline{\psi}\not{e}\gamma_a$$

$$\delta\omega^a = d\lambda^a + \epsilon^a_{bc}\omega^b\lambda^c - \left(\bar{\epsilon}\gamma^a\not{e}\psi + \overline{\psi}\not{e}\gamma^a\epsilon\right)$$

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Connection

$$\mathbb{A} = A\mathbb{K} + \overline{\mathbb{Q}}\not{e}\psi + \overline{\psi}\not{e}\mathbb{Q} + f^a\mathbb{J}_a + \frac{1}{2}\omega^{ab}\mathbb{J}_{ab},$$

for the supercharges

$$\{\overline{\mathbb{Q}}_\alpha, \mathbb{Q}^\beta\} = -\frac{i}{s}(\Gamma^a)^\beta_\alpha\mathbb{J}_a + \frac{i}{2}(\Gamma^{ab})^\beta_\alpha\mathbb{J}_{ab} - \delta^\beta_\alpha\mathbb{K}\,,$$

- In $D = 4$ we do not have the Chern Simons form as in $D = 3$

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- The curvature $\mathbb{F} = d\mathbb{A} + \mathbb{A}\mathbb{A}$ takes the form

$$\mathbb{F} = F_0 \mathbb{K} + \overline{\mathbb{Q}}_\alpha \mathcal{F}^\alpha + \overline{\mathcal{F}}_\alpha \mathbb{Q}^\alpha + F^a \mathbb{J}_a + \frac{1}{2} F^{ab} \mathbb{J}_{ab}$$

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- But this does not give field equations: it is an invariant so when we perform a variation it vanish identically
- We can break the symmetry, introducing some kind of dual $\circledast \mathbb{F}$

$$\circledast \mathbb{F} = *F_0 \mathbb{K} + (\overline{\mathbb{Q}})_\alpha (\Gamma_5 \mathcal{F})^\alpha + (\overline{\mathcal{F}})_\alpha (\Gamma_5 \mathbb{Q})^\alpha + \hat{\Gamma} \left[F^a \mathbb{J}_a + \frac{1}{2} F^{ab} \mathbb{J}_{ab} \right],$$

with

$$\hat{\Gamma} = \left[\begin{array}{cccc|cc} & & & & 0 & 0 \\ & \Gamma_5 & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

With this rules we write the Lagrangian as

$$L = -\frac{1}{4}\langle \mathbb{F} \circledast \mathbb{F} \rangle$$

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A natural way to avoid parity violation is to impose $f^a \wedge e_a = 0$ which means[1]
$f^a = \mu e^a$

with μ a constant of canonical dimension L^{-1}

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Doing the computations we obtain

$$\begin{aligned} L &= -\frac{1}{16}\epsilon_{abcd}(R^{ab} - \mu^2 e^a e^b)(R^{cd} - \mu^2 e^c e^d) \\ &\quad + \frac{1}{2}\left[\overline{\psi}(\overleftarrow{\nabla\!\!\!/} - \overrightarrow{\nabla\!\!\!/})\psi + 4\mu i \overline{\psi}\psi\right] + t^\mu \overline{\psi}\Gamma_5\Gamma_\mu\psi \\ &\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}|e|d^4x - \frac{1}{3\mu^2}\left[(\overline{\psi}\psi)^2 - (\overline{\psi}\Gamma_5\psi)^2\right]. \end{aligned}$$

with
$t^\mu \equiv -\frac{1}{3!}\varepsilon^{\mu\nu\rho\tau} e^a_\nu T_{a\rho\tau}|e|$, and

$$\begin{aligned} \overrightarrow{\nabla\!\!\!/}\psi &\equiv (\partial\!\!\!/ - iA\!\!\!/ + \frac{1}{2}\psi\!\!\!/)\psi \\ \overline{\psi}\overleftarrow{\nabla\!\!\!/} &\equiv \overline{\psi}(\overleftarrow{\partial\!\!\!/} + iA\!\!\!/ - \frac{1}{2}\psi\!\!\!/) \end{aligned}$$

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- Can we construct the Lagrangian in $D = 4$ in a more subtle way?
  Starting with a CS form in $D = 5$?

Thanks!

SUSY algebra $D = 4$

$$[\mathbb{J}_a, \mathbb{J}_b] = s^2 \mathbb{J}_{ab}\,, \quad [\mathbb{J}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{J}_c - \eta_{ac}\mathbb{J}_b\,,$$
$$[\mathbb{J}_{ab}, \mathbb{J}_{cd}] = \eta_{ad}\mathbb{J}_{bc} - \eta_{ac}\mathbb{J}_{bd} + \eta_{bc}\mathbb{J}_{ad} - \eta_{bd}\mathbb{J}_{ac}\,,$$

$$[\mathbb{J}_a, \mathbb{Q}^\alpha] = -\tfrac{s}{2}(\Gamma_a)^\alpha{}_\beta \mathbb{Q}^\beta, \quad [\mathbb{J}_a, \overline{\mathbb{Q}}_\alpha] = \tfrac{s}{2}\overline{\mathbb{Q}}_\beta (\Gamma_a)^\beta{}_\alpha,$$
$$[\mathbb{J}_{ab}, \mathbb{Q}^\alpha] = -\tfrac{1}{2}(\Gamma_{ab})^\alpha{}_\beta \mathbb{Q}^\beta, \quad [\mathbb{J}_{ab}, \overline{\mathbb{Q}}_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}_\beta (\Gamma_{ab})^\beta{}_\alpha.$$

$$[\mathbb{K}, \mathbb{Q}^\alpha] = i\mathbb{Q}^\alpha\,, \qquad [\mathbb{K}, \overline{\mathbb{Q}}_\alpha] = -i\overline{\mathbb{Q}}_\alpha\,.$$

$$\{\mathbb{Q}^\alpha, \overline{\mathbb{Q}}_\beta\} = -\frac{i}{s}(\Gamma^a)^\alpha_\beta \mathbb{J}_a + \frac{i}{2}(\Gamma^{ab})^\alpha_\beta \mathbb{J}_{ab} - \delta^\alpha_\beta \mathbb{K}.$$

Explicit Form of the Generators $D = 4$

$$(\mathbb{Q}^{\alpha})^{A}_{B} = -i \left[\begin{array}{c|cc} 0_{4\times 4} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} & C^{\alpha A} \\ \hline \delta^{\alpha}_{B} & 0 & 0 \\ 0\;0\;0\;0 & 0 & 0 \end{array} \right] = -i(\delta^{A}_{5}\delta^{\alpha}_{B} + C^{\alpha A}\delta^{6}_{B}) (1)$$

$$(\overline{\mathbb{Q}}_{\alpha})^{A}_{B} = \left[\begin{array}{c|cc} 0_{4\times 4} & \delta^{A}_{\alpha} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0\;0\;0\;0 & 0 & 0 \\ C_{\alpha B} & 0 & 0 \end{array} \right] = \delta^{A}_{\alpha}\delta^{5}_{B} + \delta^{A}_{6} C_{\alpha B} \,, \qquad (2)$$

Explicit Form of the Generators $D = 4$

$$(\mathbb{J}_a)^A{}_B = \left[\begin{array}{cccc|cc} & & & & 0 & 0 \\ & \frac{s}{2}\Gamma_a & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] = \frac{s}{2}(\Gamma_a)^\alpha{}_\beta \delta^A_\alpha \delta^\beta_B \quad (3)$$

$$(\mathbb{J}_{ab})^A{}_B = \left[\begin{array}{cccc|cc} & & & & 0 & 0 \\ & \frac{1}{2}\Gamma_{ab} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] = \frac{1}{2}(\Gamma_{ab})^\alpha{}_\beta \delta^A_\alpha \delta^\beta_B \quad (4)$$

SUSY Transformations $D = 4$

$$(\mathbb{K})^{A}{}_{B} = \left[\begin{array}{cccc|cc} & & & & 0 & 0 \\ & 0_{4\times 4} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \end{array}\right] = i(\delta^{A}_{5}\delta^{5}_{B} - \delta^{A}_{6}\delta^{6}_{B}), \qquad (5)$$

SUSY Transformations $D = 4$

The variation of $\mathbb{A}$ under supersymmetry generated by $\Lambda = \overline{\mathbb{Q}}\epsilon - \bar{\epsilon}\mathbb{Q}$ is

$$\delta\mathbb{A} = d\Lambda + [\mathbb{A}, \Lambda].$$

Using the (anti-) commuting relations of the superalgebra, one finds

$$\delta A_\mu = -\left(\bar{\epsilon}\Gamma_\mu\psi + \overline{\psi}\Gamma_\mu\epsilon\right) \tag{6}$$

$$\delta f^a = -\frac{i}{s}\left(\bar{\epsilon}\Gamma^a \not{e}\psi + \overline{\psi}\not{e}\Gamma^a\epsilon\right) \tag{7}$$

$$\delta\omega^{ab} = i\left(\bar{\epsilon}\Gamma^{ab}\not{e}\psi + \overline{\psi}\not{e}\Gamma^{ab}\epsilon\right) \tag{8}$$

$$\delta\left[\Gamma_c e^c\psi\right] = \left[d - iA + \frac{1}{2}sf^a\Gamma_a + \frac{1}{4}\omega^{ab}\Gamma_{ab}\right]\epsilon\,. \tag{9}$$

Relations Between Constants in $D = 3$ and $D = 4$

In three dimensions,

Electric charge	$e = 1$
Newton's constant	$G = 1$
Cosmological constant	$\Lambda = -\mu^2$
Fermion mass	$m = \mu$

And in four dimensions,

Electric charge	$e = 1$
Newton's constant	$G = -s^2(4\pi\mu^2)^{-1}$
Cosmological constant	$\Lambda = -s^2\mu^2$
Nambu–Jona-Lasinio coupling	$g = (3\mu)^{-2}$