Generalizing the geometry of space-time:
from gravity to supergravity
and beyond...?

Mariana Graña
CEA/Saclay

Aldazabal, Berman, Cederwall, M.G., Coimbra, Hohm, Jeon, Kleinschmidt, Lee, Marques,
Park, Perry, Rosabal, Samtleben, Strickland-Constable, Thomson, Waldram, West, Zweibach, ... 2011-2013

Based on Hitchin's generalized geometry 2001

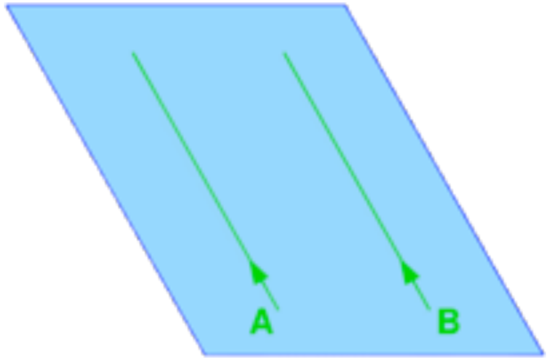
Hull and coll double field theory 2006

SUSY 2013, Trieste

Gravity by Einstein

Gravity by Einstein

- Classical mechanics



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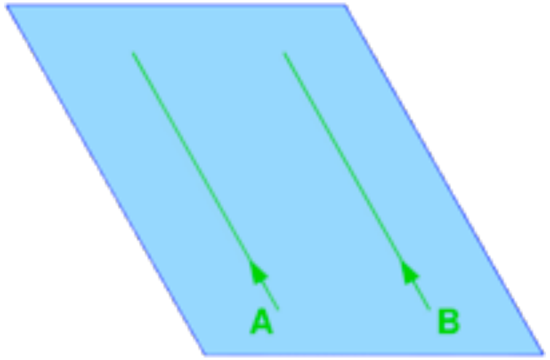

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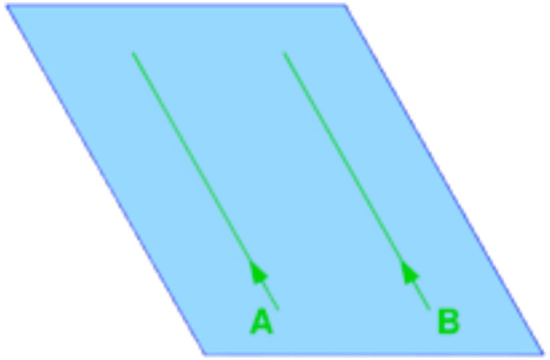

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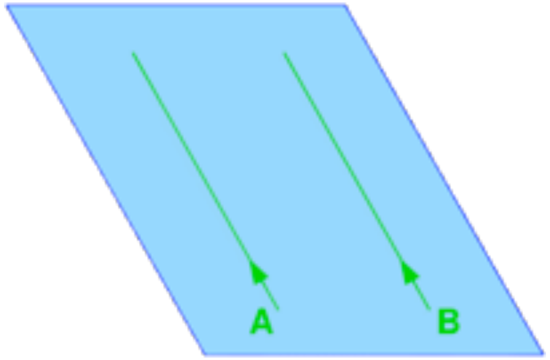

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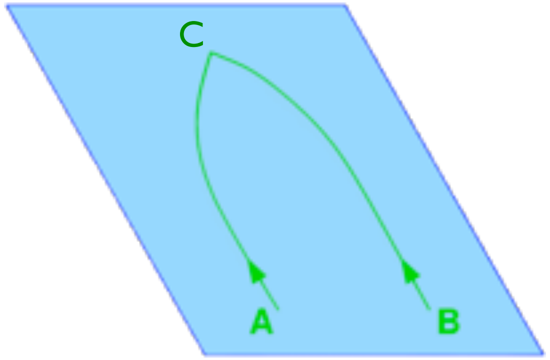

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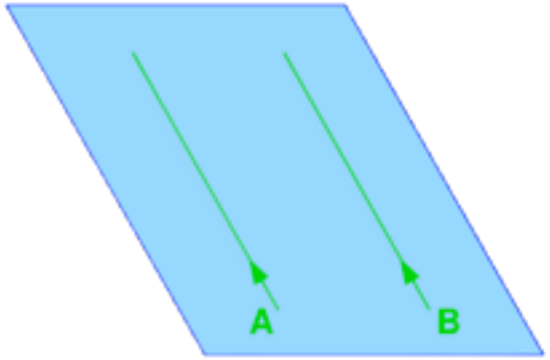

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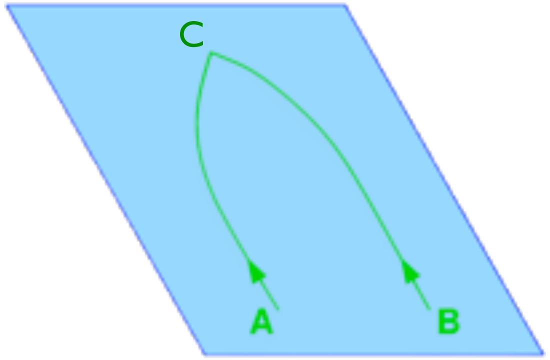

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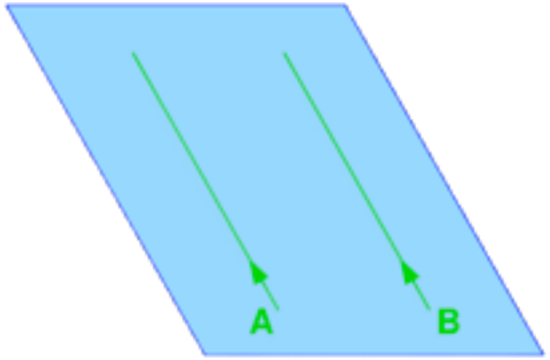

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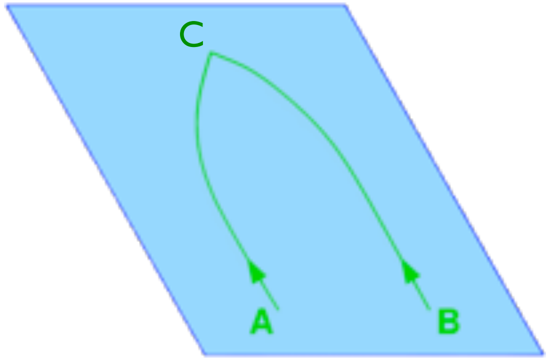

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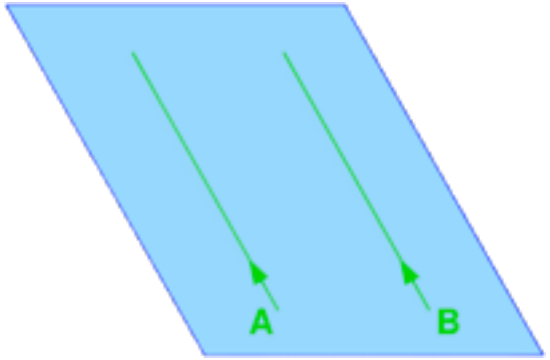

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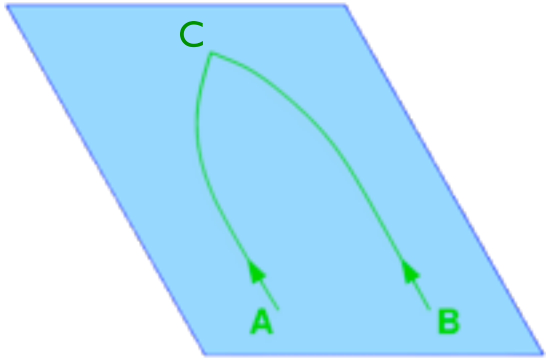

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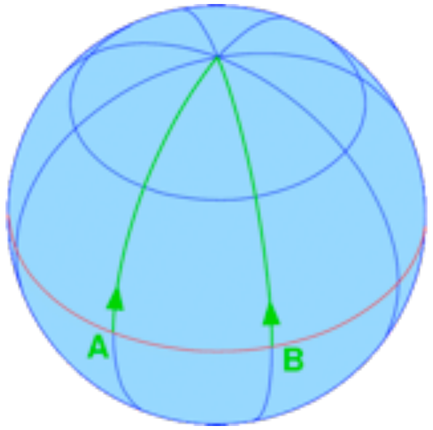

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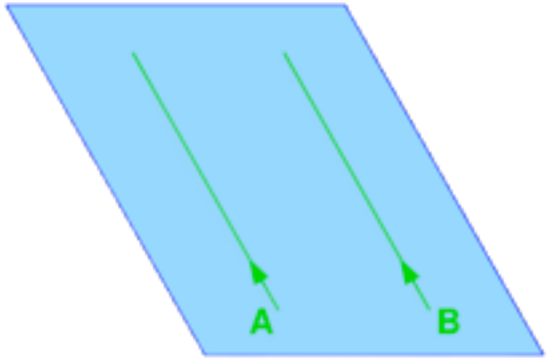

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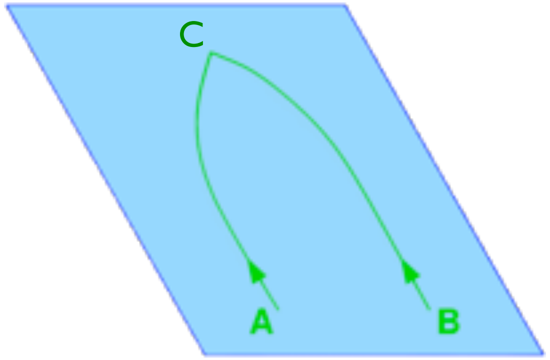

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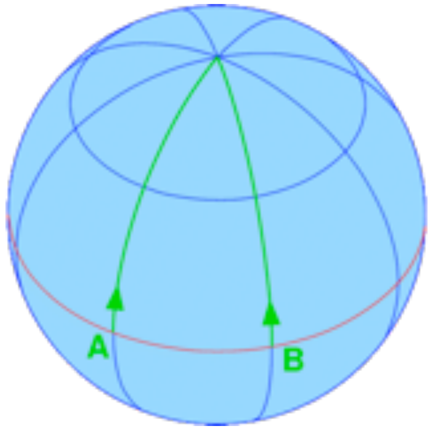

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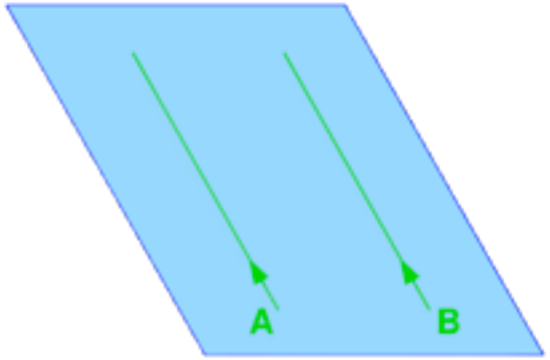

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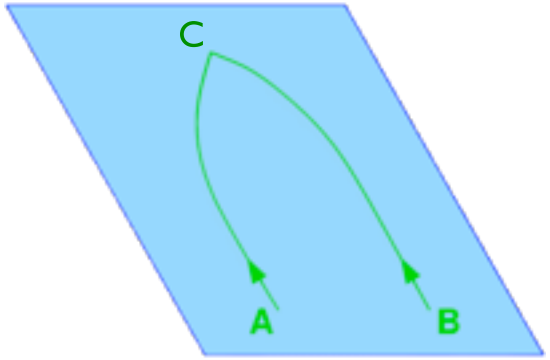

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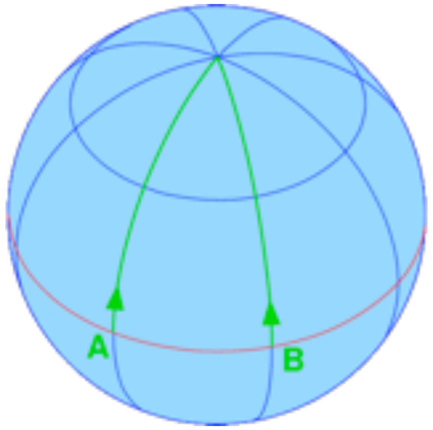

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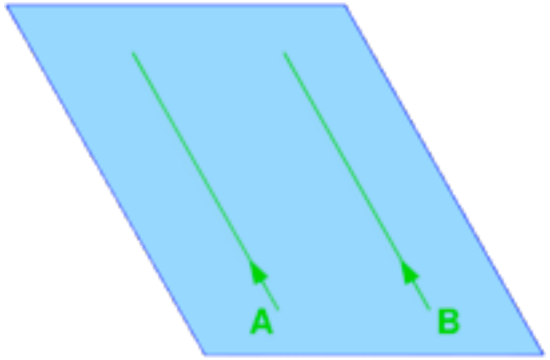

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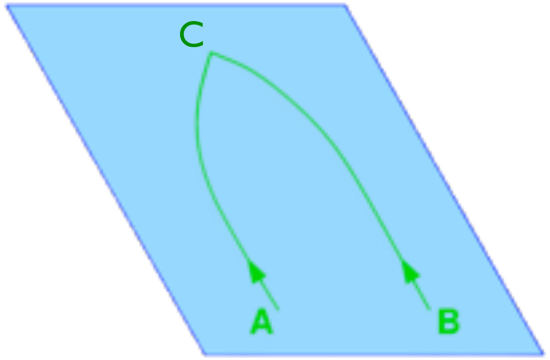

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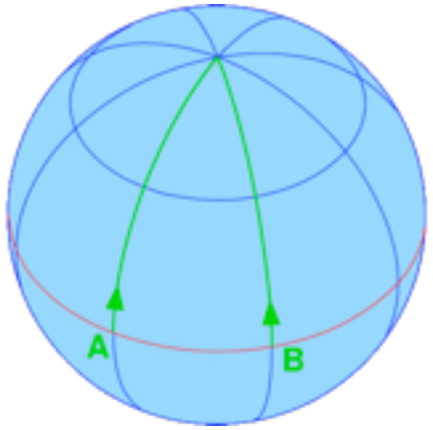

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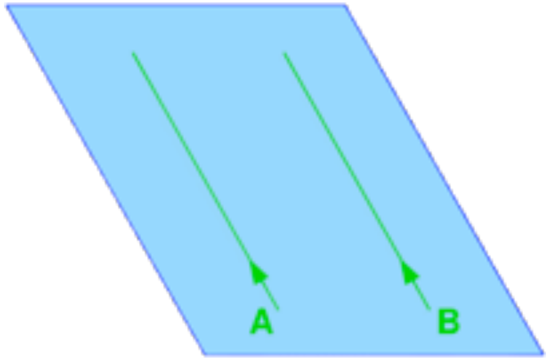

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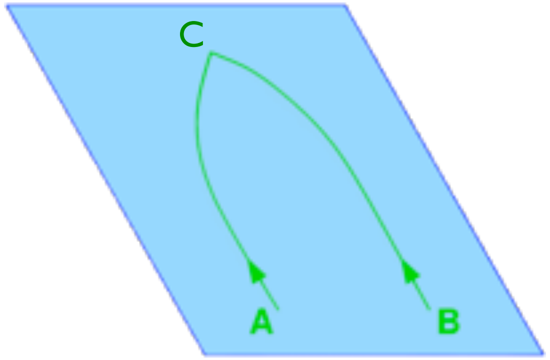

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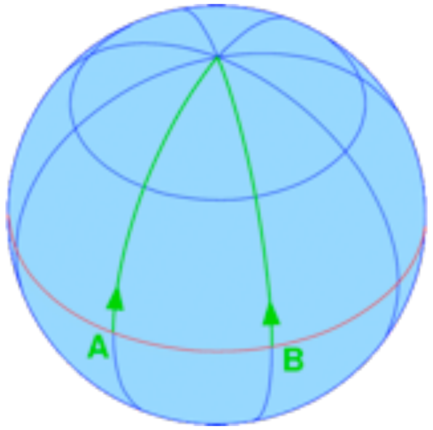

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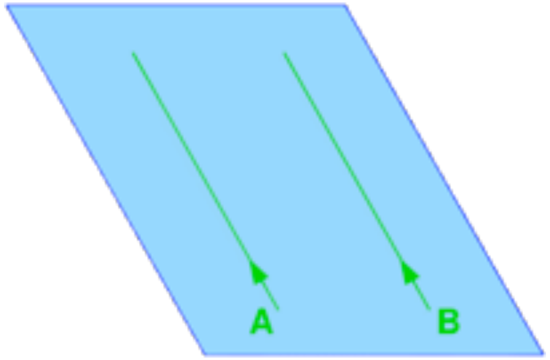

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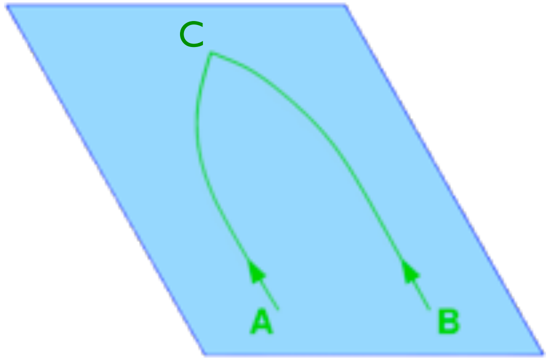

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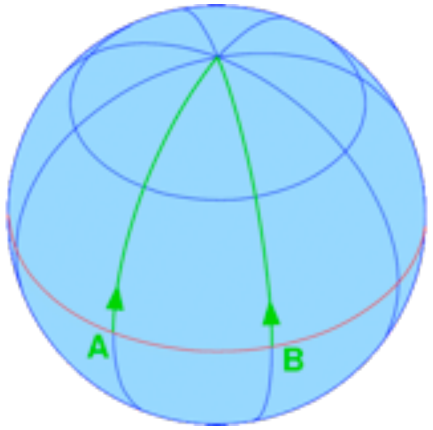

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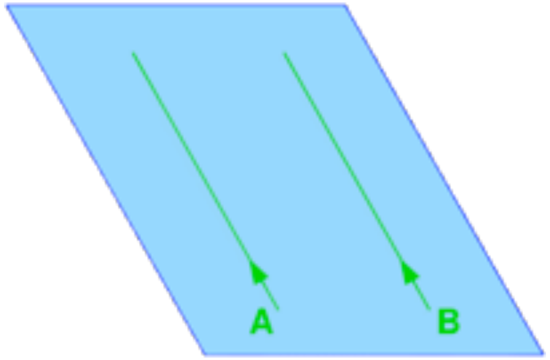

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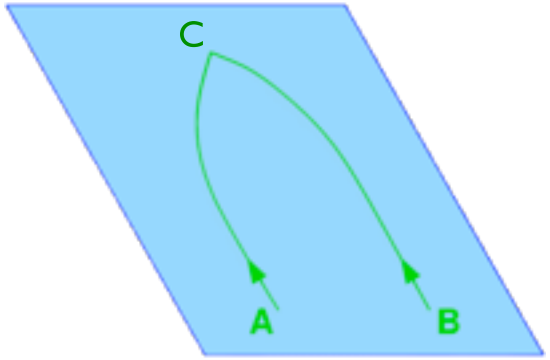

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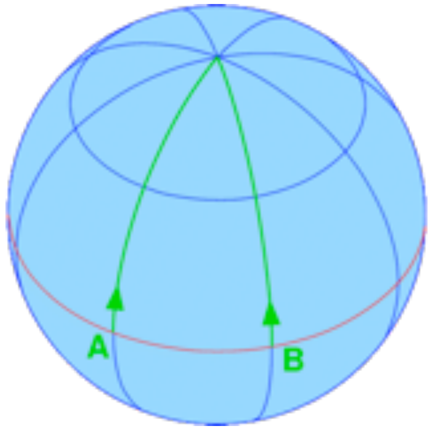

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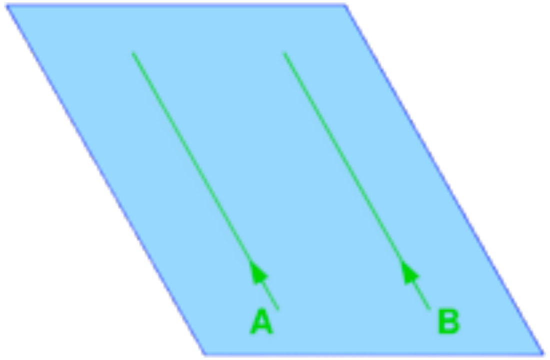

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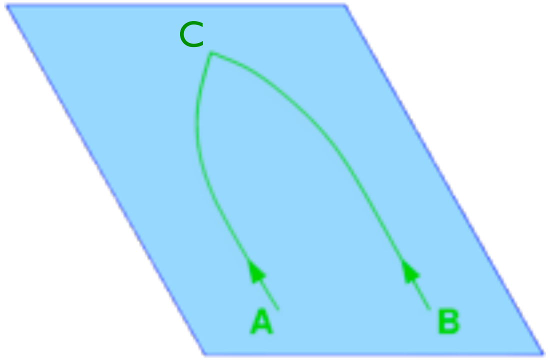

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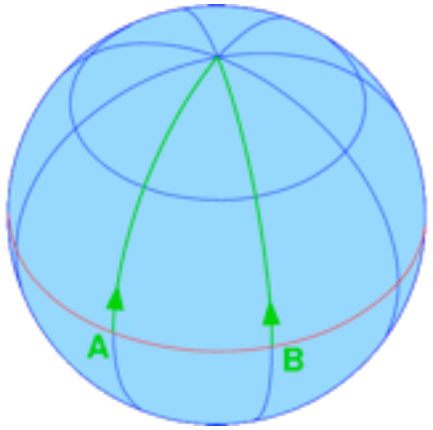

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energy-momentum tensor ↓

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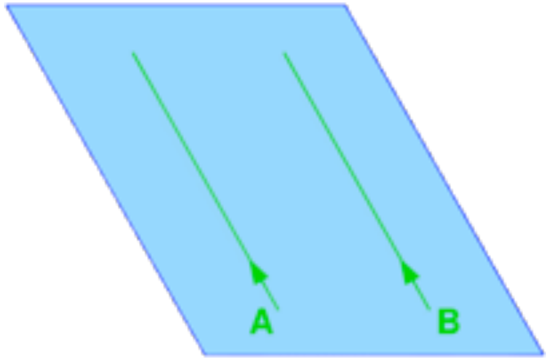

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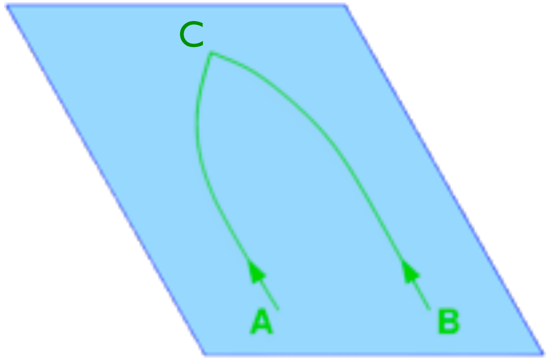

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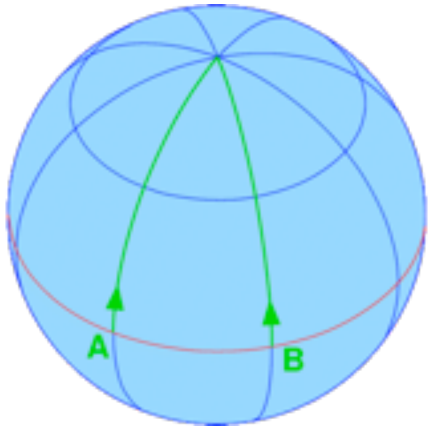

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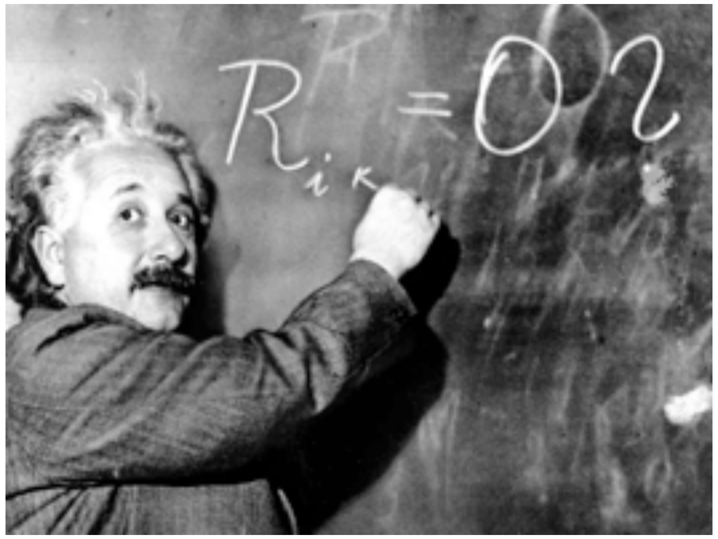
Gravity = Geometry



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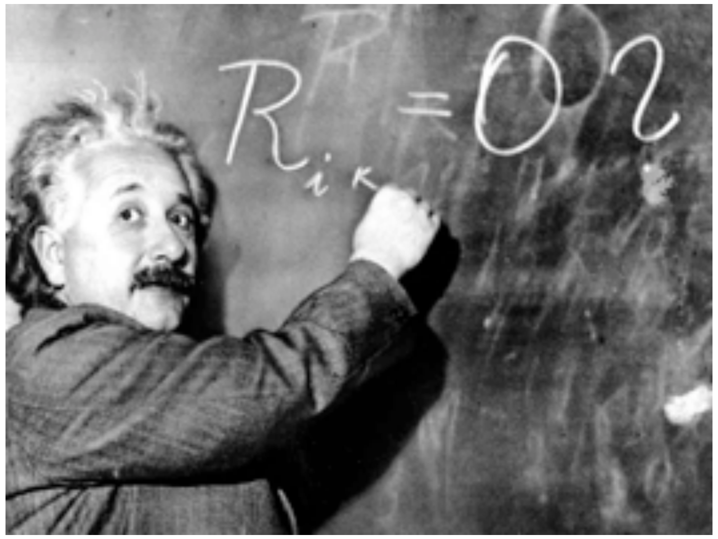
gravity geometry



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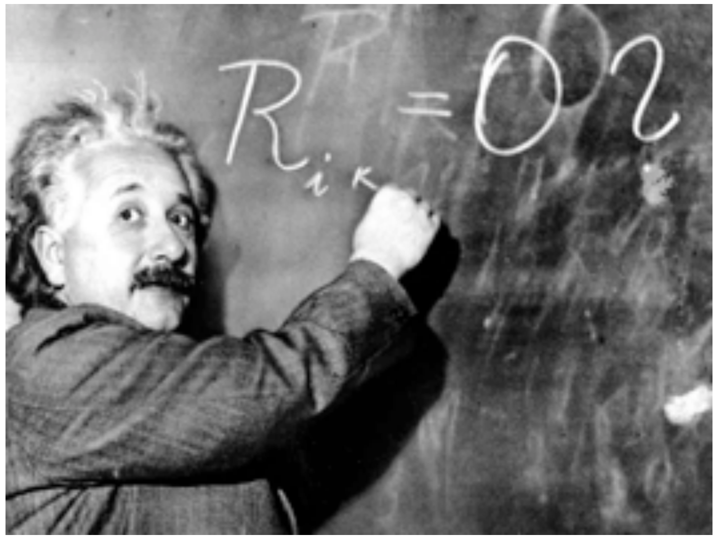

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super
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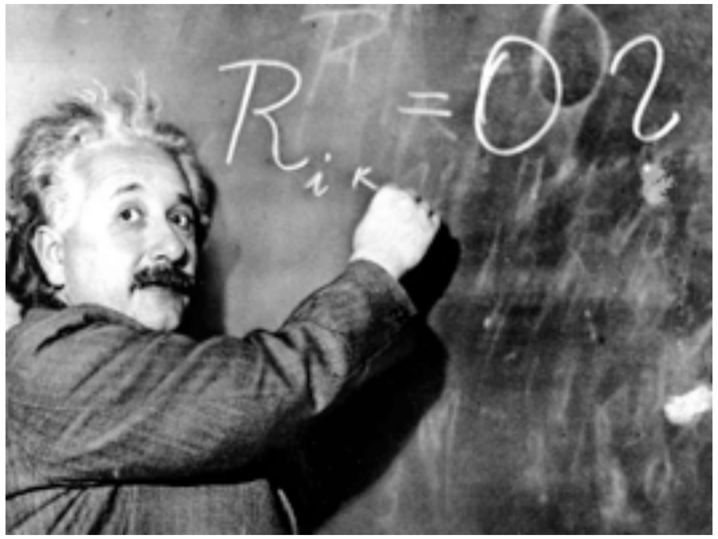

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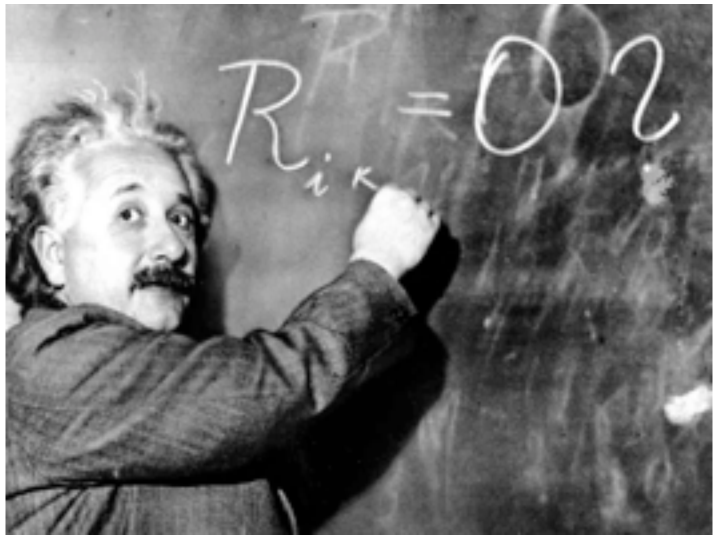

gravity = geometry

↓ SUSY

super gravity = generalized geometry

via superstring theory

Outline

- Gravity as geometry
- Supergravity and superstring theory
- Generalized geometry
- Supergravity as generalized geometry
- Beyond supergravity and generalized geometry?
- Conclusions

Gravity : general relativity

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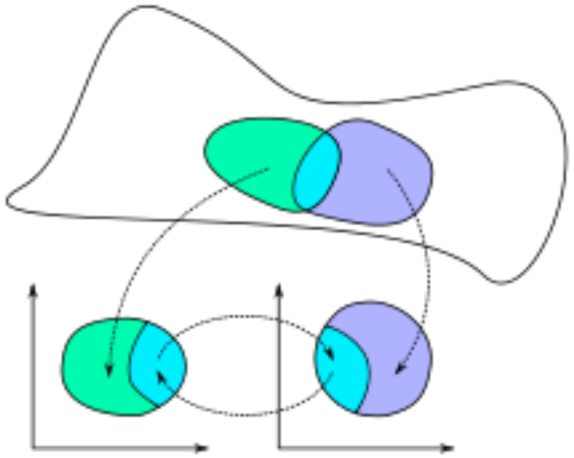
- Manifold M



space that looks
locally like $\mathbb{R}^n$

Gravity : general relativity

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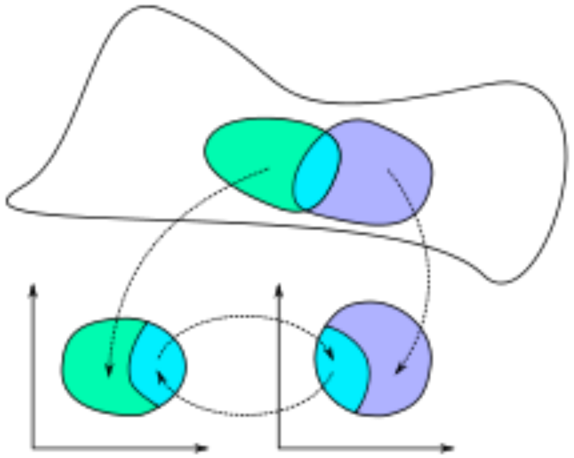

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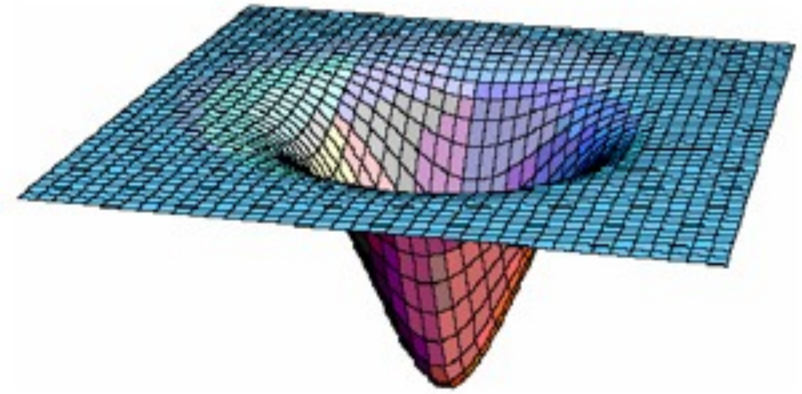

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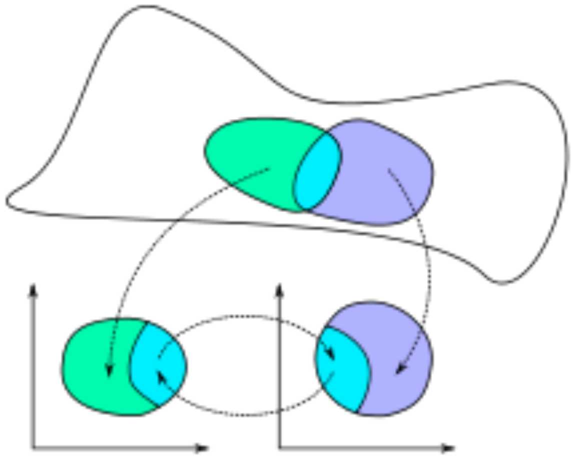

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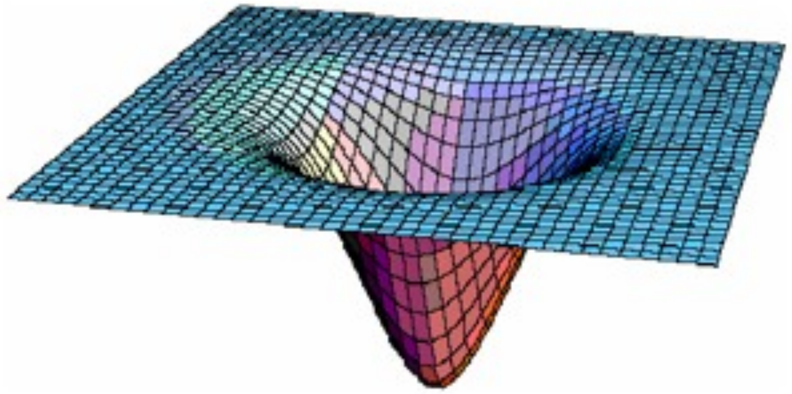

- Metric g

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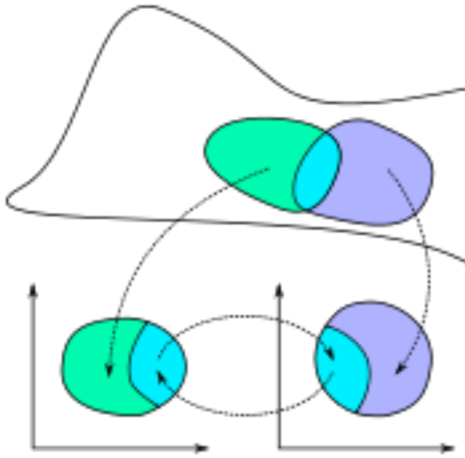

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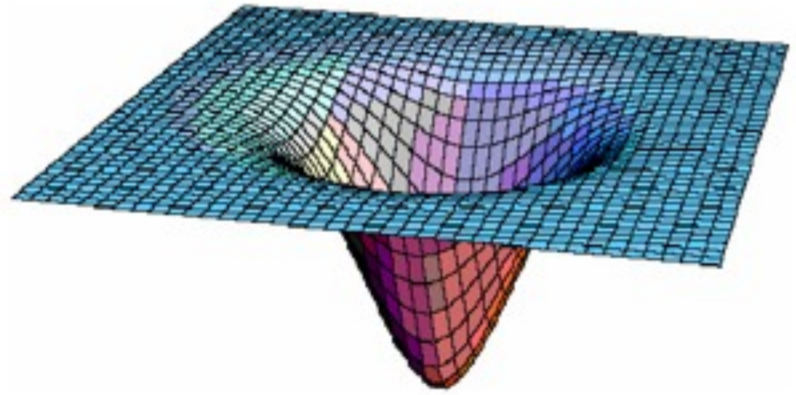

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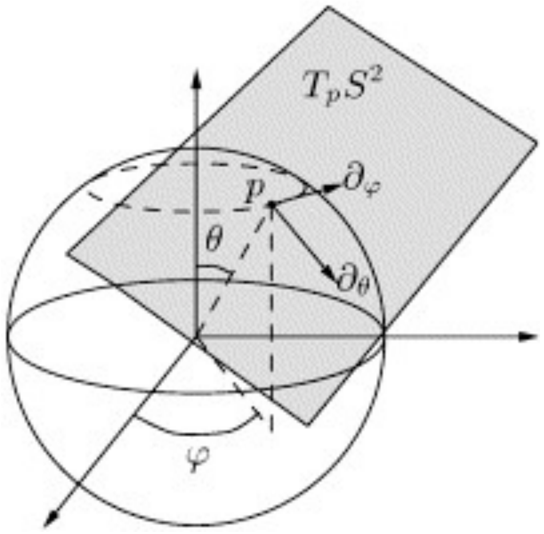

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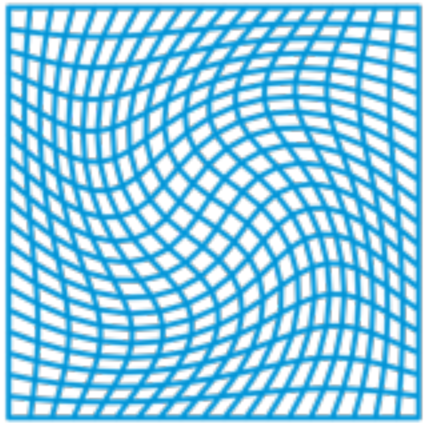

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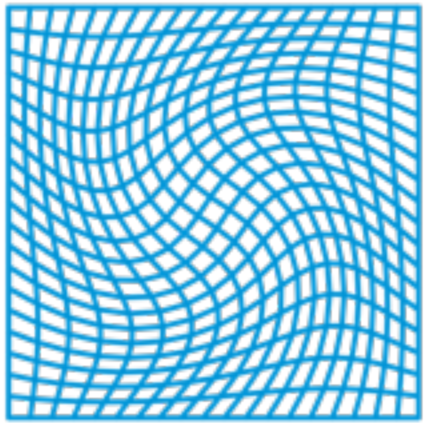

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infinitesimal generators: vector fields $v^\mu \frac{\partial}{\partial x^\mu}$

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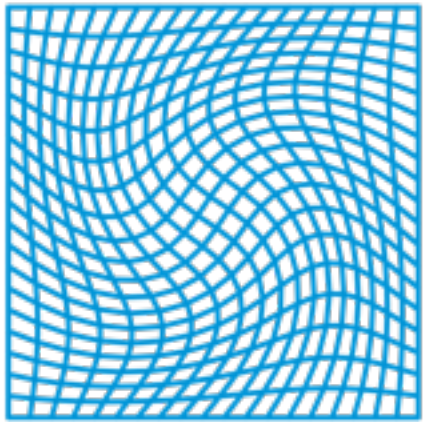

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Lie derivative

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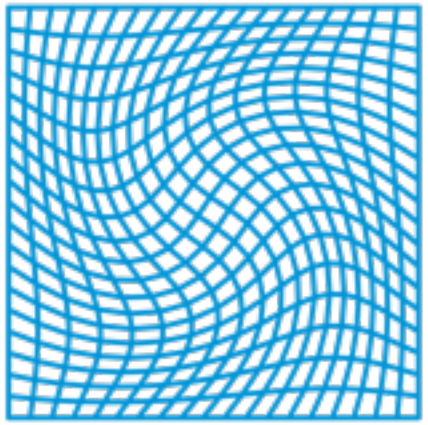

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Algebra $[\mathcal{L}_v, \mathcal{L}_w] = \mathcal{L}_{[v,w]}$

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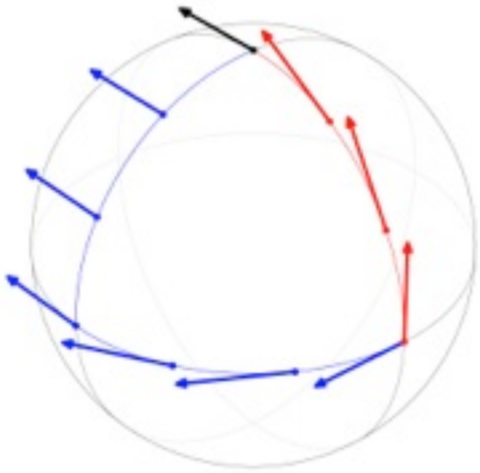

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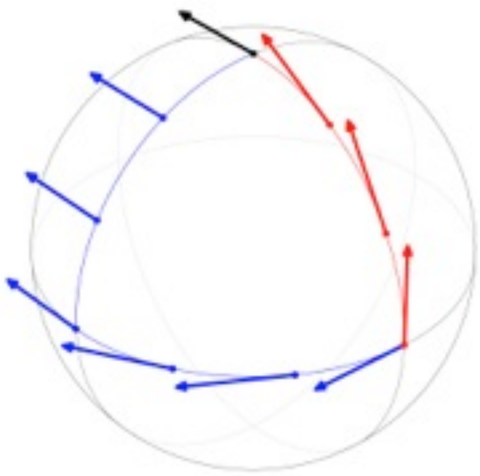

Parallel transport $\nabla_\mu v^\rho = 0$

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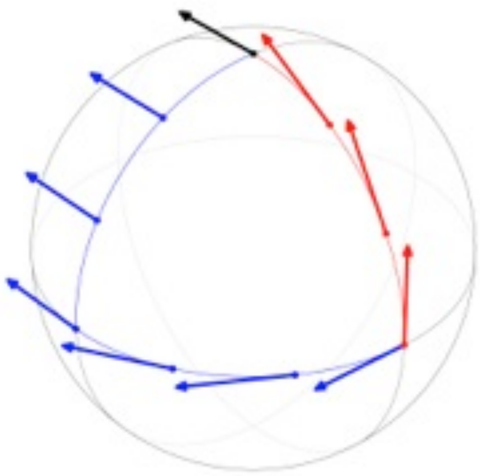

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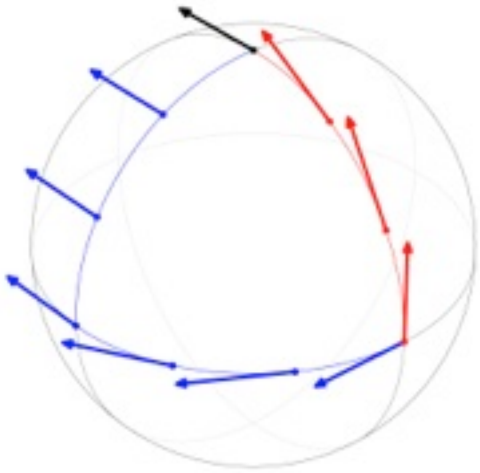

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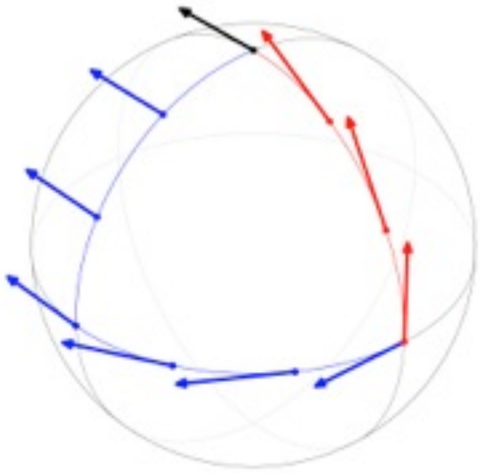

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metric compatible $\nabla g = 0$
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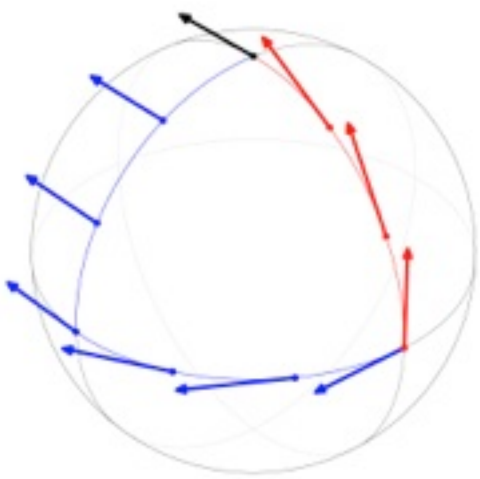

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- Curvature and torsion

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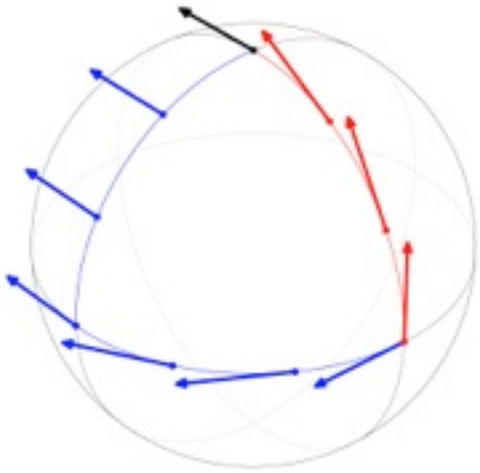
Riemann Torsion

- Connection ∇



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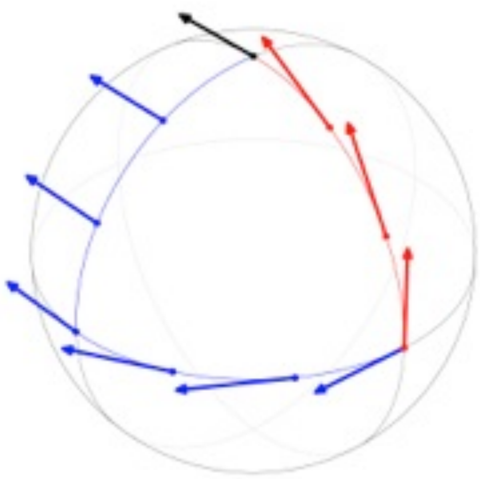

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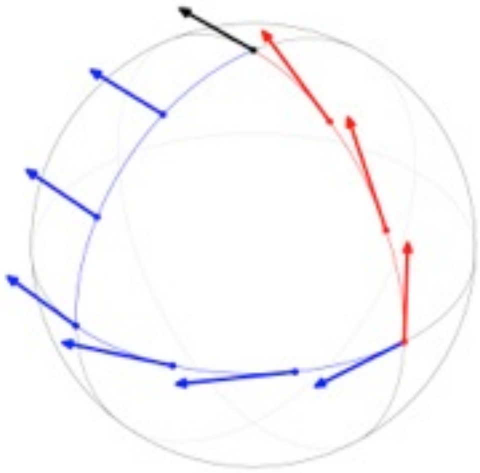

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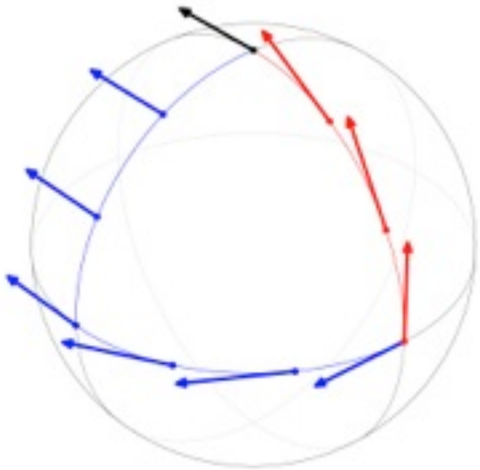
•Connection ∇



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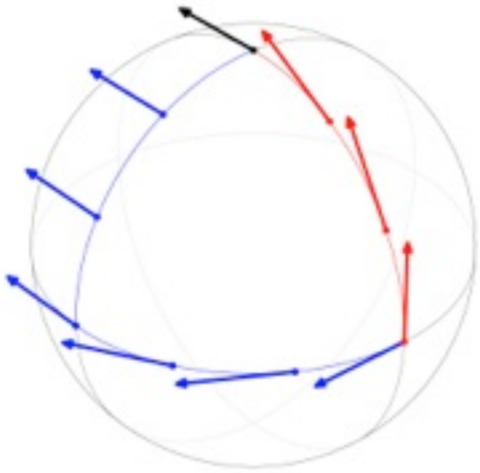

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$R_{\mu\lambda} = R_{\mu\nu\lambda}{}^\nu$ Ricci tensor

$R = R_\mu{}^\mu$ Ricci scalar

- Action

- Action

$$S = \int \sqrt{-g}\, R$$

- Action

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- Equations of motion : Einstein equations

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No matter $\quad R_{\mu\nu} = 0$

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No matter $\qquad R_{\mu\nu} = 0 \qquad$ Ricci-flat

Supergravity

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- Extension of Einstein gravity that incorporates SUSY

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- Strongly constrained and intricate theory : unique in D=11 (max. dimension)

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$$g \ , B \ , \Phi$$

metric dilaton

2-form $B_{\mu\nu}$

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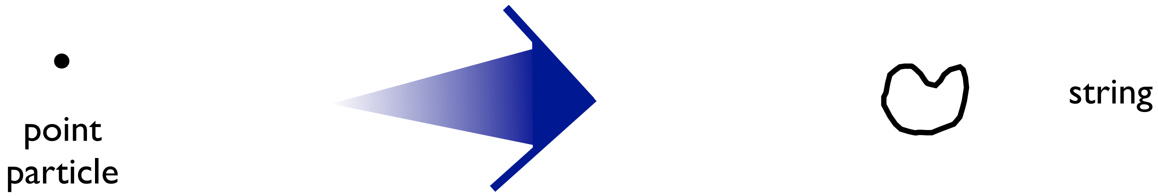
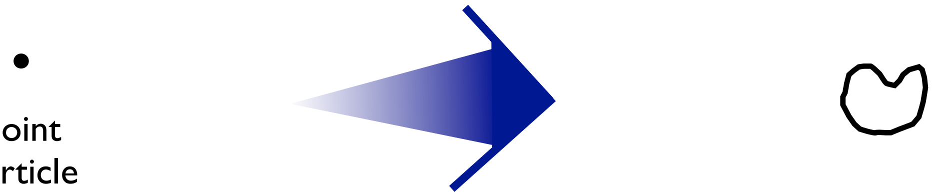
- Low energy limit of superstring theory

Superstring theory

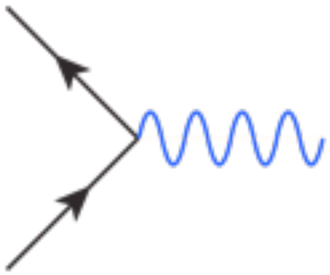


Superstring theory



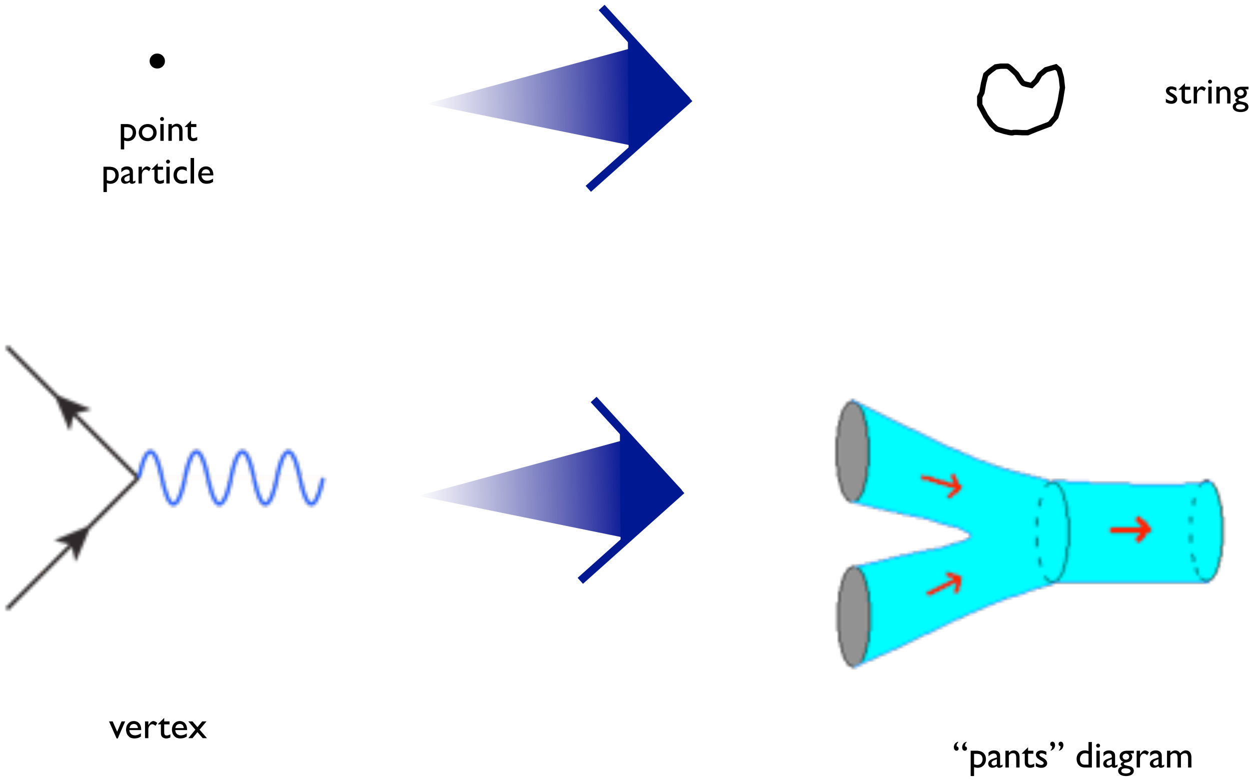
point
particle

string



vertex

Superstring theory



point particle

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“pants” diagram

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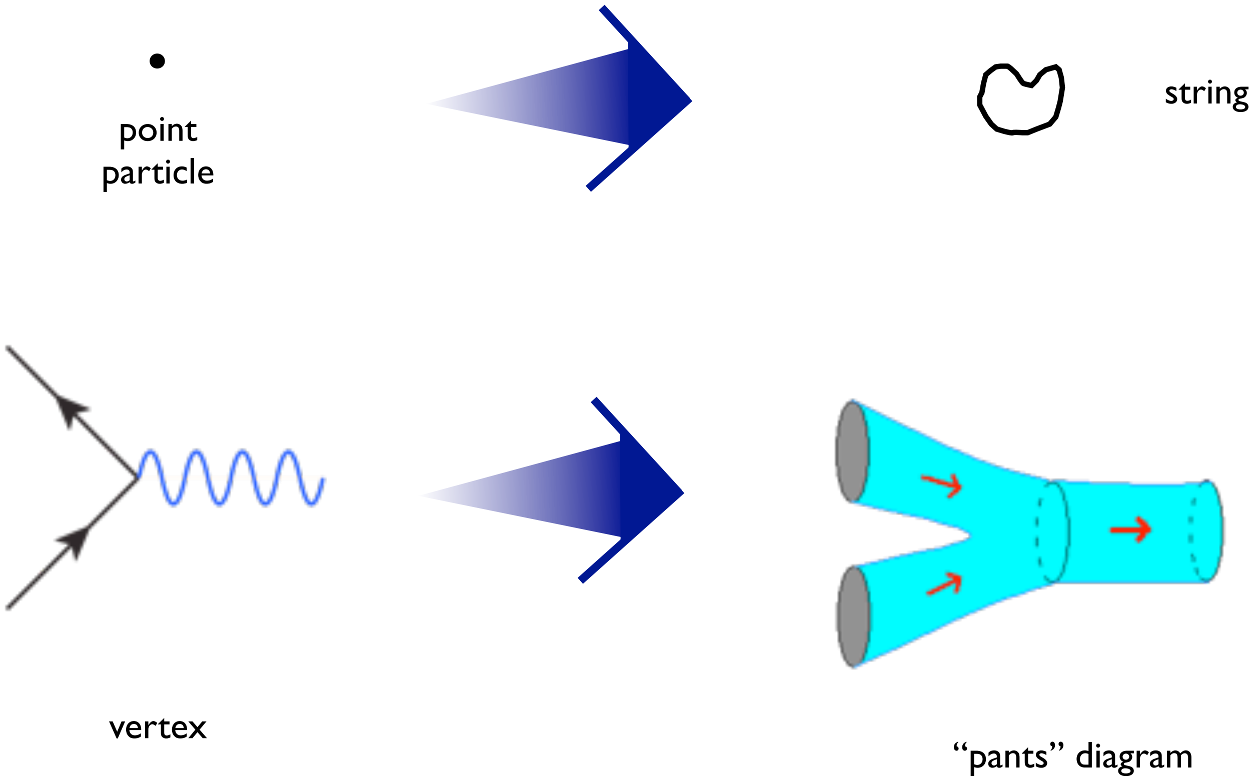
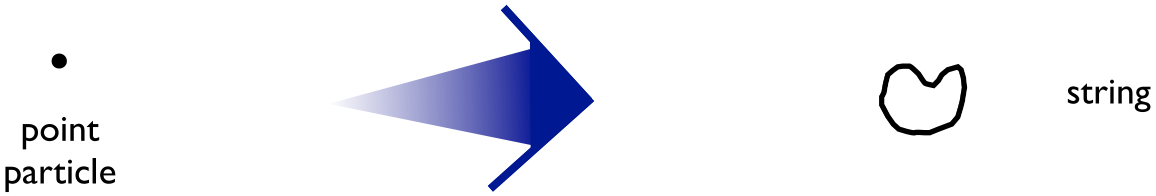
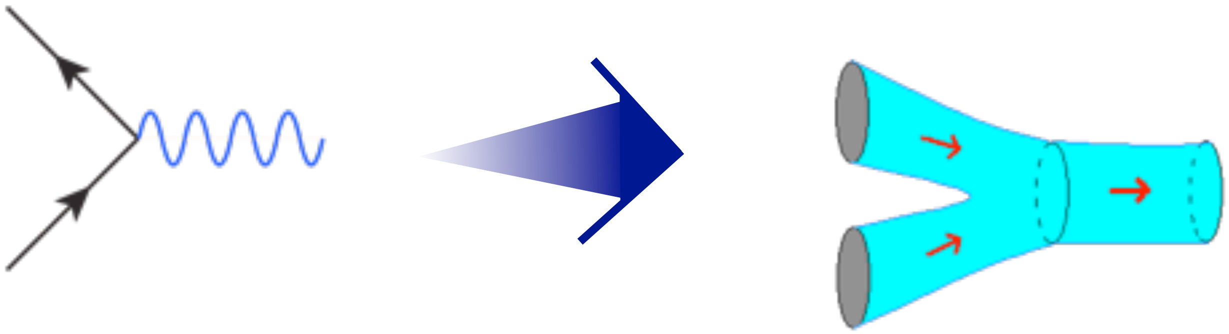


singularities get resolved

Superstring theory





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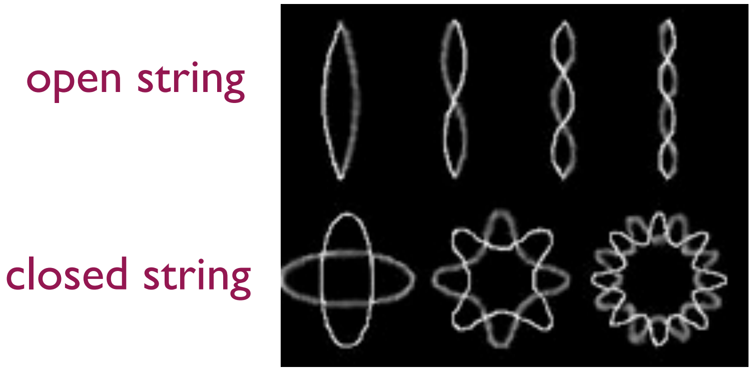
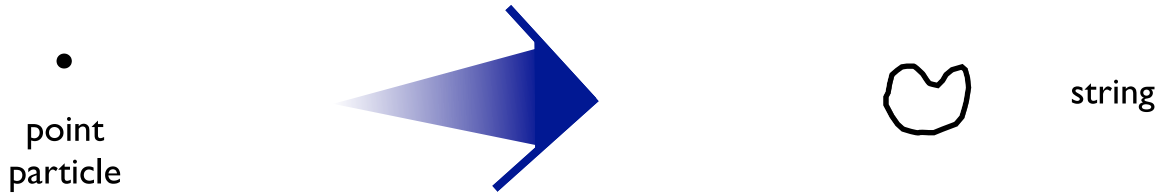
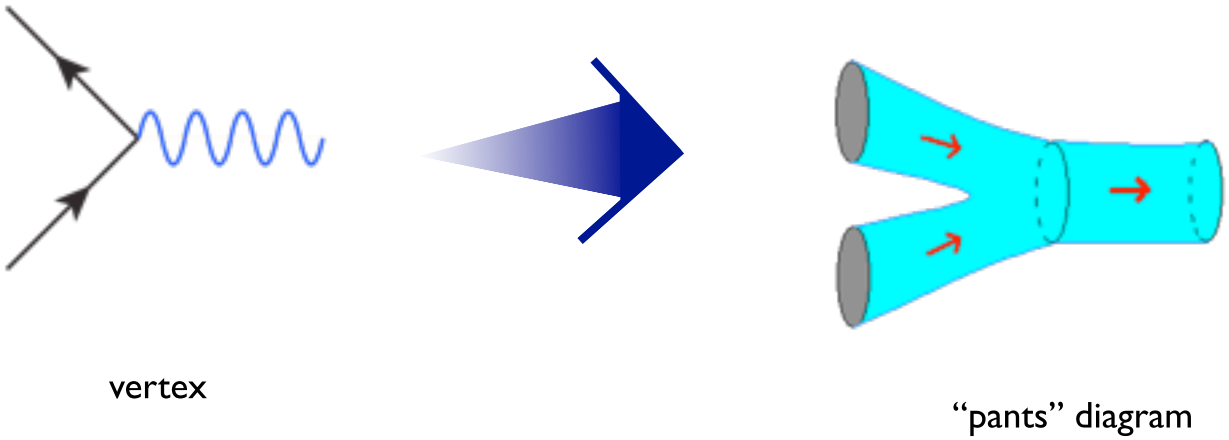


Oscillation modes:
different particles

Superstring theory





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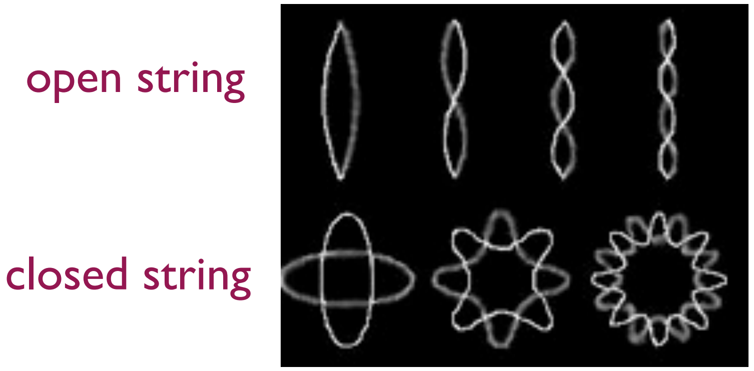

Oscillation modes:
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Closed string massless excitation modes:

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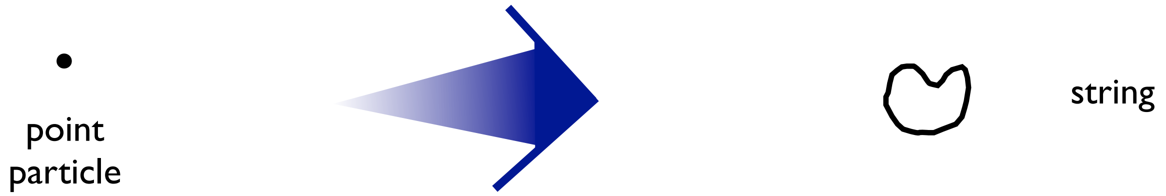
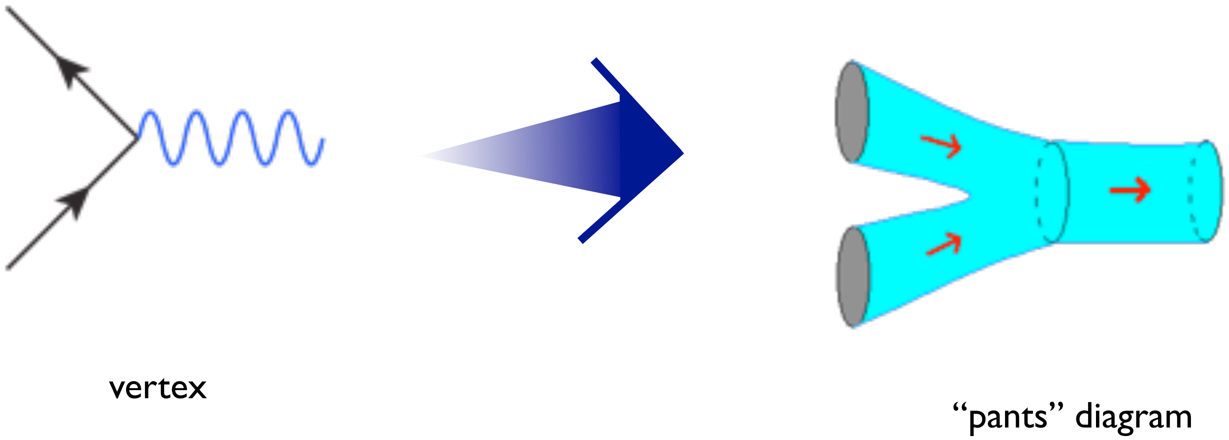
Superstring theory





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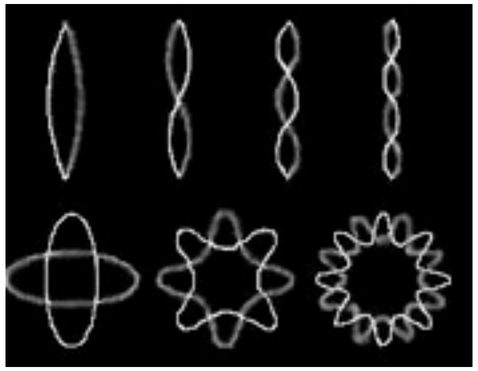

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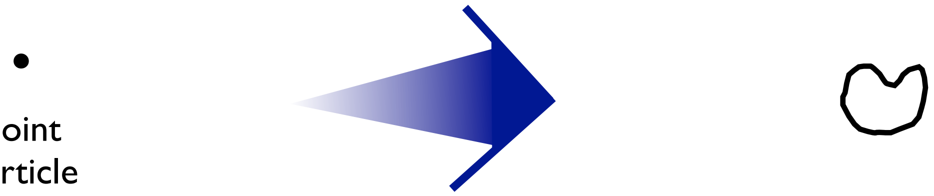

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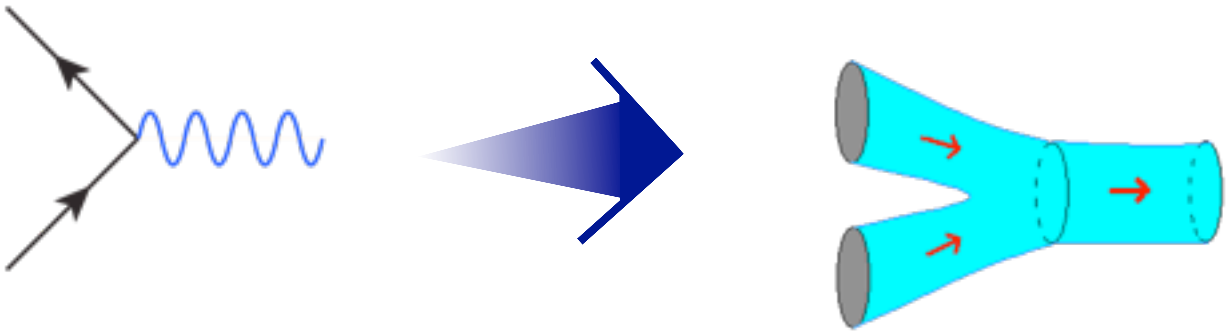


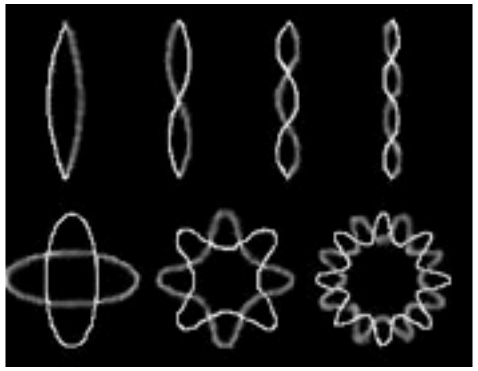
vertex

“pants” diagram

singularities get resolved

open string

closed string



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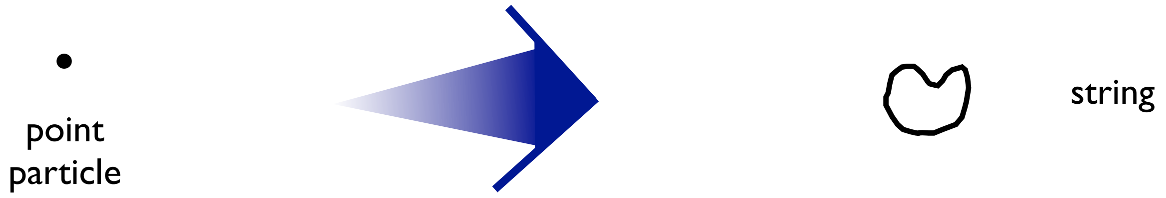
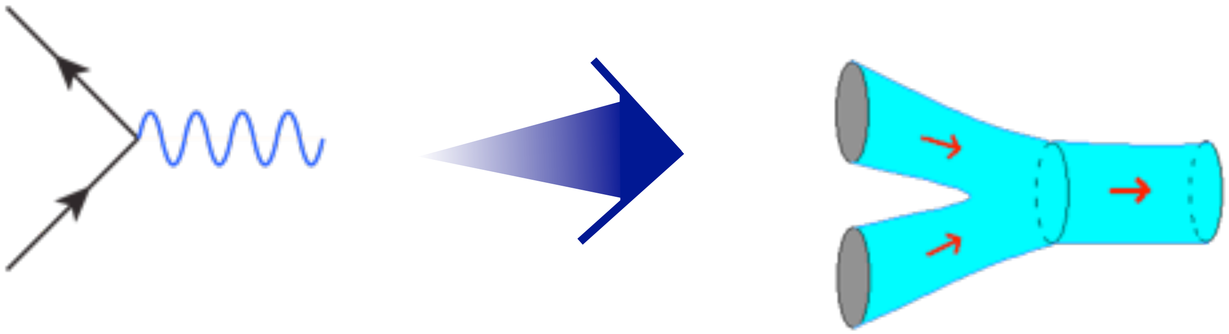
Superstring theory





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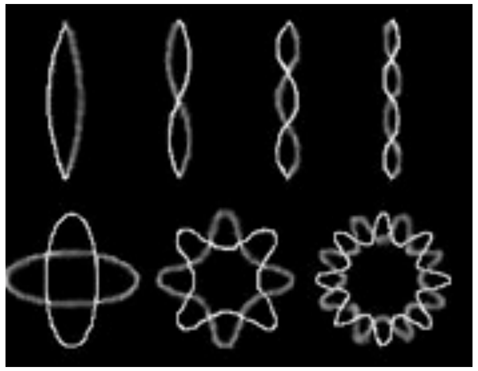

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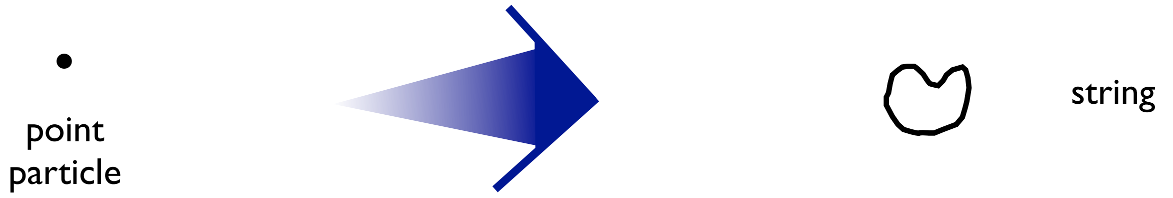
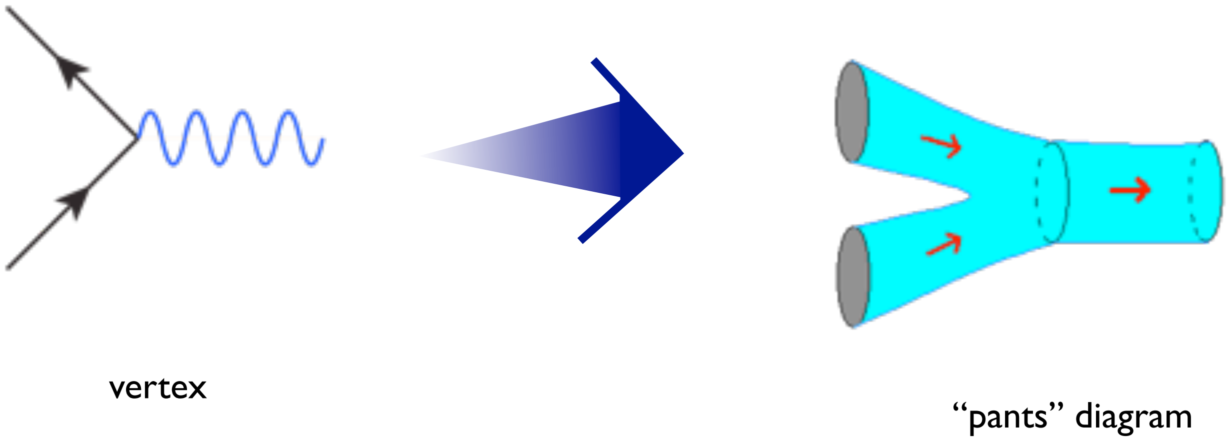
$\xi_{(\mu\nu)}$

Superstring theory





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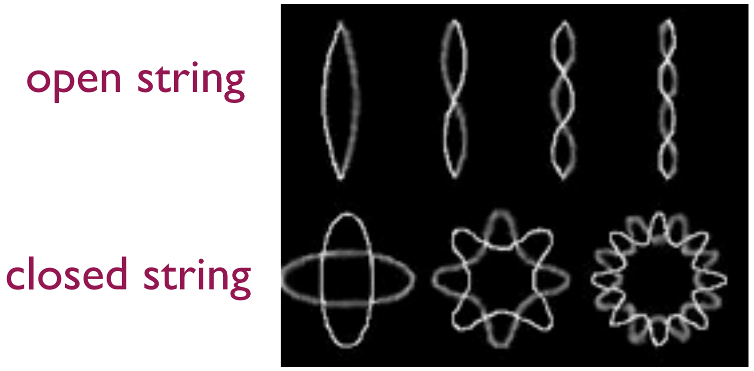

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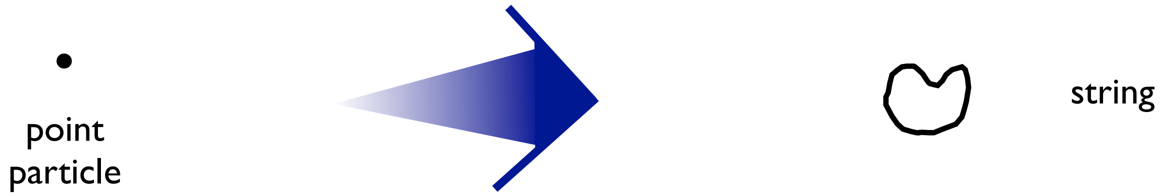
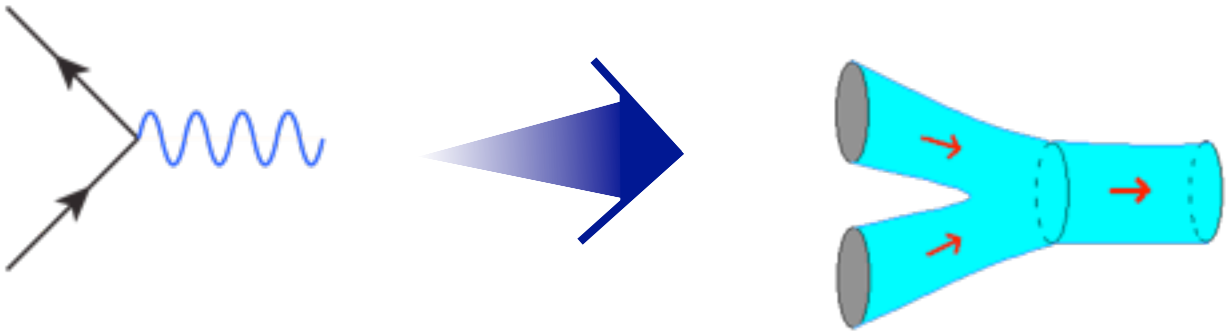
$\xi_{(\mu\nu)}$ $\xi_{[\mu\nu]}$

Superstring theory

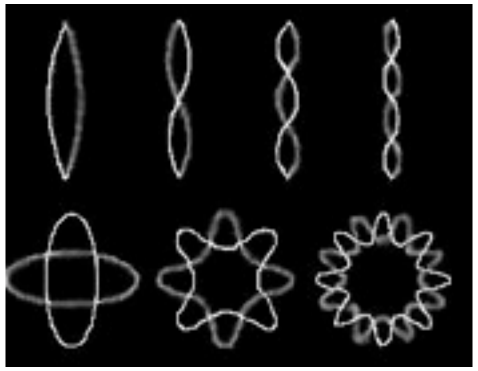




singularities
get resolved

open string

closed string



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different particles

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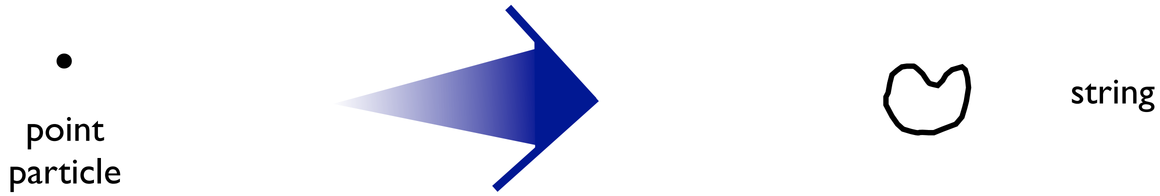
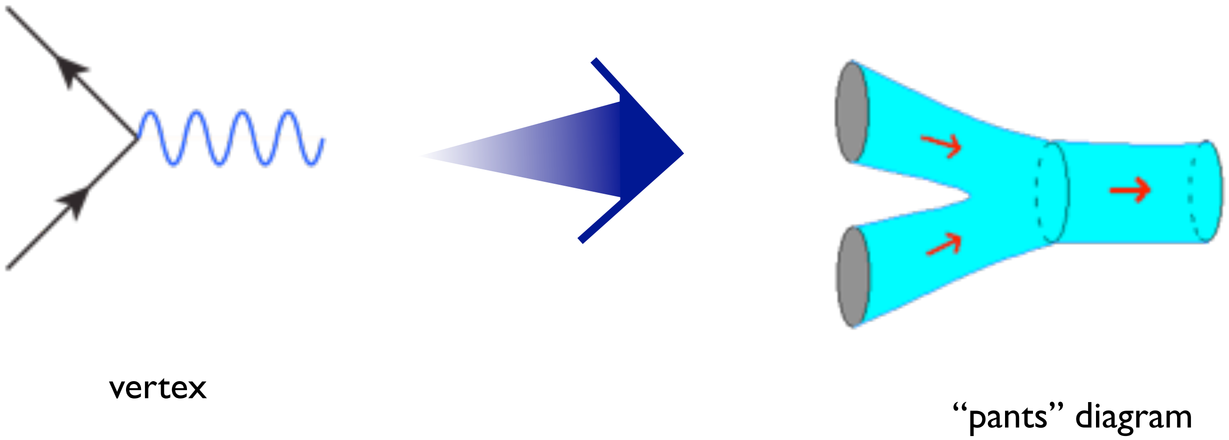
left-moving
osc. mode

right-moving
osc. mode

$\xi_{(\mu\nu)}$ $\xi_{[\mu\nu]}$ $\delta^{\mu\nu}\xi_{\mu\nu}$

Superstring theory





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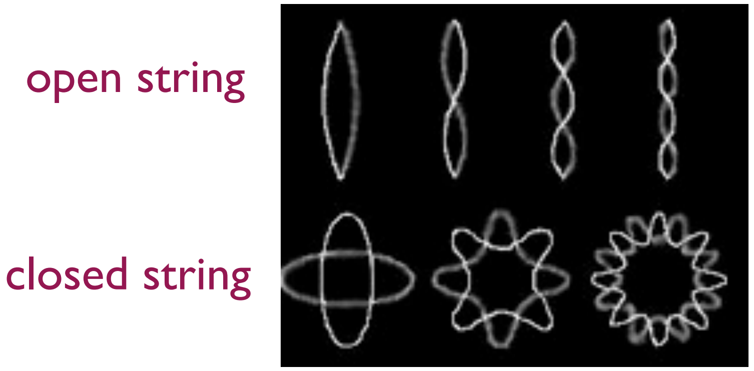

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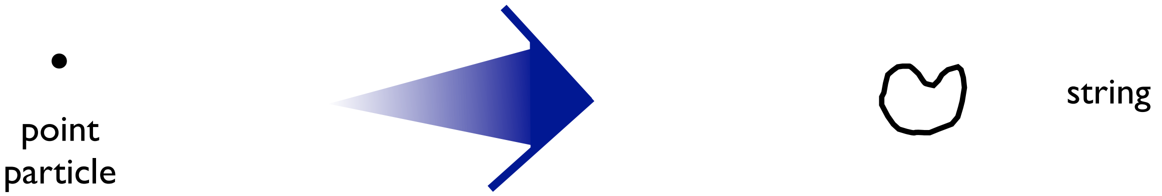
$$\xi_{(\mu\nu)} \qquad \xi_{[\mu\nu]} \qquad \delta^{\mu\nu}\xi_{\mu\nu}$$

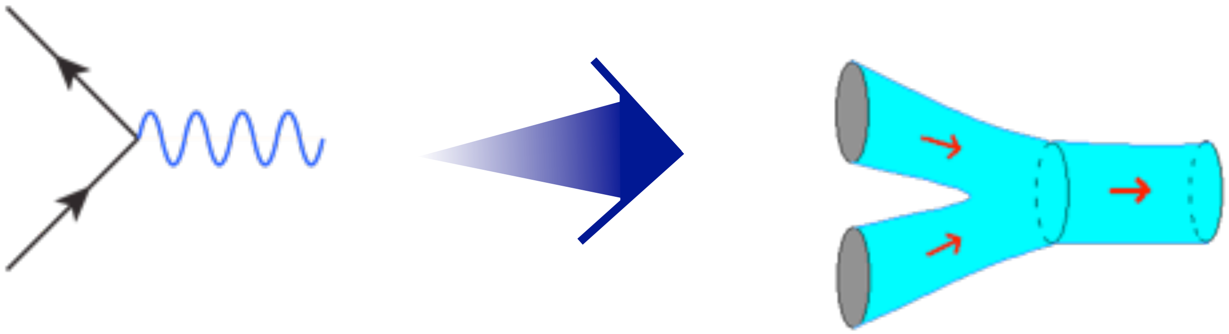
$$\downarrow$$

$$g_{\mu\nu}$$

metric

Superstring theory





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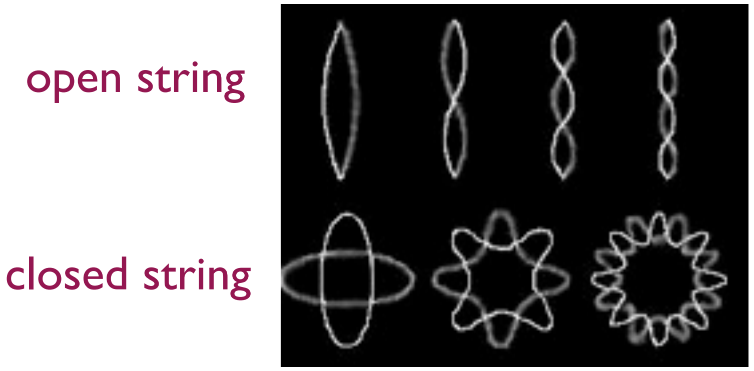

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different particles

Closed string massless excitation modes:

$$|\xi_{\mu\nu} >= \xi_\mu\tilde{\xi}_\nu\underset{\substack{\uparrow\\ \text{left-moving}\\ \text{osc. mode}}}{\alpha_1^\mu}\underset{\substack{\uparrow\\ \text{right-moving}\\ \text{osc. mode}}}{\tilde{\alpha}_1^\nu}|0 >$$

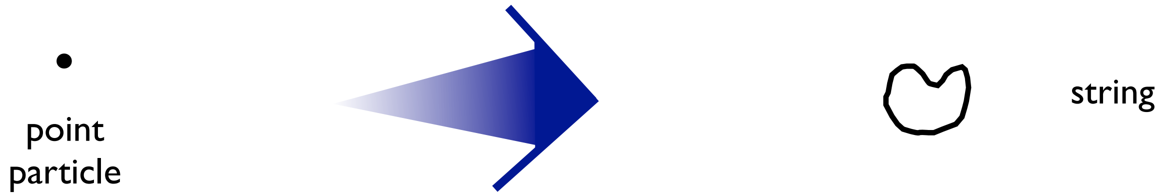
$$\xi_{(\mu\nu)} \qquad \xi_{[\mu\nu]} \qquad \delta^{\mu\nu}\xi_{\mu\nu}$$

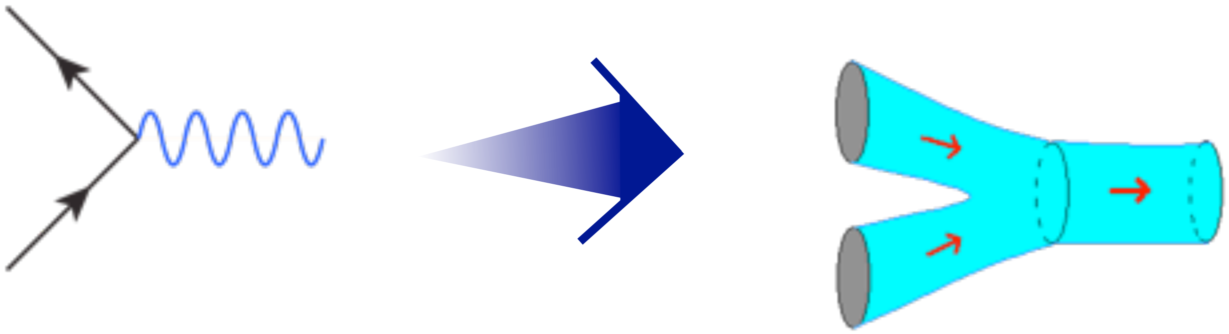
$$\downarrow \qquad \downarrow$$

$$g_{\mu\nu} \qquad B_{\mu\nu}$$

metric   B-field

Superstring theory

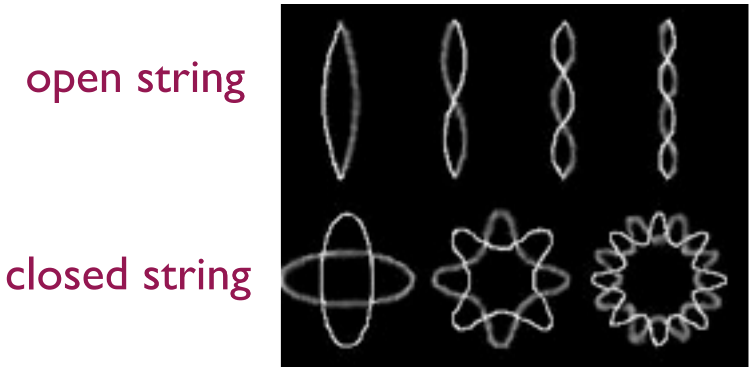




vertex

"pants" diagram

singularities
get resolved



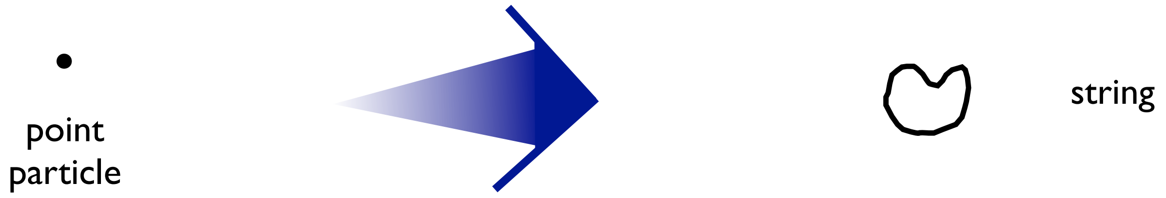
Oscillation modes:
different particles

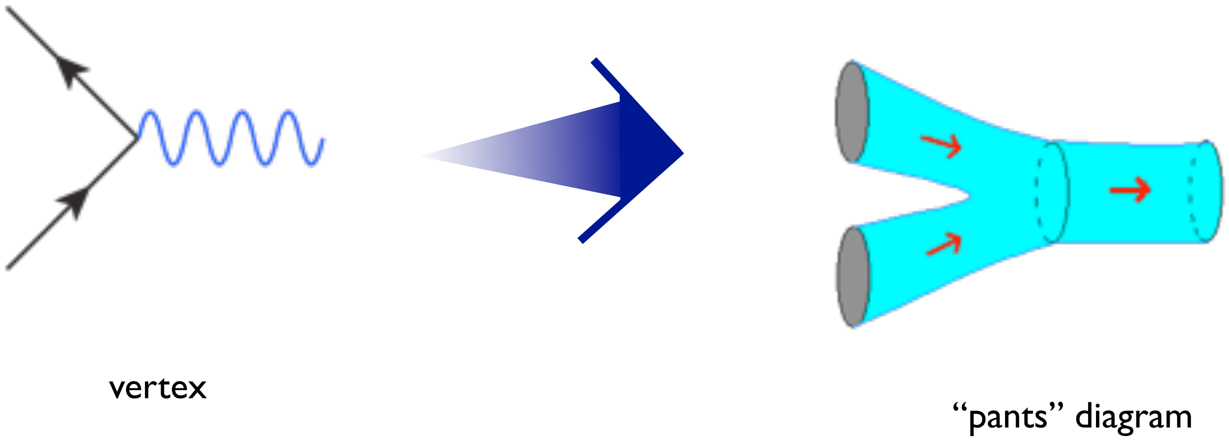
Closed string massless excitation modes:

$$|\xi_{\mu\nu} >= \xi_\mu\tilde{\xi}_\nu \underset{\substack{\uparrow\\ \text{left-moving}\\ \text{osc. mode}}}{\alpha_1^\mu} \underset{\substack{\uparrow\\ \text{right-moving}\\ \text{osc. mode}}}{\tilde{\alpha}_1^\nu}|0 >$$

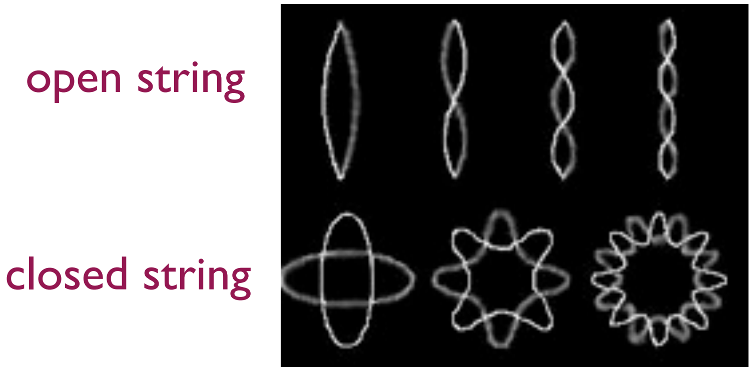
$$\begin{array}{ccc} \xi_{(\mu\nu)} & \xi_{[\mu\nu]} & \delta^{\mu\nu}\xi_{\mu\nu} \\ \downarrow & \downarrow & \downarrow \\ g_{\mu\nu} & B_{\mu\nu} & \phi \\ \text{metric} & \text{B-field} & \text{dilaton} \end{array}$$

Superstring theory





singularities get resolved



Oscillation modes:
different particles

Closed string massless excitation modes:

$$|\xi_{\mu\nu} >= \xi_\mu \tilde{\xi}_\nu \underset{\uparrow}{\alpha_1^\mu} \underset{\uparrow}{\tilde{\alpha}_1^\nu} |0 >$$

left-moving
osc. mode

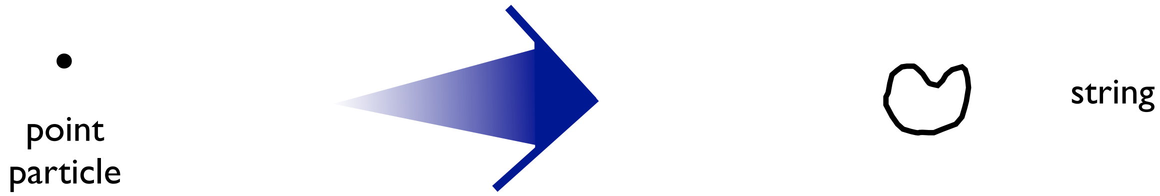
right-moving
osc. mode

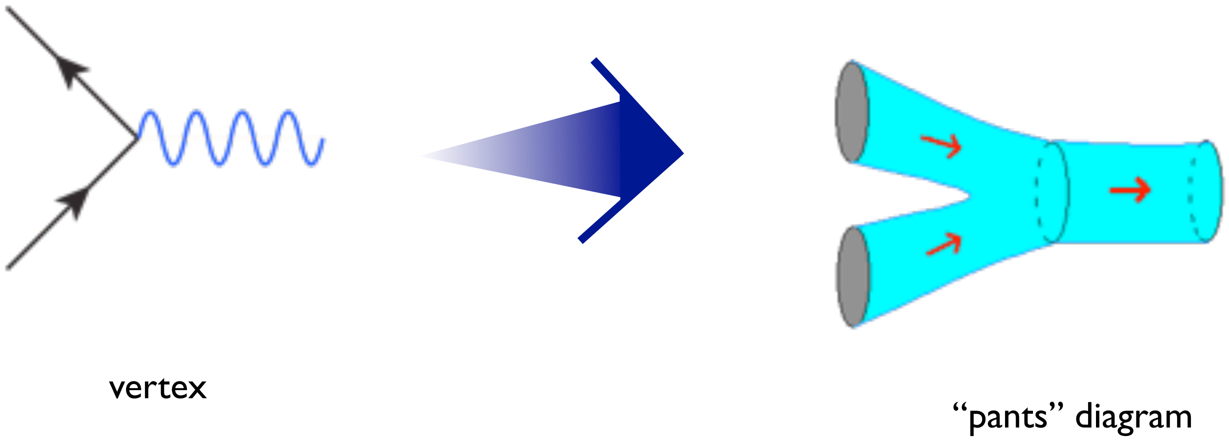
$$\begin{array}{ccc} \xi_{(\mu\nu)} & \xi_{[\mu\nu]} & \delta^{\mu\nu}\xi_{\mu\nu} \\ \downarrow & \downarrow & \downarrow \\ g_{\mu\nu} & B_{\mu\nu} & \phi \end{array}$$

metric B-field dilaton

Supergravity : keep only massless modes

Superstring theory

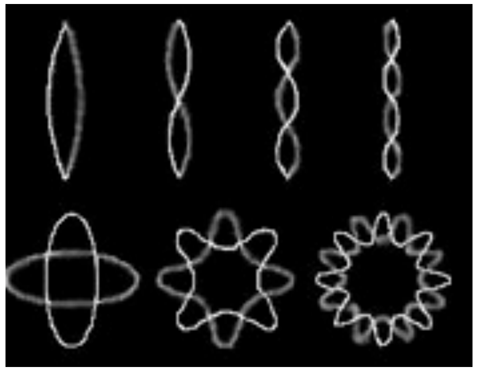




singularities
get resolved

open string

closed string



Oscillation modes:
different particles

Closed string massless excitation modes:

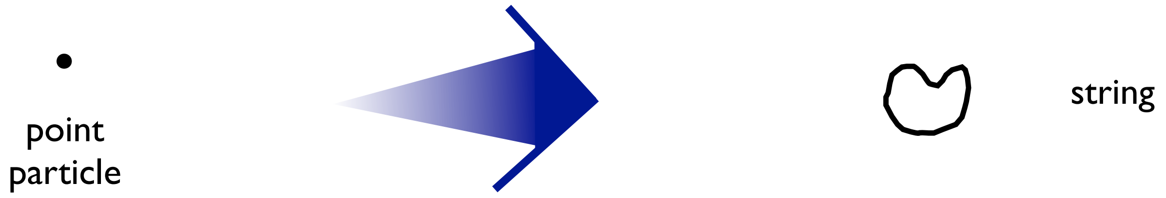
$$|\xi_{\mu\nu}>=\xi_\mu\tilde{\xi}_\nu\underset{\underset{\text{left-moving osc. mode}}{\uparrow}}{\alpha_1^\mu}\underset{\underset{\text{right-moving osc. mode}}{\uparrow}}{\tilde{\alpha}_1^\nu}|0>$$

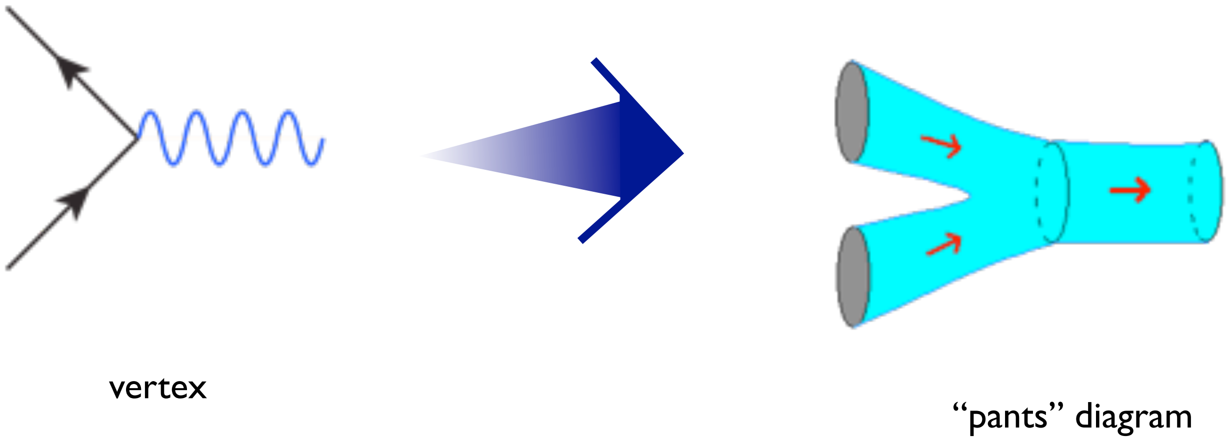
$$\begin{array}{ccc} \xi_{(\mu\nu)} & \xi_{[\mu\nu]} & \delta^{\mu\nu}\xi_{\mu\nu} \\ \downarrow & \downarrow & \downarrow \\ g_{\mu\nu} & B_{\mu\nu} & \phi \\ \text{metric} & \text{B-field} & \text{dilaton} \end{array}$$

Supergravity : keep only massless modes

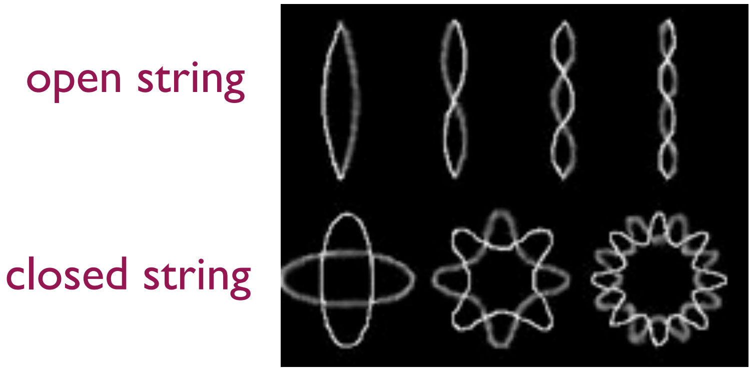
Consistently defined in 10D

Superstring theory





singularities get resolved



Oscillation modes:
different particles

Supergravity : keep only massless modes

Closed string massless excitation modes:

$$|\xi_{\mu\nu} >= \xi_\mu\tilde{\xi}_\nu\underset{\substack{\uparrow\\ \text{left-moving}\\ \text{osc. mode}}}{\alpha_1^\mu}\underset{\substack{\uparrow\\ \text{right-moving}\\ \text{osc. mode}}}{\tilde{\alpha}_1^\nu}|0 >$$

$$\begin{array}{ccc} \xi_{(\mu\nu)} & \xi_{[\mu\nu]} & \delta^{\mu\nu}\xi_{\mu\nu} \\ \downarrow & \downarrow & \downarrow \\ g_{\mu\nu} & B_{\mu\nu} & \phi \\ \text{metric} & \text{B-field} & \text{dilaton} \end{array}$$

Consistently defined in 10D $\mu, \nu = 0, ..., 9$

- Action

- Action

$$\mathrm{S} = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \qquad\qquad \right]$$

$$H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$$

- Action

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$$H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$$

- Equations of motion

- Action

$$S = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \qquad\qquad \right]$$

$$H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$$

- Equations of motion

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{4}H_{\mu\lambda\rho}H_\nu{}^{\lambda\rho} + 2\nabla_\mu\nabla_\nu\phi &= 0 \\ \nabla^\mu\left(e^{-2\phi}H_{\mu\nu\lambda}\right) &= 0 \\ \nabla^2\phi - (\nabla\phi)^2 + \tfrac{1}{4}R - \tfrac{1}{48}H^2 &= 0 \end{aligned}$$

- Action

$$S = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \qquad \right]$$

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Adding RR fields

- Action

$$\mathrm{S} = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + ...\right]$$

$$H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$$

- Equations of motion

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{4}H_{\mu\lambda\rho}H_\nu{}^{\lambda\rho} + 2\nabla_\mu\nabla_\nu\phi &= 0 \\ \nabla^\mu\left(\mathrm{e}^{-2\phi}H_{\mu\nu\lambda}\right) &= 0 \\ \nabla^2\phi - (\nabla\phi)^2 + \tfrac{1}{4}R - \tfrac{1}{48}H^2 &= 0 \end{aligned}$$

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$$\mathrm{S} = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + ...\right]$$

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$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{4}H_{\mu\lambda\rho}H_\nu{}^{\lambda\rho} + 2\nabla_\mu\nabla_\nu\phi + \ldots &= 0 \\ \nabla^\mu\left(\mathrm{e}^{-2\phi}H_{\mu\nu\lambda}\right) + \ldots &= 0 \\ \nabla^2\phi - (\nabla\phi)^2 + \tfrac{1}{4}R - \tfrac{1}{48}H^2 &= 0 \end{aligned}$$

Adding RR fields

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$$\mathrm{S} = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + ...\right]$$

$$H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$$

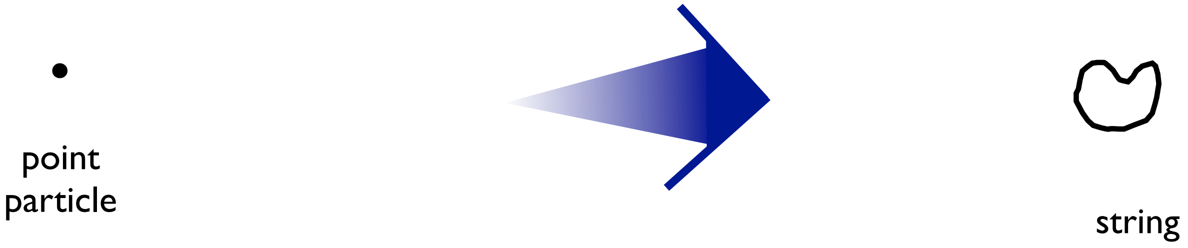
- Equations of motion

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{4}H_{\mu\lambda\rho}H_{\nu}{}^{\lambda\rho} + 2\nabla_\mu\nabla_\nu\phi + \ldots &= 0 \\ \nabla^\mu\left(\mathrm{e}^{-2\phi}H_{\mu\nu\lambda}\right) + \ldots &= 0 \\ \nabla^2\phi - (\nabla\phi)^2 + \tfrac{1}{4}R - \tfrac{1}{48}H^2 &= 0 \\ \ldots \end{aligned}$$

Adding RR fields

gravity
=
geometry

supergravity
=
generalized geometry



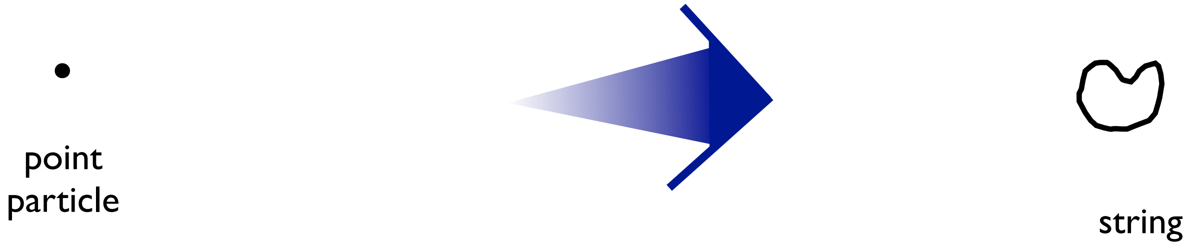
point
particle

string

Superstring theory

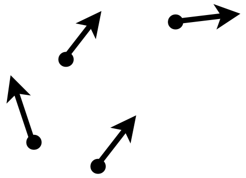
gravity
=
geometry

supergravity
=
generalized geometry



Superstring theory

point particles

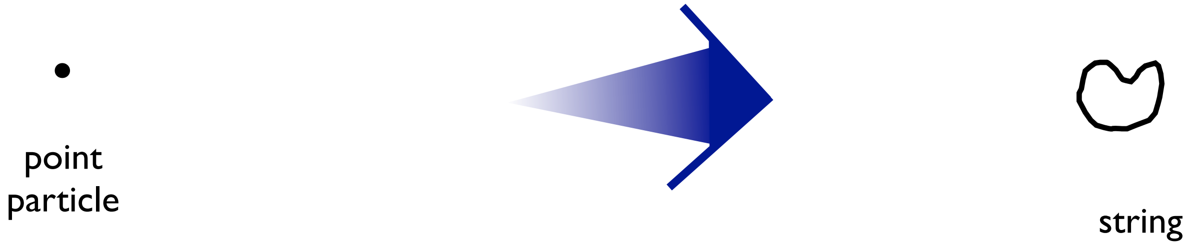


$\vec{p}$

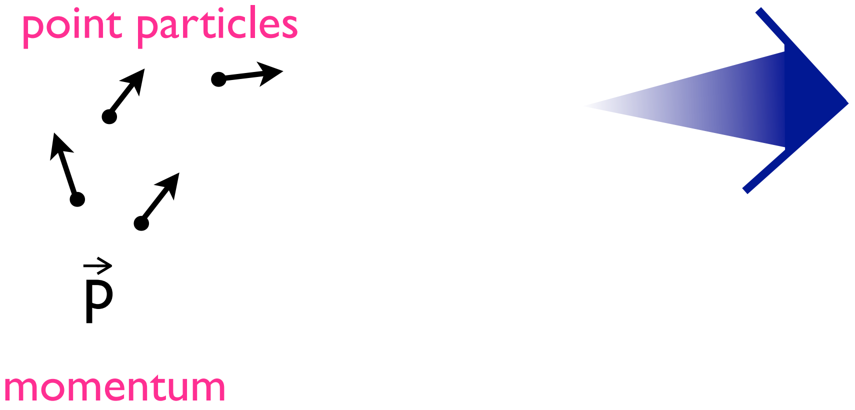
momentum

gravity
=
geometry

supergravity
=
generalized geometry

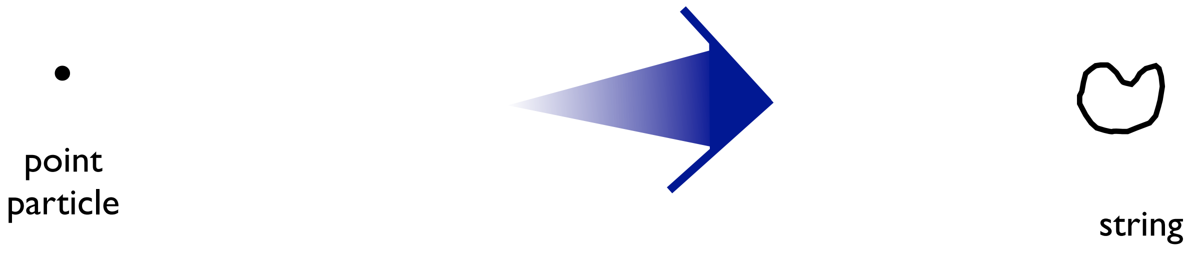


Superstring theory



gravity = geometry

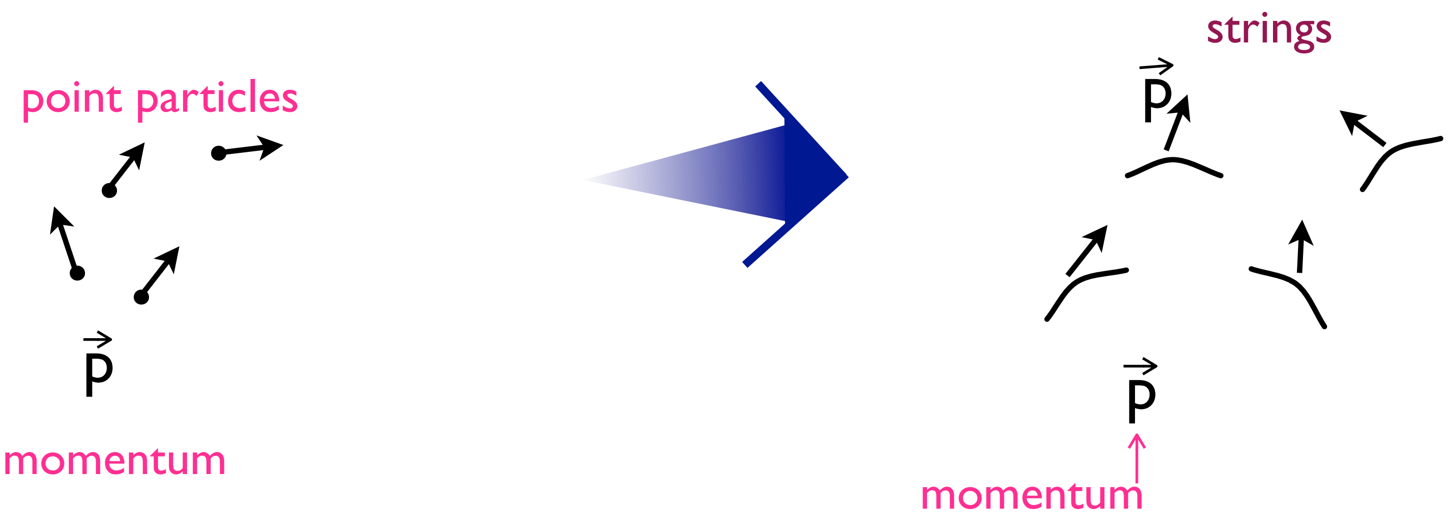
supergravity = generalized geometry



point particle

string

Superstring theory



point particles

$\vec{p}$

momentum

strings

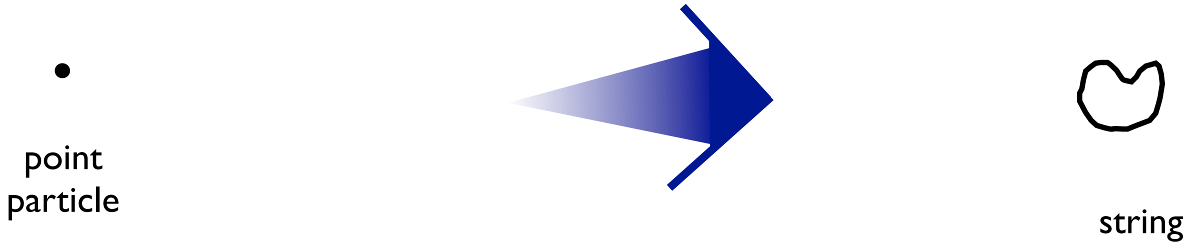
$\vec{p}$

$\vec{p}$

momentum

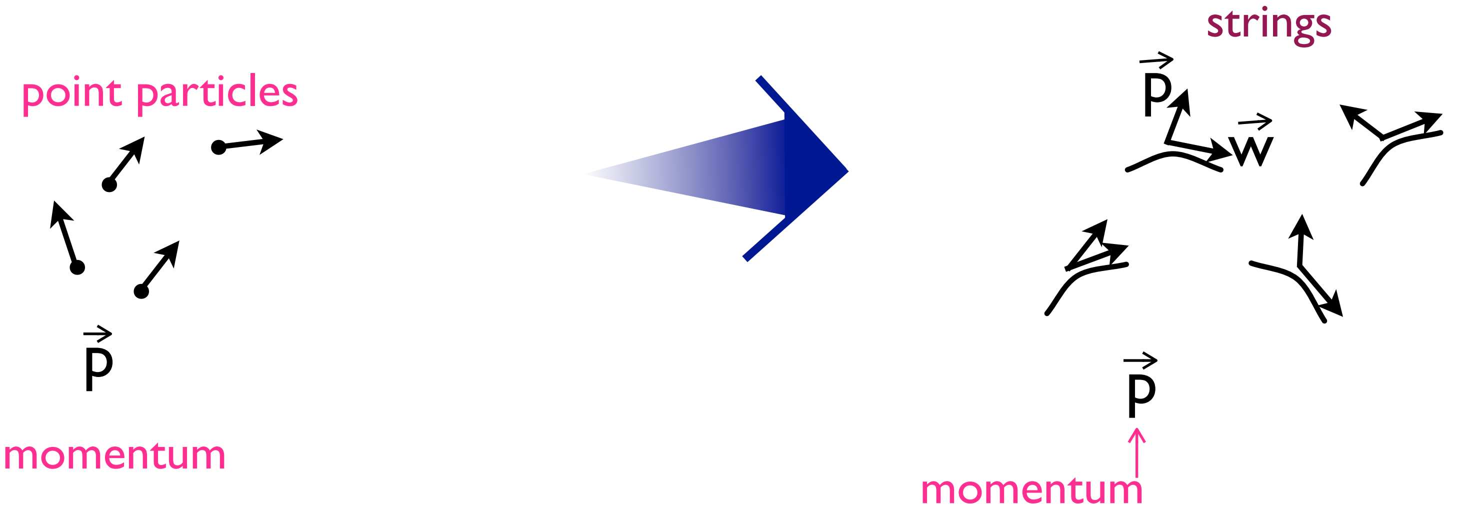
gravity
=
geometry

supergravity
=
generalized geometry


point particle

string

Superstring theory


point particles

$\vec{p}$

momentum

strings

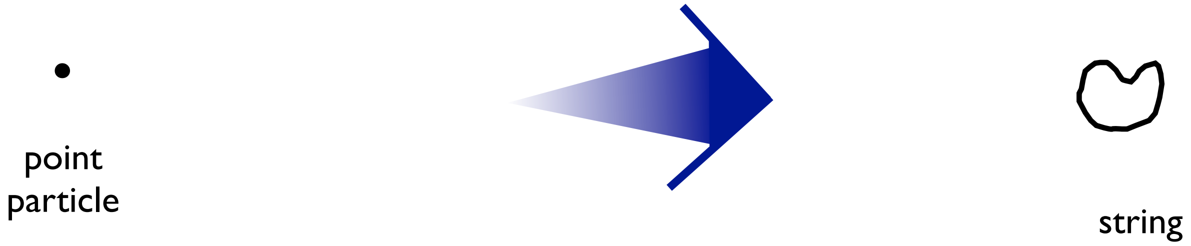
$\vec{p}$ $\vec{w}$

$\vec{p}$

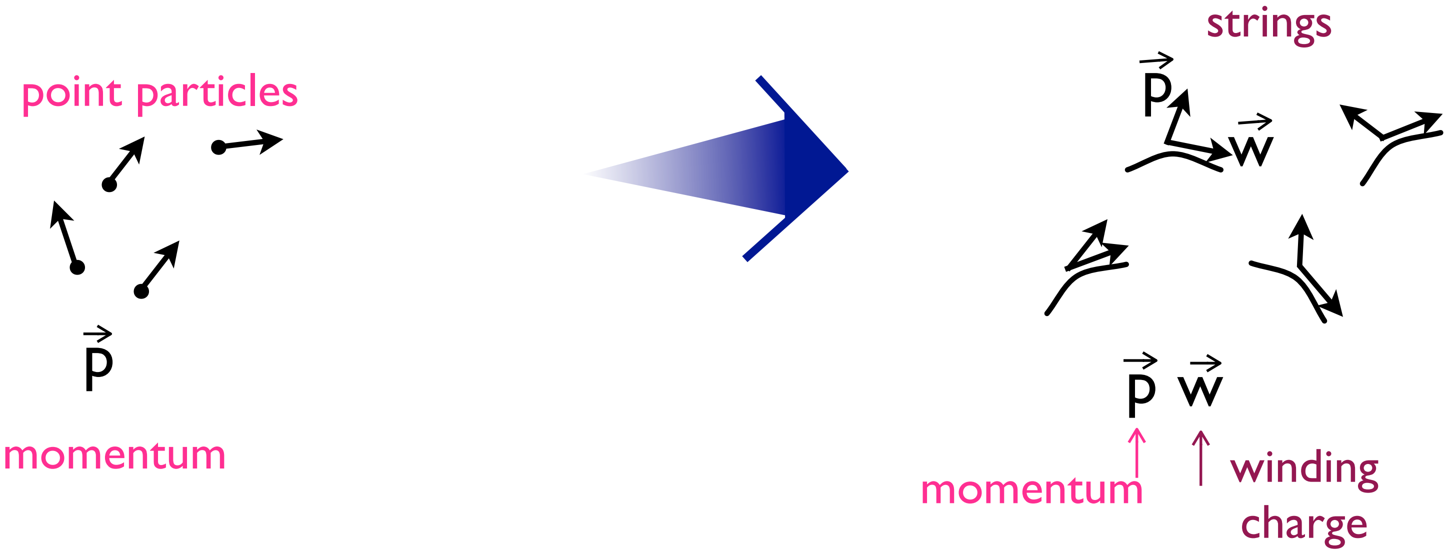
momentum

gravity = geometry

supergravity = generalized geometry

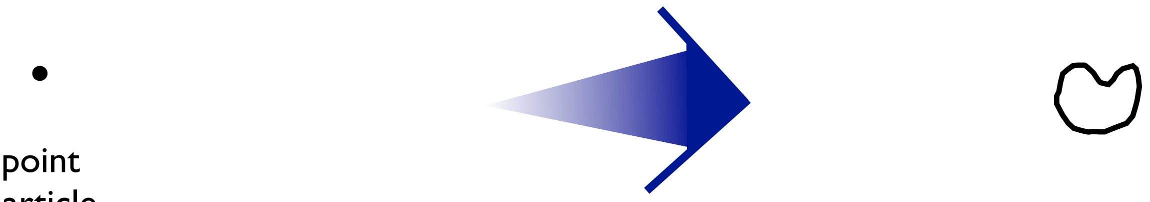


Superstring theory



gravity = geometry

supergravity = generalized geometry

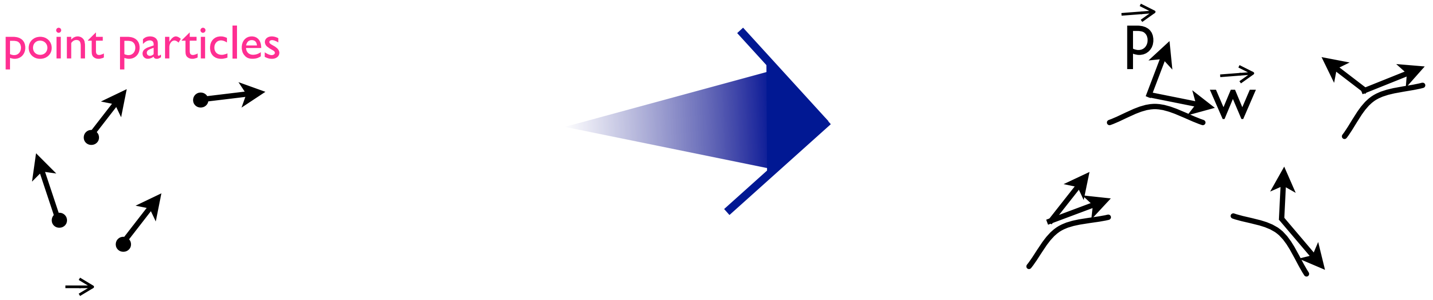


point particle

string

Superstring theory

point particles



strings

$\vec{p}$

momentum

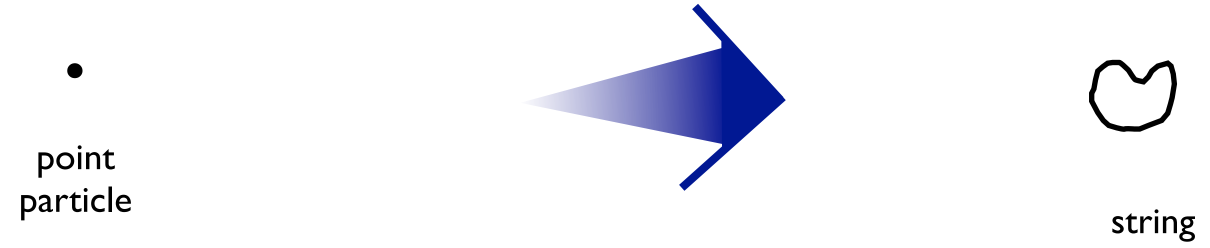
$(\vec{p},\vec{w})$

↑ momentum

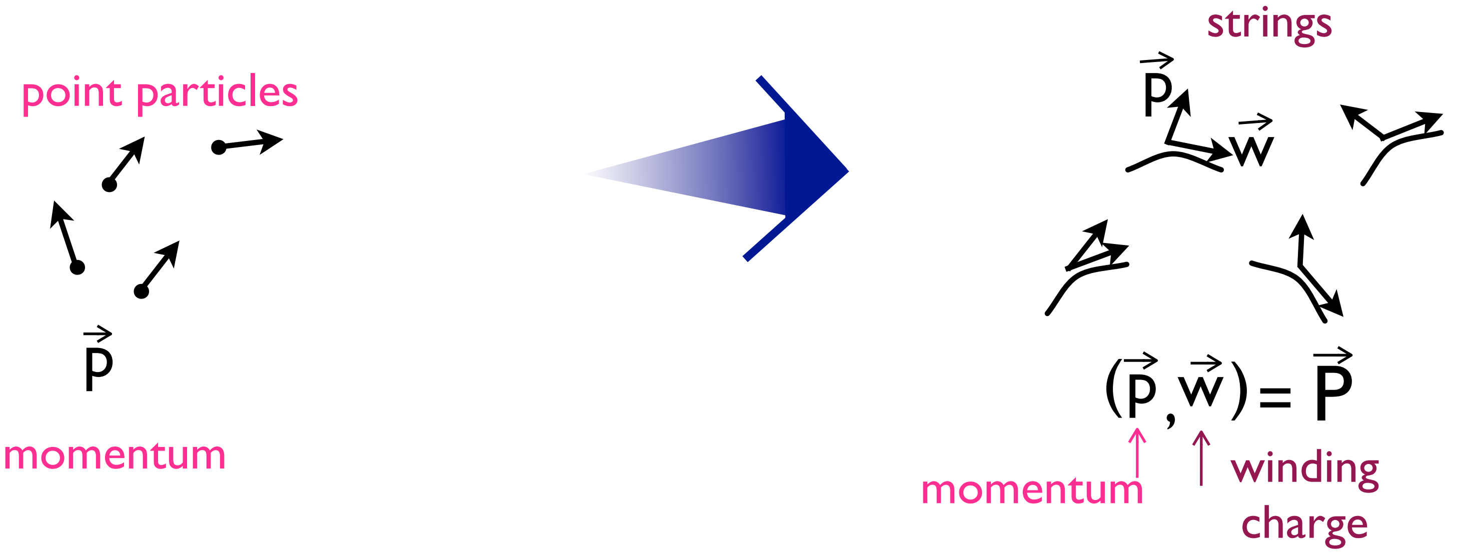
↑ winding charge

gravity = geometry

supergravity = generalized geometry



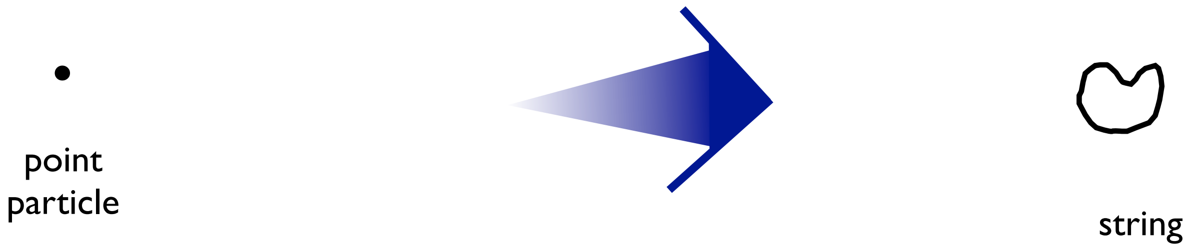
Superstring theory



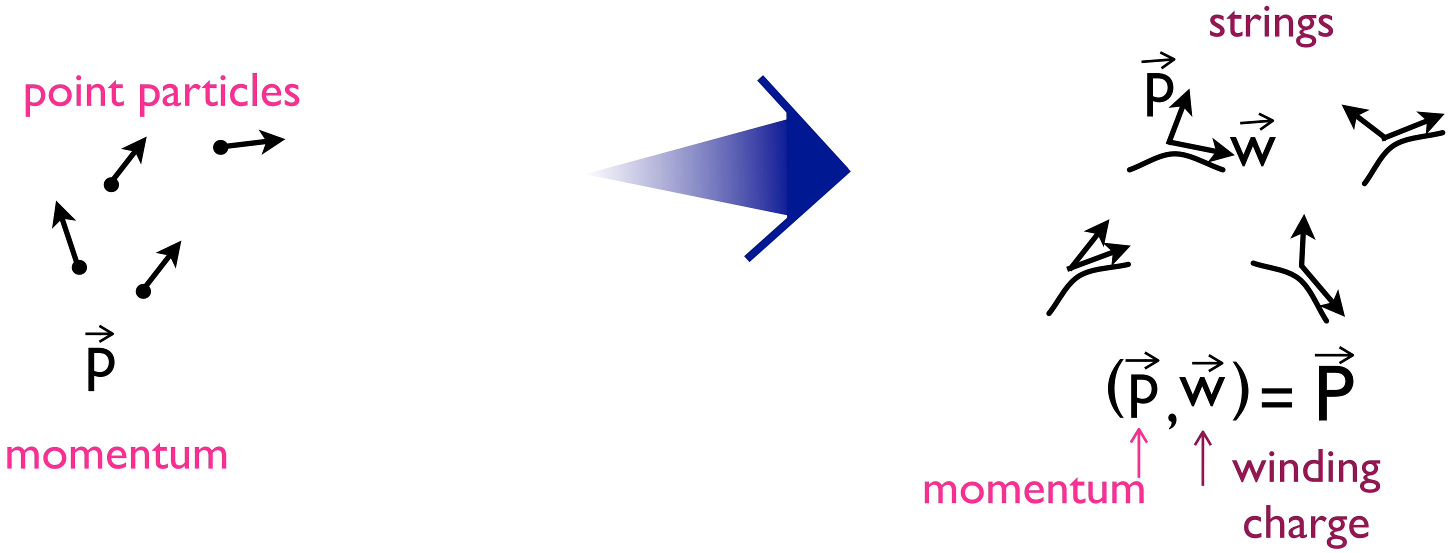
$$(\vec{p},\vec{w}) = \vec{P}$$

gravity
=
geometry

supergravity
=
generalized geometry



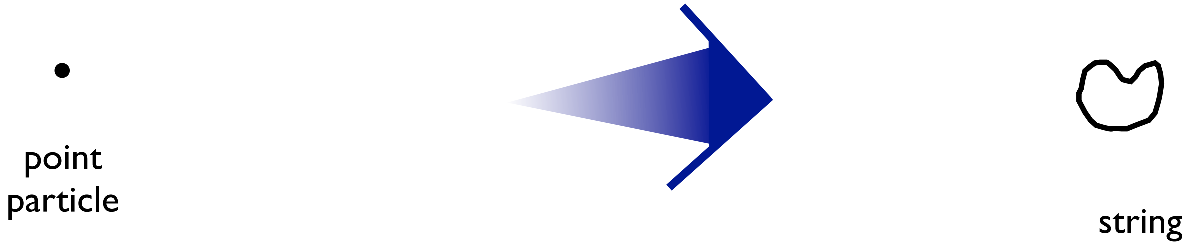
Superstring theory



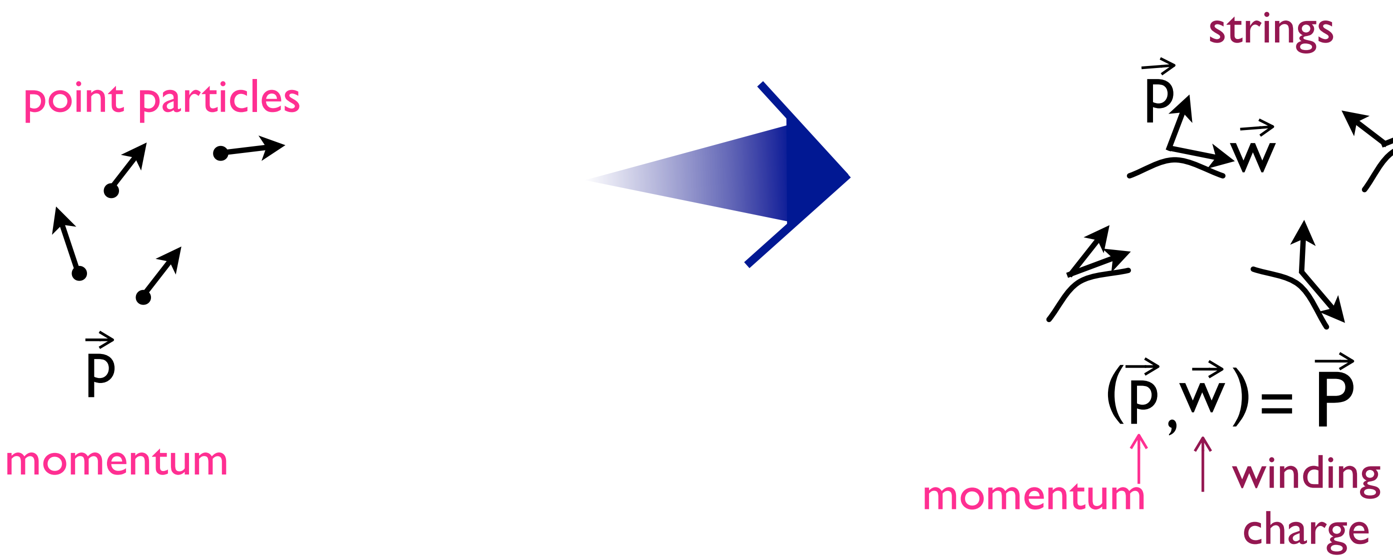
Do pure geometry on
a double tangent
space (20D)

gravity = geometry

supergravity = generalized geometry



Superstring theory



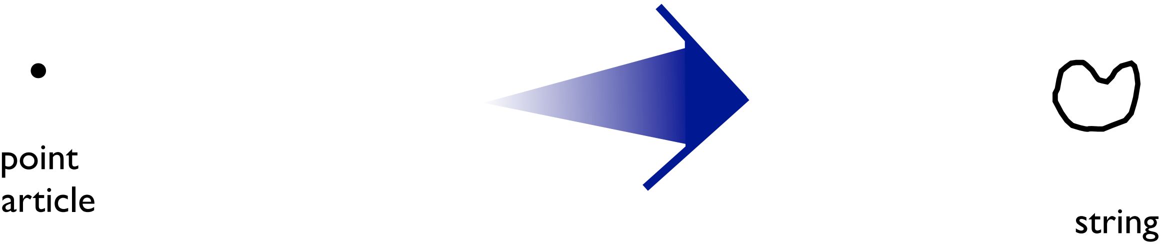
$$(\vec{p}, \vec{w}) = \vec{P}$$

Do pure geometry on
a double tangent
space (20D)

Tseytlin 1991
Siegel 1993

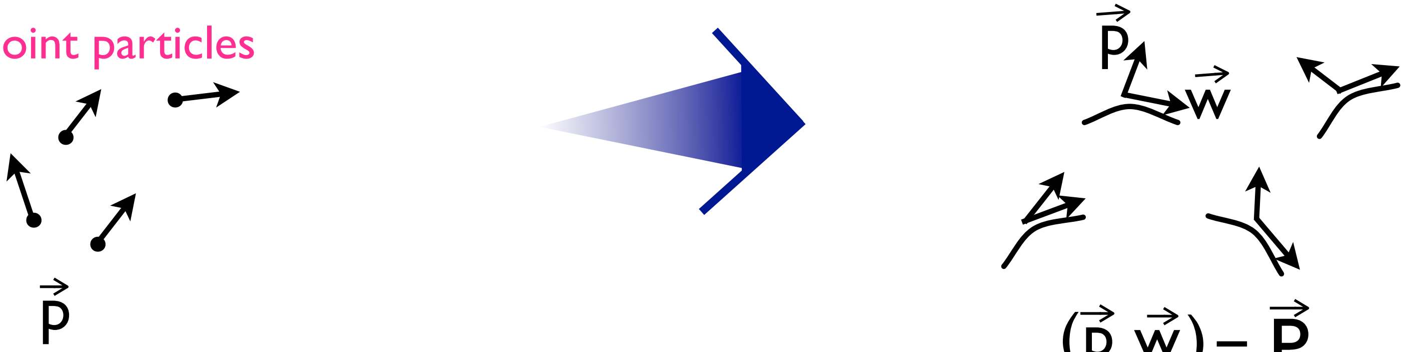
gravity = geometry

supergravity = generalized geometry



Superstring theory

point particles



momentum

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\underset{\substack{\uparrow\\ \text{winding charge}}}{\vec{w}}) = \vec{P}$$

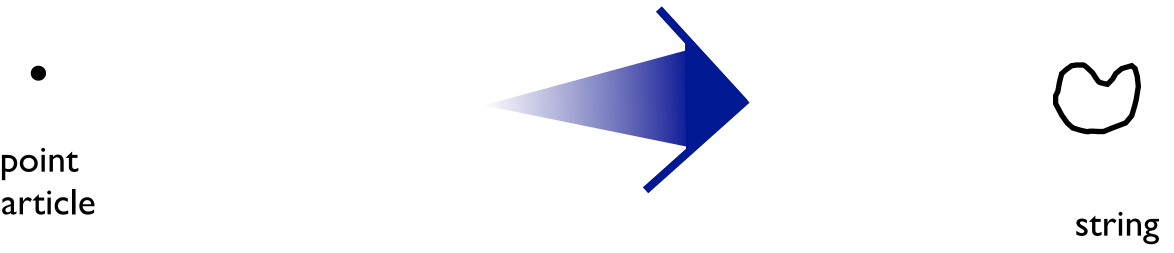
Do pure geometry on a double tangent space (20D)

Tseytlin 1991
Siegel 1993

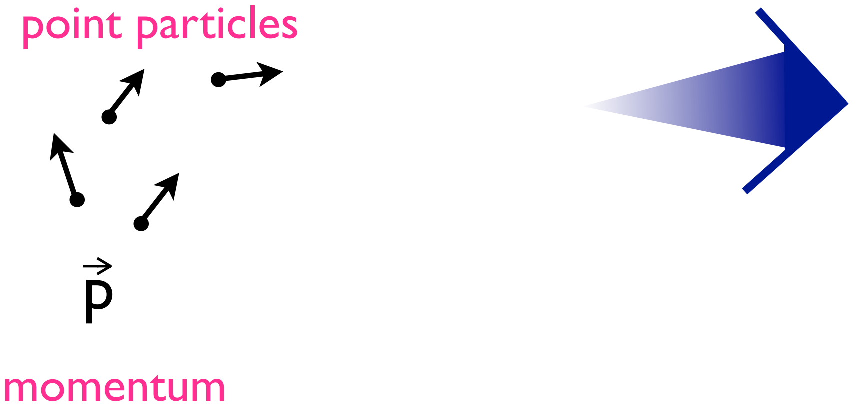
Left-moving

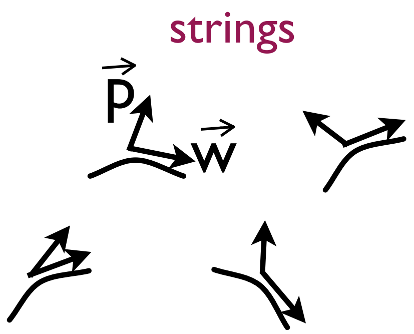
gravity = geometry

supergravity = generalized geometry



Superstring theory





$$(\vec{p},\vec{w}) = \vec{P}$$

momentum ↑ ↑ winding charge

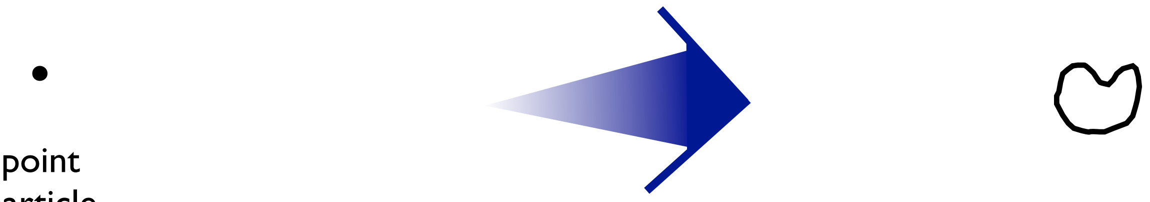
Left-moving + Right-moving

Do pure geometry on a double tangent space (20D)

Tseytlin 1991
Siegel 1993

gravity = geometry

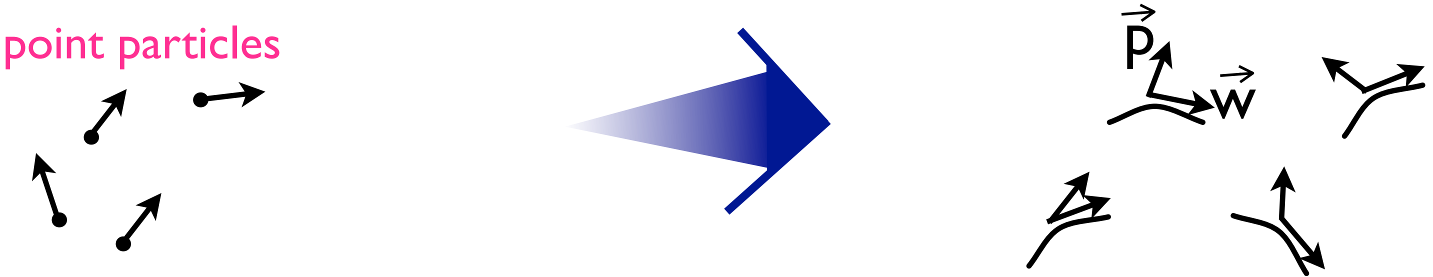
supergravity = generalized geometry



point particle

string

Superstring theory



$\vec{p}$

momentum

$$(\vec{p},\vec{w}) = \vec{P}$$

momentum ↑ ↑ winding charge

Do pure geometry on a double tangent space (20D)

Tseytlin 1991
Siegel 1993

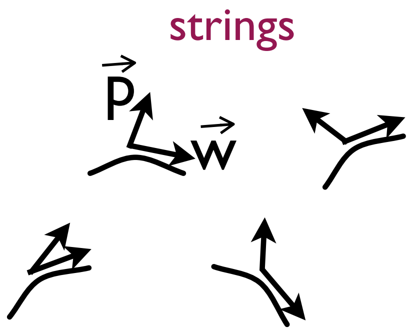
Left-moving + Right-moving

Metric + B-field+dilaton

Hitchin 2001

Generalized geometry

Generalized geometry



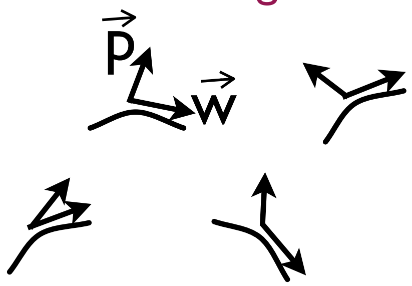
$$(\underset{\uparrow\atop\text{momentum}}{\vec{p}},\underset{\uparrow\atop\text{winding charge}}{\vec{w}}) = \vec{P}$$

Left-moving + Right-moving

Metric + B-field

Generalized geometry

strings



$$(\vec{p},\vec{w}) = \vec{P}$$

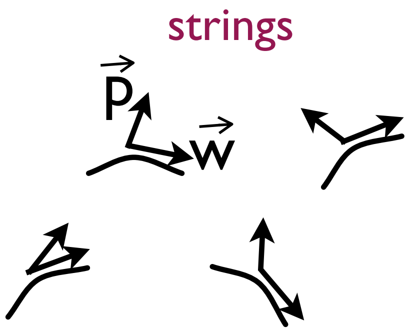
momentum ↑ ↑ winding charge

Left-moving + Right-moving

Metric + B-field

Generalized geometry

- Symmetries: diffeomorphisms



$$(\vec{p},\vec{w}) = \vec{P}$$

momentum

winding charge

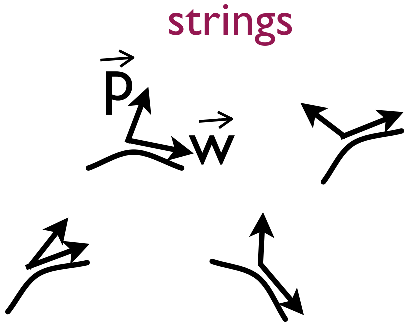
Left-moving + Right-moving

$\boxed{\text{Metric + B-field}}$

Generalized geometry

- Symmetries: diffeomorphisms

$$\delta g = \mathcal{L}_v g$$



$(\vec{p}, \vec{w}) = \vec{P}$

momentum ↑ ↑ winding charge

Left-moving + Right-moving

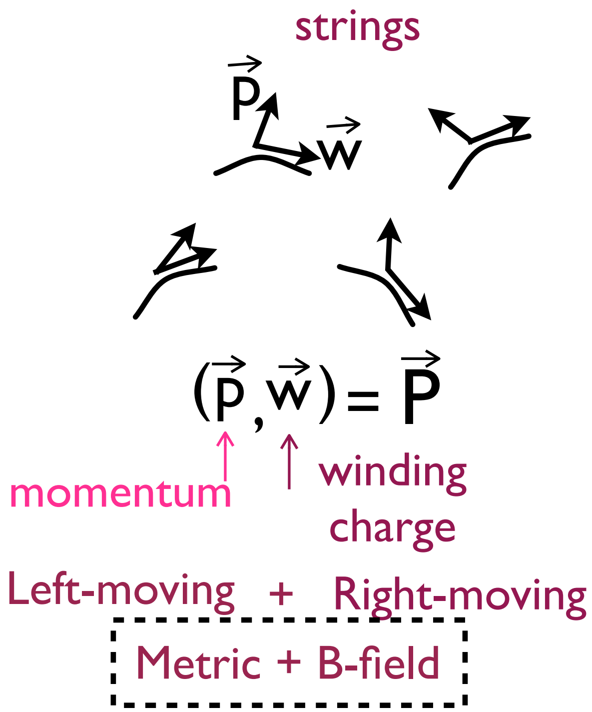
Metric + B-field

Generalized geometry

- Symmetries: diffeomorphisms

$$\delta g = \mathcal{L}_v g$$

$$\delta B = \mathcal{L}_v B$$



$(\vec{p}, \vec{w}) = \vec{P}$

momentum

winding charge

Left-moving + Right-moving

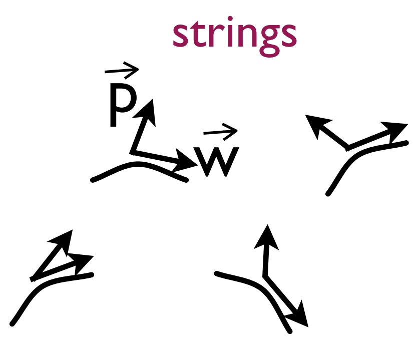
Metric + B-field

Generalized geometry

- Symmetries: diffeomorphisms

$$\delta g = \mathcal{L}_v g \qquad\qquad v \in TM$$

$$\delta B = \mathcal{L}_v B$$



$$(\vec{p},\vec{w}) = \vec{P}$$

momentum ↑ ↑ winding charge

Left-moving + Right-moving

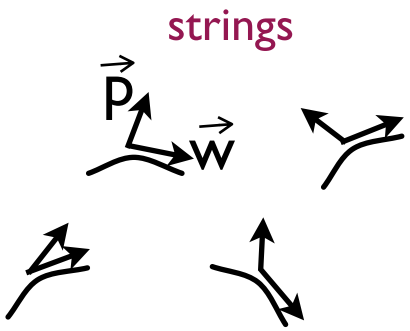
$\boxed{\text{Metric + B-field}}$

Generalized geometry

- Symmetries: diffeomorphisms + gauge transformations

$$\delta g = \mathcal{L}_v g \qquad v \in TM$$

$$\delta B = \mathcal{L}_v B + \mathrm{d}\lambda$$



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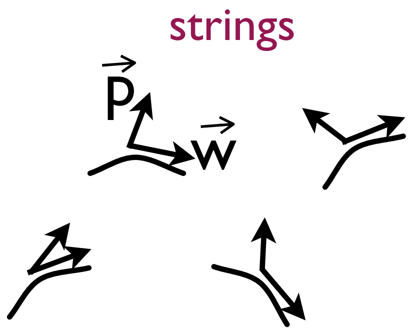

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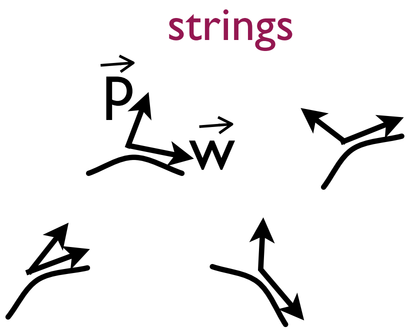

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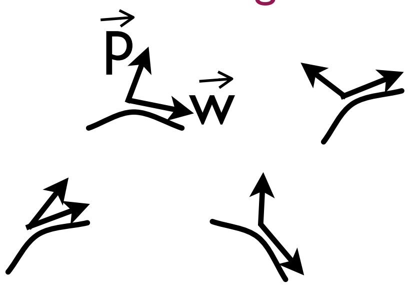
$$\begin{array}{l} \quad v \in TM \\ + \\ \quad \lambda \in T^*M \\ \hline V \in TM \oplus T^*M \end{array}$$

generalized Vectors

strings



$(\vec{p}, \vec{w}) = \vec{P}$

$\uparrow$ momentum

$\uparrow$ winding charge

Left-moving + Right-moving

Metric + B-field

Generalized geometry

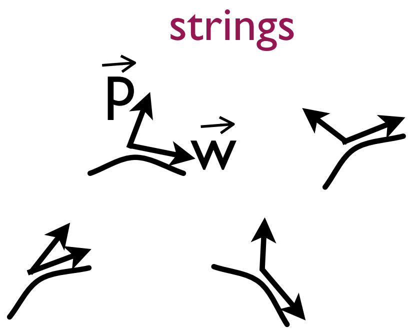
- Symmetries: diffeomorphisms + gauge transformations

$$\delta g = \mathcal{L}_v g$$

$$\delta B = \mathcal{L}_v B + \mathrm{d}\lambda$$

$$\begin{array}{rl} & v \in TM \\ + & \lambda \in T^*M \\ \hline & V \in TM \oplus T^*M \end{array} \quad \sim \quad \begin{array}{l} p \in TM \\ \omega \in T^*M \end{array}$$

generalized Vectors



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Generalized geometry

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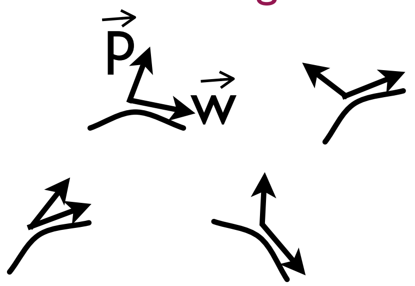
$$\begin{array}{ccc} v \in TM & & p \in TM \\ +\ \lambda \in T^*M & \sim & \omega \in T^*M \\ \hline V \in TM \oplus T^*M & \sim & \vec{P} \in TM \oplus T^*M \end{array}$$

generalized Vectors



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Left-moving + Right-moving

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Generalized geometry

- Symmetries: diffeomorphisms + gauge transformations

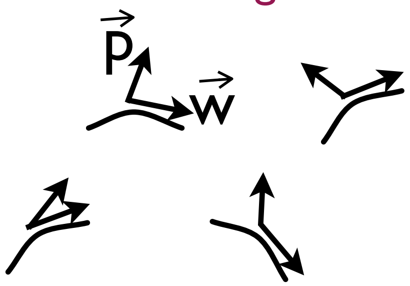
$$\delta g = \mathcal{L}_v g$$

$$\delta B = \mathcal{L}_v B + \mathrm{d}\lambda$$

$$\frac{\begin{matrix} v \in TM \\ + \ \lambda \in T^*M \end{matrix}}{V \in TM \oplus T^*M} \sim \frac{\begin{matrix} p \in TM \\ \omega \in T^*M \end{matrix}}{\vec{P} \in TM \oplus T^*M}$$

generalized Vectors

strings



$$(\vec{p}, \vec{w}) = \vec{P}$$

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Left-moving + Right-moving

$$\boxed{\text{Metric + B-field}}$$

$$V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$$

Generalized geometry

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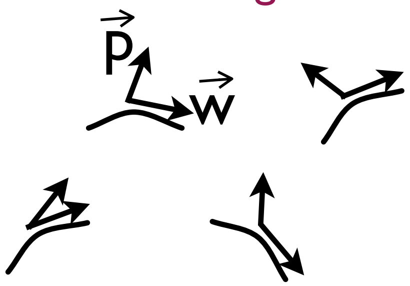
$$\begin{array}{c} v \in TM \\ + \; \lambda \in T^*M \\ \hline V \in TM \oplus T^*M \end{array} \quad \sim \quad \begin{array}{c} p \in TM \\ \omega \in T^*M \\ \hline \vec{\mathrm{P}} \in TM \oplus T^*M \end{array}$$

generalized Vectors

strings



$$(\vec{p},\vec{w}) = \vec{P}$$

momentum ↑ ↑ winding charge

Left-moving + Right-moving

Metric + B-field

$$V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$$

Fund of O(10,10)

Generalized geometry

- Symmetries: diffeomorphisms + gauge transformations

$$\delta g = \mathcal{L}_v g$$

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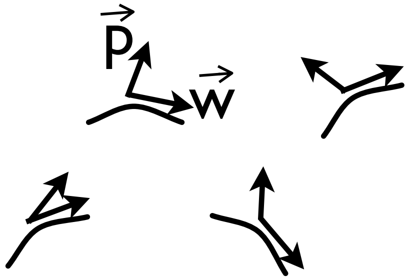
$$\begin{array}{c} v \in TM \\ + \; \lambda \in T^*M \\ \hline V \in TM \oplus T^*M \end{array} \sim \begin{array}{c} p \in TM \\ \omega \in T^*M \\ \hline \vec{\mathrm{P}} \in TM \oplus T^*M \end{array}$$

generalized Vectors

strings



$$(\underset{\uparrow \atop \text{momentum}}{\vec{p}}, \underset{\uparrow \atop \text{winding charge}}{\vec{w}}) = \vec{P}$$

Left-moving + Right-moving

$$\boxed{\text{Metric + B-field}}$$

$$V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$$

Natural inner product

Fund of O(10,10)

Generalized geometry

- Symmetries: diffeomorphisms + gauge transformations

$$\delta g = \mathcal{L}_v g$$

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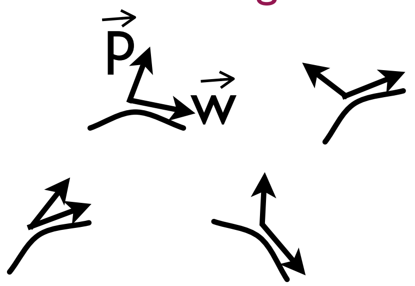
$$\begin{array}{c} v \in TM \\ +\ \lambda \in T^*M \\ \hline \end{array} \sim \begin{array}{c} p \in TM \\ \omega \in T^*M \\ \hline \end{array}$$

$$V \in TM \oplus T^*M \sim \vec{P} \in TM \oplus T^*M$$

generalized Vectors

strings



$(\vec{p},\vec{w}) = \vec{P}$

momentum $\uparrow$ $\uparrow$ winding charge

Left-moving + Right-moving

Metric + B-field

$$V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$$

Fund of O(10,10)

Natural inner product $V^M \eta_{MN} U^N = v^\mu \xi_\mu + u^\mu \lambda_\mu$ $\qquad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

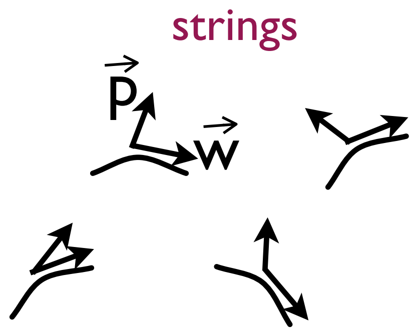
V=v+λ

U=u+ξ

$$M = 1, ..., 20$$

Generalized geometry

- Symmetries: diffeomorphisms + gauge transformations



$$\delta g = \mathcal{L}_v g$$

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$$\begin{array}{c} v \in TM \\ +\ \lambda \in T^*M \\ \hline V \in TM \oplus T^*M \end{array} \quad \sim \quad \begin{array}{c} p \in TM \\ \omega \in T^*M \\ \hline \vec{P} \in TM \oplus T^*M \end{array}$$

generalized Vectors

$$(\underset{\substack{\uparrow \\ \text{momentum}}}{\vec{p}}, \underset{\substack{\uparrow \\ \text{winding charge}}}{\vec{w}}) = \vec{P}$$

Left-moving + Right-moving

Metric + B-field

$$V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$$

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Natural inner product $V^M \eta_{MN} U^N = v^\mu \xi_\mu + u^\mu \lambda_\mu$ $\quad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

V=v+λ
U=u+ξ

$$M = 1, \ldots, 20$$

$$L_V U = \mathcal{L}_v u + \mathcal{L}_v \xi - \iota_u d\lambda$$

generalized diffeomorphism generated by V on U

Generalized geometry

- Symmetries: diffeomorphisms + gauge transformations

$$\delta g = \mathcal{L}_v g$$

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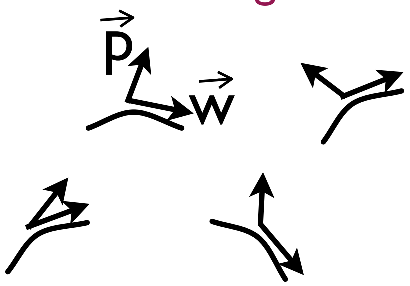
$$\begin{array}{c} v \in TM \\ + \; \lambda \in T^*M \\ \hline \end{array} \sim \begin{array}{c} p \in TM \\ \omega \in T^*M \\ \hline \end{array}$$

$$V \in TM \oplus T^*M \sim \vec{\mathsf{P}} \in TM \oplus T^*M$$

generalized Vectors

strings



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Fund of O(10,10)

V=v+λ
U=u+ξ

$$M = 1, ..., 20$$

$L_V U = \mathcal{L}_v u + \mathcal{L}_v \xi - \iota_u d\lambda$ generalized diffeomorphism generated by V on U

Algebra : Courant bracket $[[U, V]] = \frac{1}{2}(L_U V - L_V U)$

- Generalized metric

- Generalized metric $V^M \mathcal{G}_{MN} U^N$

- Generalized metric $V^M \mathcal{G}_{MN} U^N$

V=v+λ

- Generalized metric $V^M \mathcal{G}_{MN} U^N$

$\mathcal{G} =$

V=v+λ

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

$$\mathcal{G} = \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix}$$

V=v+λ

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

$$\mathcal{G} = \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix}$$

V=v+λ

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix}$$

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Contains g, B

V=v+λ

- Generalized metric

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Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\, \mathcal{G})$$

V=v+λ

Projects onto left-moving
and right moving

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\, \mathcal{G})$$

$$P_\pm V = \tfrac{1}{2}\left[(v \qquad) + (\quad \pm gv \qquad)\right]$$

V=v

Projects onto left-moving
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- Generalized metric

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Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\, \mathcal{G})$$

$$P_\pm V = \tfrac{1}{2}\left[(v \pm g^{-1}(\lambda \qquad)) + (\lambda \pm gv \qquad)\right]$$

V=v+λ

Projects onto left-moving
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- Generalized metric

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$$P_\pm V = \tfrac{1}{2}\left[(v \pm g^{-1}(\lambda + Bv)) + (\lambda \pm gv \mp Bg^{-1}(\lambda + Bv))\right]$$

$V=v+\lambda$

Projects onto left-moving
and right moving

- Generalized metric $$V^M \mathcal{G}_{MN} U^N$$

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V=v+λ

Projects onto left-moving
and right moving

Can add dilaton

$$\mathcal{G} = (e^{-2\phi}\sqrt{-g})^2 \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

- Generalized metric $V^M \mathcal{G}_{MN} U^N$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

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V=v+λ

Projects onto left-moving and right moving

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- Connection ∇

- Generalized metric $$V^M \mathcal{G}_{MN} U^N$$

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Projects onto left-moving
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- Connection ∇

$$\nabla_M V^P = \partial_M V^P + \Gamma^P_{MN} V^N$$

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

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Contains g, B

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V=v+λ

Projects onto left-moving
and right moving

Can add dilaton

$$\mathcal{G} = (e^{-2\phi}\sqrt{-g})^2 \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

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demand torsion-free & metric compatible

- Generalized metric $V^M \mathcal{G}_{MN} U^N$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Contains g, B

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Projects onto left-moving
and right moving

Can add dilaton

$$\mathcal{G} = (e^{-2\phi}\sqrt{-g})^2 \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

- Connection ∇

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demand torsion-free & metric compatible

∃ but is not unique

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

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Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\, \mathcal{G})$$

Projects onto left-moving
and right moving

$$P_\pm V = \tfrac{1}{2}\left[(v \pm g^{-1}(\lambda + Bv)) + (\lambda \pm gv \mp Bg^{-1}(\lambda + Bv))\right]$$

V=v+λ

Can add dilaton

$$\mathcal{G} = (e^{-2\phi}\sqrt{-g})^2 \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

- Connection ∇

$$\nabla_M V^P = \partial_M V^P + \Gamma^P_{MN} V^N$$

demand torsion-free & metric compatible

↑

$$\Gamma_{[MNP]} = 0$$

∃ but is not unique

- **Generalized metric** $V^M \mathcal{G}_{MN} U^N$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\,\mathcal{G})$$

Projects onto left-moving and right moving

$$P_\pm V = \tfrac{1}{2}\left[(v \pm g^{-1}(\lambda + Bv)) + (\lambda \pm gv \mp Bg^{-1}(\lambda + Bv))\right]$$

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- **Connection** ∇

$$\nabla_M V^P = \partial_M V^P + \Gamma^P_{MN} V^N$$

demand torsion-free & metric compatible

∃ but is not unique

↑ $\Gamma_{[MNP]} = 0$

↑ determines (2,1) + (1,2) pieces of Γ in terms of $\partial\mathcal{G}$

- **Generalized metric** $V^M \mathcal{G}_{MN} U^N$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\, \mathcal{G})$$

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V=v+λ

Projects onto left-moving
and right moving

Can add dilaton

$$\mathcal{G} = (e^{-2\phi}\sqrt{-g})^2 \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

- **Connection** ∇

$$\nabla_M V^P = \partial_M V^P + \Gamma^P_{MN} V^N$$

demand torsion-free & metric compatible

∃ but is not unique

↑ ↑

$\Gamma_{[MNP]} = 0$

determines (2,1) + (1,2)
pieces of Γ in terms of
$\partial\mathcal{G}$

⟵

(2,1) : 2 right-moving,
1 left-moving index

- Generalized metric

$$V^M \mathcal{G}_{MN} U^N$$

$$\mathcal{G} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Contains g, B

$$P_\pm = \tfrac{1}{2}(\delta \pm \eta\, \mathcal{G})$$

Projects onto left-moving and right moving

$$P_\pm V = \tfrac{1}{2}\left[(v \pm g^{-1}(\lambda + Bv)) + (\lambda \pm gv \mp Bg^{-1}(\lambda + Bv))\right]$$

V=v+λ

Can add dilaton

$$\mathcal{G} = (e^{-2\phi}\sqrt{-g})^2 \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

- Connection ∇

$$\nabla_M V^P = \partial_M V^P + \Gamma^P_{MN} V^N$$

demand torsion-free & metric compatible

∃ but is not unique

↑

$\Gamma_{[MNP]} = 0$

↑

determines (2,1) + (1,2) pieces of Γ in terms of $\partial\mathcal{G}$ ⟵ (2,1) : 2 right-moving, 1 left-moving index

$$P_{+M}{}^Q\, P_{+N}{}^R\, P_{-P}{}^S\, \Gamma_{QRS}$$

- Generalized metric $V^M \mathcal{G}_{MN} U^N$

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pieces of Γ in terms of
$\partial\mathcal{G}$

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$$P_{+M}{}^Q \, P_{+N}{}^R \, P_{-P}{}^S \, \Gamma_{QRS}$$

$$\equiv \Gamma_{\underline{MN}\overline{S}}$$

- Curvature

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$$R_{MNP}{}^{R} = \partial_M \Gamma^R_{NP} - \partial_N \Gamma^R_{MP} + \Gamma^R_{MQ}\Gamma^Q_{NP} - \Gamma^R_{NQ}\Gamma^Q_{MP}$$

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not tensorial (under gen. diffeos)

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$$\mathcal{R}_{\underline{M}\overline{N}} = \mathcal{R}_{\underline{MK}\overline{N}}{}^{\underline{K}}$$

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tensorial, not uniquely defined

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$$\mathcal{R}_{\underline{M}\overline{N}} = \mathcal{R}_{\underline{MK}\overline{N}}{}^{\underline{K}}$$

tensorial, uniquely defined

Ricci scalar

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- Curvature

$$R_{MNP}{}^{R} = \partial_M \Gamma^R_{NP} - \partial_N \Gamma^R_{MP} + \Gamma^R_{MQ}\Gamma^Q_{NP} - \Gamma^R_{NQ}\Gamma^Q_{MP}$$

not tensorial (under gen. diffeos)

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tensorial, not uniquely defined

Ricci tensor

$$\mathcal{R}_{\underline{M}\overline{N}} = \mathcal{R}_{\underline{MK}\overline{N}}{}^{\underline{K}}$$

tensorial, uniquely defined

Ricci scalar

$$\mathcal{R} = \mathcal{R}^{\underline{MN}}{}_{\underline{MN}}$$

scalar, uniquely defined

Gravity

Supergravity

Action $S = \int \sqrt{-g}\, R$

$$\mathrm{S} = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \ldots\right]$$

EOM $R_{\mu\nu} = 0$

Ricci-flat

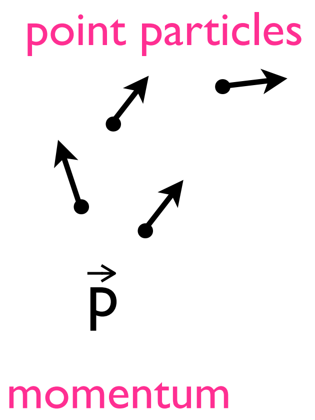
$$\mathcal{R}_{\mu\nu} - \tfrac{1}{4}H_{\mu\lambda\rho}H_{\nu}{}^{\lambda\rho} + 2\nabla_\mu\nabla_\nu\phi + \ldots = 0$$

$$\nabla^\mu\left(\mathrm{e}^{-2\phi}H_{\mu\nu\lambda}\right) + \ldots = 0$$

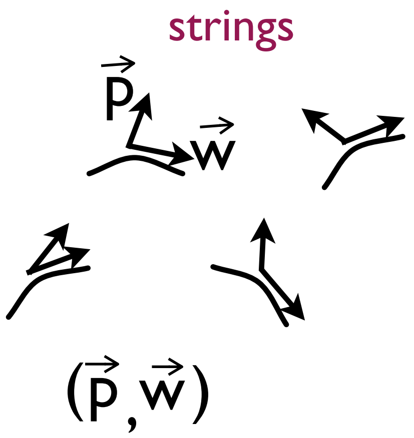
$$\nabla^2\phi - (\nabla\phi)^2 + \tfrac{1}{4}\mathcal{R} - \tfrac{1}{48}H^2 = 0$$

$$\ldots$$

Gravity



Supergravity



Action $S = \int \sqrt{-g}\, R$

$$S = \int e^{-2\phi}\sqrt{-g}\left[R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + ...\right]$$

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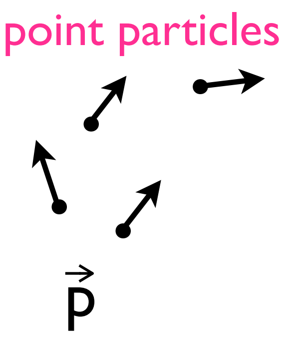
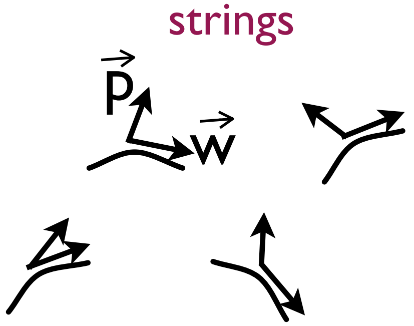


momentum

Supergravity



$(\vec{p},\vec{w}) = P$

$\uparrow$ momentum $\uparrow$ winding charge

Left-moving + Right-moving

Metric + B-field+dilaton

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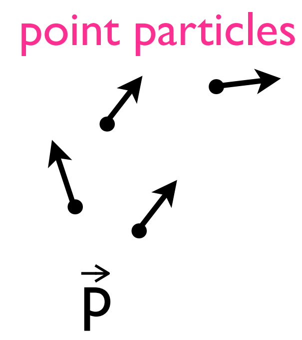
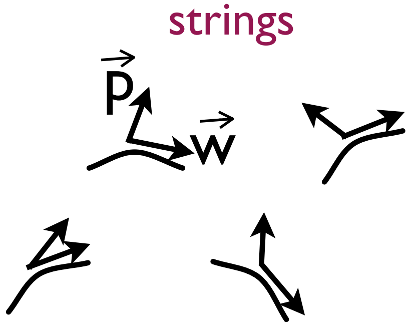
Gravity



momentum

Supergravity



$(\vec{p},\vec{w}) = P$

momentum $\uparrow$ $\uparrow$ winding charge

Do pure geometry on a double space

Left-moving + Right-moving

Metric + B-field+dilaton

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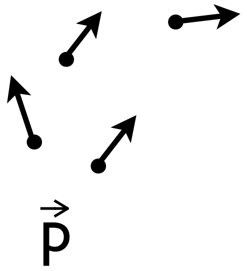
Gravity

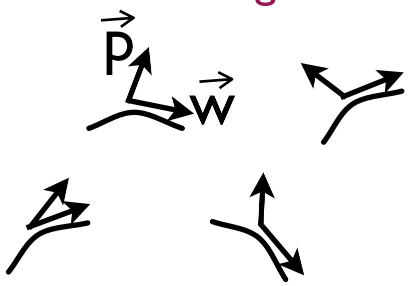
point particles



momentum

Supergravity

strings



$(\vec{p},\vec{w}) = P$

$\uparrow$ momentum $\uparrow$ winding charge

Do pure geometry on a double space

Left-moving + Right-moving

Metric + B-field+dilaton

Action $S = \int \sqrt{-g}\, R$

$$S = \int \sqrt{\mathcal{G}}^{1/20}\, \mathcal{R}$$

EOM $R_{\mu\nu} = 0$

Ricci-flat

$$\mathcal{R}_{\mu\nu} - \tfrac{1}{4} H_{\mu\lambda\rho} H_{\nu}{}^{\lambda\rho} + 2\nabla_\mu \nabla_\nu \phi + \ldots = 0$$

$$\nabla^\mu \left(\mathrm{e}^{-2\phi} H_{\mu\nu\lambda} \right) + \ldots = 0$$

$$\nabla^2 \phi - (\nabla\phi)^2 + \tfrac{1}{4}\mathcal{R} - \tfrac{1}{48} H^2 = 0$$

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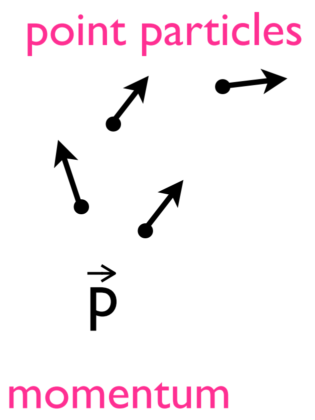
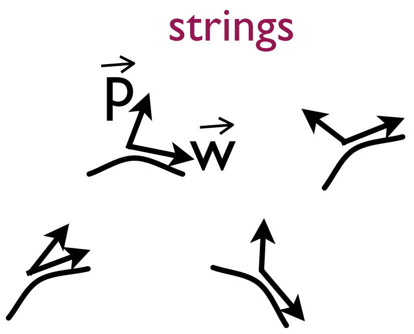


Supergravity



$(\vec{p},\vec{w}) = P$

momentum

winding charge

Do pure geometry on a double space

Left-moving + Right-moving
Metric + B-field+dilaton

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Ricci-flat

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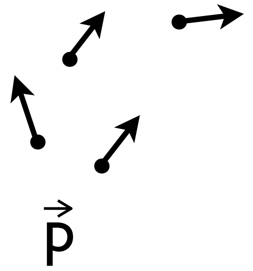
Gravity

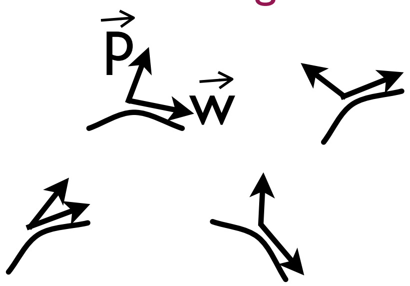
point particles



momentum

Supergravity

strings



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Ricci-flat

$$\mathcal{R}_{\underline{M}\overline{N}} = 0$$

Generalized Ricci-flat

To account for the + ...

To account for the $+ \overset{\text{(RR fields)}}{\ldots}$

To account for the + (RR fields) ...

More charges
in string theory

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

More charges
in string theory

$$\underset{\text{momentum}}{\underset{\uparrow}{\vec{p}}}\,,\ \underset{\substack{\text{string}\\\text{charge}}}{\underset{\uparrow}{\vec{w}}}$$

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$\big(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\vec{\tilde{q}}}\ \big)$$

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

More charges
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$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\vec{\tilde{q}}}\ ) = \vec{P}$$

To account for the + (RR fields) ...

More charges
in string theory

$$(\underset{\substack{\uparrow\\\text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\\text{string}\\\text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\\text{brane}\\\text{charges}}}{\vec{\tilde{q}}}\ ) = \vec{P}$$

Do pure geometry on a larger tangent space

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$\Big(\underset{\text{momentum}}{\underset{\uparrow}{\vec{p}}},\ \underset{\text{string charge}}{\underset{\uparrow}{\vec{w}}},\ \underset{\text{brane charges}}{\underset{\uparrow}{\vec{\tilde{q}}}}\ \Big) = \vec{P}$$

Do pure geometry on a larger tangent space

Cannot do it (yet) for the full 10D theory

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\vec{p}, \vec{w}, \vec{\tilde{q}}\,) = \vec{P}$$

momentum

string
charge

brane
charges

Do pure geometry on a larger tangent space

Cannot do it (yet) for the full 10D theory

Apparently, need ∞-dim. space

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

More charges
in string theory

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane charges}}}{\vec{\tilde{q}}}\ ) = \vec{P}$$

Do pure geometry on a larger tangent space

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Apparently, need ∞-dim. space

Can do it for d≤6

To account for the + (RR fields) …

More charges
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$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec p},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec w},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\vec{\tilde q}}\ ) = \vec P$$

Do pure geometry on a larger tangent space

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Apparently, need ∞-dim. space

Can do it for $d \leq 6$

For $d=6$:

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec p},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec w},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\tilde{\vec q}}\ ) = \vec P$$

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For $d=6$:

$\vec p,\ \vec w$

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

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$$\underset{\substack{\uparrow\\ \text{momentum (6)}}}{\vec{p}}\ , \underset{\substack{\uparrow\\ \text{string}\\ \text{charge (6)}}}{\vec{w}}$$

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To account for the + (RR fields) …

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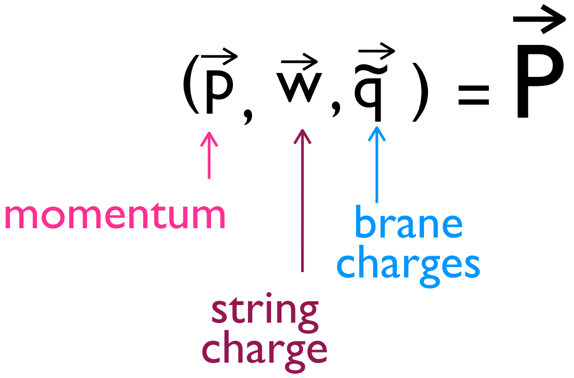

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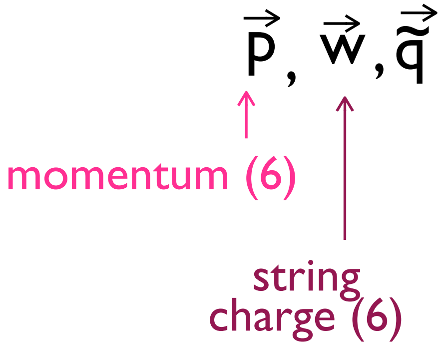

$$\vec{p}, \vec{w}, \vec{\tilde{q}}$$

To account for the + (RR fields) ...

More charges
in string theory

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\tilde{\vec{q}}}\ ) = \vec{P}$$

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Can do it for d≤6

For d=6 :

$$\underset{\substack{\uparrow\\ \text{momentum (6)}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge (6)}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{D1, D3, D5}\\ \text{brane charges}\\ 6+20+6}}{\tilde{\vec{q}}}$$

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\vec{\tilde{q}}}\ ) = \vec{P}$$

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To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
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To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
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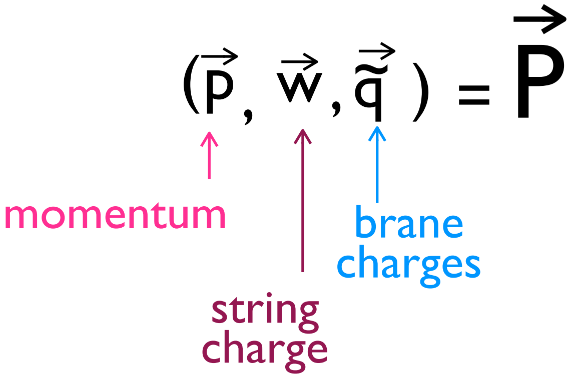
To account for the $+ \overset{\text{(RR fields)}}{\dots}$

More charges
in string theory



$$(\vec{p}, \vec{w}, \vec{\tilde{q}}\,) = \vec{P}$$

momentum

string
charge

brane
charges

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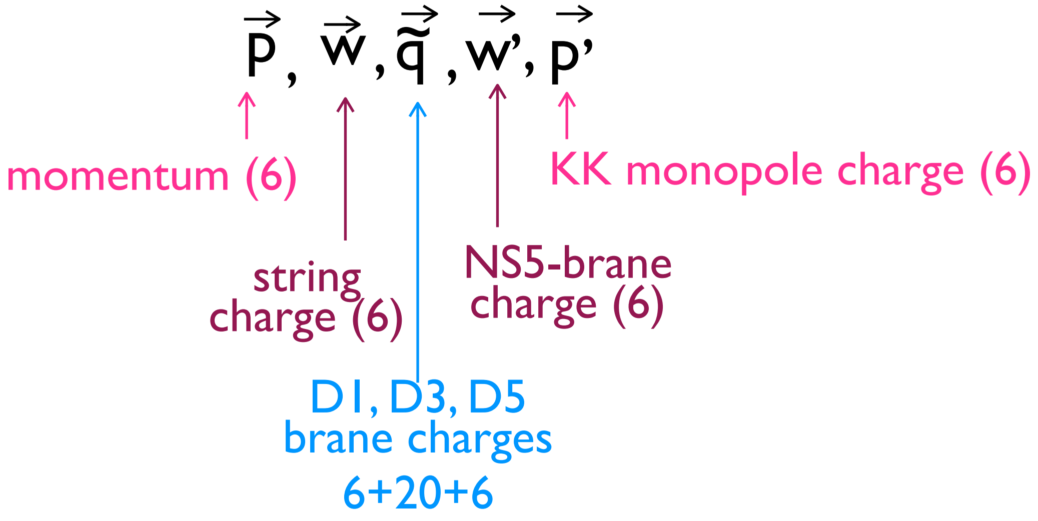

$$\vec{p}, \vec{w}, \vec{\tilde{q}}, \vec{w}', \vec{p}'$$

momentum (6)

string
charge (6)

D1, D3, D5
brane charges
6+20+6

NS5-brane
charge (6)

KK monopole charge (6)

To account for the + $\overset{\text{(RR fields)}}{\dots}$

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D1, D3, D5
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charge (6)

KK monopole charge (6)

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\vec{\tilde{q}}}\ ) = \vec{P}$$

Do pure geometry on a larger tangent space

Cannot do it (yet) for the full 10D theory

Apparently, need ∞-dim. space

Can do it for d≤6

For d=6 :

$$(\underset{\substack{\uparrow\\ \text{momentum (6)}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge (6)}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{D1, D3, D5}\\ \text{brane charges}\\ 6+20+6}}{\vec{\tilde{q}}},\ \underset{\substack{\uparrow\\ \text{NS5-brane}\\ \text{charge (6)}}}{\vec{w}'},\ \underset{\substack{\uparrow\\ \text{KK monopole charge (6)}}}{\vec{p}'}) = P$$

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56D space: fund of E_7

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\vec{p}, \vec{w}, \vec{\tilde{q}}\,) = \vec{P}$$

momentum

string
charge

brane
charges

Do pure geometry on a larger tangent space

Cannot do it (yet) for the full 10D theory

Apparently, need ∞-dim. space

Can do it for d≤6

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$$(\vec{p}, \vec{w}, \vec{\tilde{q}}, \vec{w}', \vec{p}') = P$$

momentum (6)

string
charge (6)

D1, D3, D5
brane charges
6+20+6

NS5-brane
charge (6)

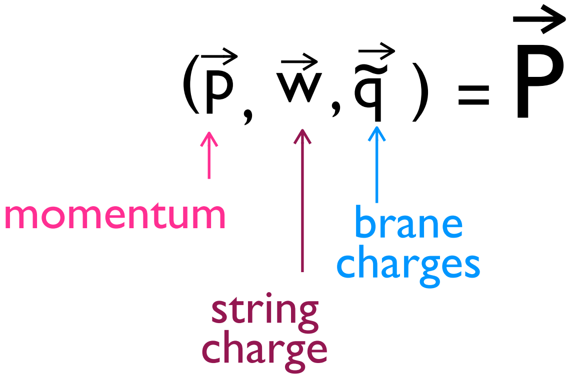
KK monopole charge (6)

56D space: fund of E_7

Everything works analogously to O(d,d)

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

More charges
in string theory



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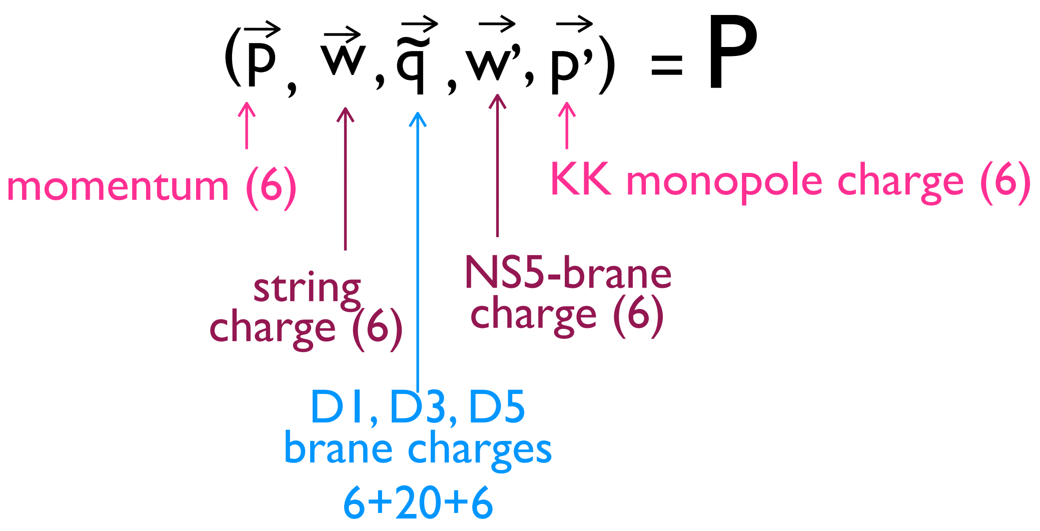

$$(\vec{p}, \vec{w}, \vec{\tilde{q}}, \vec{w}', \vec{p}') = P$$

56D space: fund of E_7

- Metric $\mathcal{G}(g, B, \Phi, C)$

Everything works analogously to O(d,d)

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\underset{\substack{\uparrow\\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow\\ \text{string}\\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow\\ \text{brane}\\ \text{charges}}}{\tilde{\vec{q}}}\ ) = \vec{P}$$

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- Metric $\mathcal{G}(g, B, \Phi, C)$
- Connection

56D space: fund of E_7

Everything works analogously to O(d,d)

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

More charges
in string theory

$$(\underset{\substack{\uparrow \\ \text{momentum}}}{\vec{p}},\ \underset{\substack{\uparrow \\ \text{string} \\ \text{charge}}}{\vec{w}},\ \underset{\substack{\uparrow \\ \text{brane} \\ \text{charges}}}{\vec{\tilde{q}}}\ ) = \vec{P}$$

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56D space: fund of E_7

- Metric $\mathcal{G}(g, B, \Phi, C)$
- Connection
- Curvature

Everything works analogously to O(d,d)

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
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56D space: fund of E_7

- Metric $\mathcal{G}(g, B, \Phi, C)$
- Connection
- Curvature
- Action

Everything works analogously to O(d,d)

To account for the + $\overset{\text{(RR fields)}}{\dots}$

More charges
in string theory

$$(\vec{p},\ \vec{w},\ \vec{\tilde{q}}\ ) = \vec{P}$$

momentum

string
charge

brane
charges

Do pure geometry on a larger tangent space

Cannot do it (yet) for the full 10D theory

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momentum (6)

KK monopole charge (6)

string
charge (6)

NS5-brane
charge (6)

D1, D3, D5
brane charges
6+20+6

56D space: fund of E_7

- Metric $\mathcal{G}\,(g, B, \Phi, C)$
- Connection
- Curvature
- Action
- Equations of motion : generalized Ricci-flat

Everything works analogously to O(d,d)

To account for the + $\overset{\text{(RR fields)}}{\ldots}$

More charges
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$$(\vec{p}\,, \vec{w}, \vec{\tilde{q}}\ ) = \vec{P}$$

momentum

string
charge

brane
charges

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$$(\vec{p}\,, \vec{w}, \vec{\tilde{q}}\,, \vec{w}', \vec{p}') = P$$

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string
charge (6)

D1, D3, D5
brane charges
6+20+6

NS5-brane
charge (6)

KK monopole charge (6)

56D space: fund of E_7

- Metric $\mathcal{G}$ (g, B, Φ, C)
- Connection
- Curvature
- Action
- Equations of motion : generalized Ricci-flat
  (include RR fields)

Everything works analogously to O(d,d)

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

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Generalized geometry → doubled tangent space Hitchin 2001

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single (10D) space (coordinates)

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

Generalized geometry → doubled tangent space

single (10D) space (coordinates)

Hitchin 2001

Doubled geometry

Hull et al 2006

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

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single (10D) space (coordinates)

Doubled geometry → doubled space Hull et al 2006

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Hitchin 2001

Doubled geometry → doubled space

coordinates dual to momentum (x) & winding ($\tilde{x}$)

Hull et al 2006

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single (10D) space (coordinates)

Doubled geometry → doubled space

Hull et al 2006

coordinates dual to momentum (x) & winding $(\tilde{x})$

∂_μ

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single (10D) space (coordinates)

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coordinates dual to momentum (x) & winding $(\tilde{x})$

$\partial_\mu \rightarrow \partial_M$

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Generalized geometry → doubled tangent space

single (10D) space (coordinates)

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Doubled geometry → doubled space

coordinates dual to momentum (x) & winding ($\tilde{x}$)

$\partial_\mu \to \partial_M$ subject to constraint

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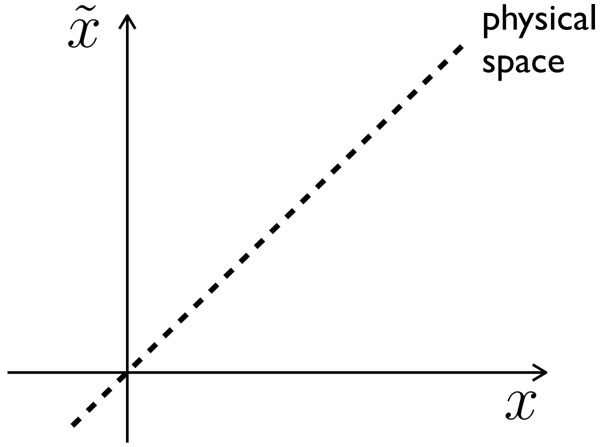

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single (10D) space (coordinates)

Hitchin 2001

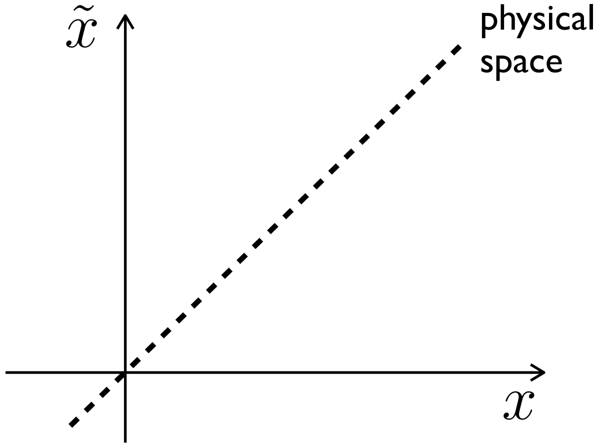
Doubled geometry → doubled space

coordinates dual to momentum (x) & winding ($\tilde{x}$)

$\partial_\mu \to \partial_M$ subject to constraint $\partial_M \partial^M = 0$

Physical space can vary as we move around

Hull et al 2006



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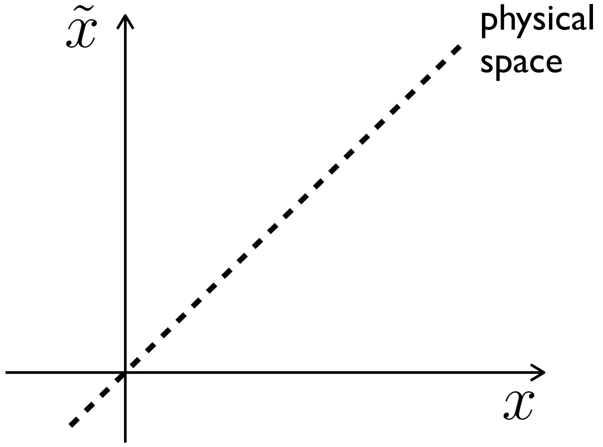


x $\tilde{x}$

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

Generalized geometry → doubled tangent space Hitchin 2001

single (10D) space (coordinates)

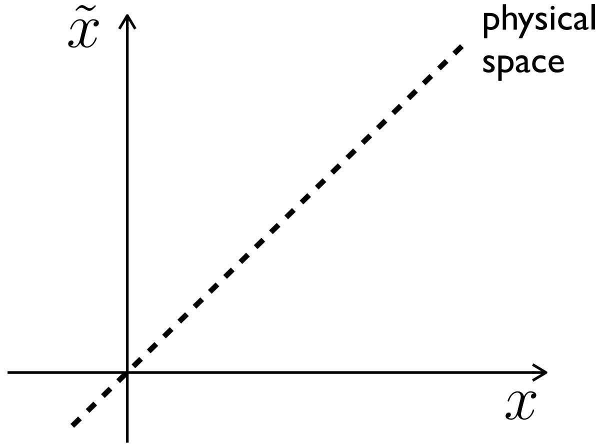
Doubled geometry → doubled space Hull et al 2006

coordinates dual to momentum (x) & winding ($\tilde{x}$)

$\partial_\mu \rightarrow \partial_M$ subject to constraint $\partial_M \, \partial^M = 0$

Physical space can vary as we move around

$$x \xleftrightarrow[\text{T-duality}]{} \tilde{x}$$



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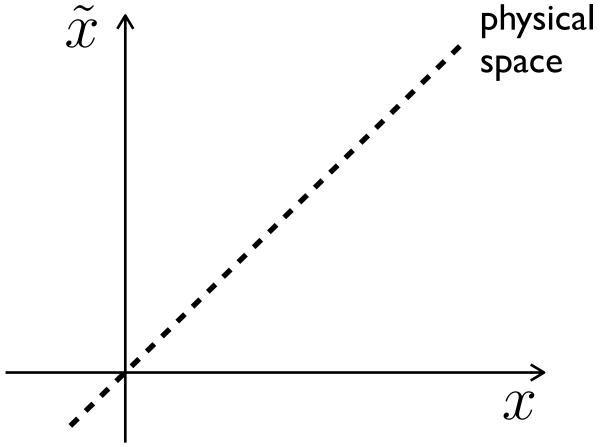
Physical space can vary as we move around

Hull et al 2006

$$x \xleftrightarrow[\text{T-duality}]{} \tilde{x}$$

$$\vec{p} \longleftrightarrow \vec{w}$$



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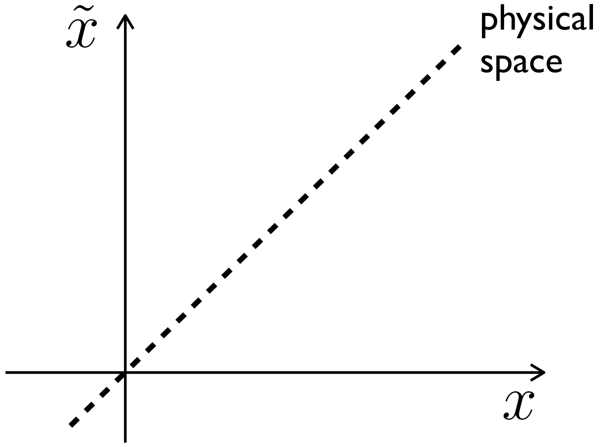

$$x \xleftrightarrow[\text{T-duality}]{\hspace{2cm}} \tilde{x}$$

$$\vec{p} \longleftrightarrow \vec{w}$$

$$E = \frac{n}{R}$$

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

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single (10D) space (coordinates)

Hitchin 2001

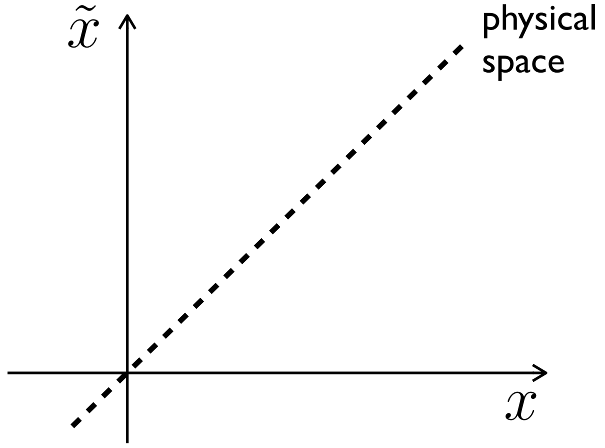
Doubled geometry → doubled space

coordinates dual to momentum (x) & winding ($\tilde{x}$)

Hull et al 2006

$\partial_\mu \rightarrow \partial_M$ subject to constraint $\partial_M \, \partial^M = 0$

Physical space can vary as we move around



$$x \longleftrightarrow \tilde{x}$$

T-duality

$$\vec{p} \longleftrightarrow \vec{w}$$

$E = \frac{n}{R}$ $E = m\tilde{R}$

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

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Doubled geometry → doubled space

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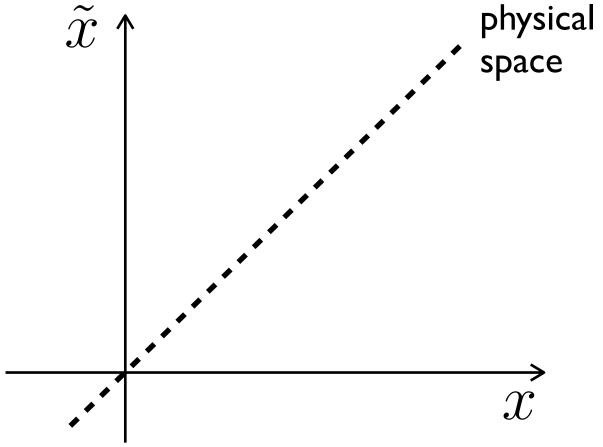
Physical space can vary as we move around

Hull et al 2006



$$x \xleftrightarrow[\text{T-duality}]{} \tilde{x}$$

$$\vec{p} \longleftrightarrow \vec{w}$$

$E = \frac{n}{R}$ $\quad$ $E = m\tilde{R}$

n

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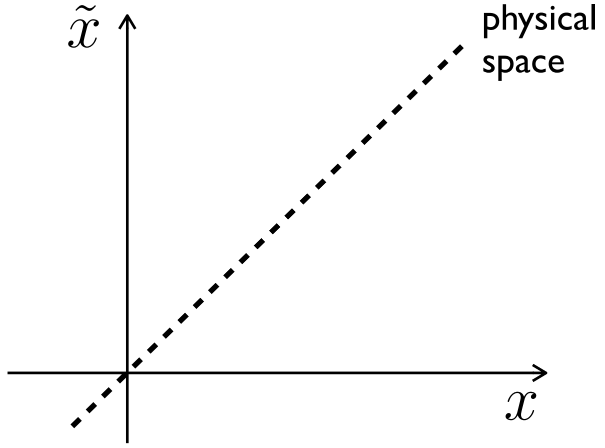

$$x \longleftrightarrow \tilde{x}$$

T-duality

$$\vec{\mathrm{p}} \longleftrightarrow \vec{\mathrm{w}}$$

$$E = \tfrac{n}{R} \qquad E = m\tilde{R}$$

$$n \qquad m.$$

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Generalized geometry → doubled tangent space

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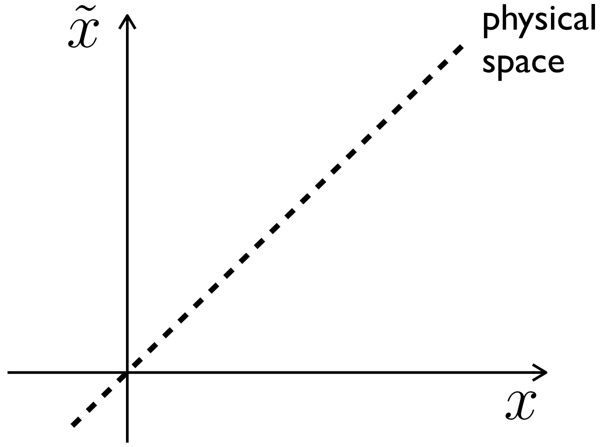

$$x \longleftrightarrow \tilde{x}$$

T-duality

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$$n\,,R \qquad\qquad m.$$

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Generalized geometry → doubled tangent space

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Hull et al 2006

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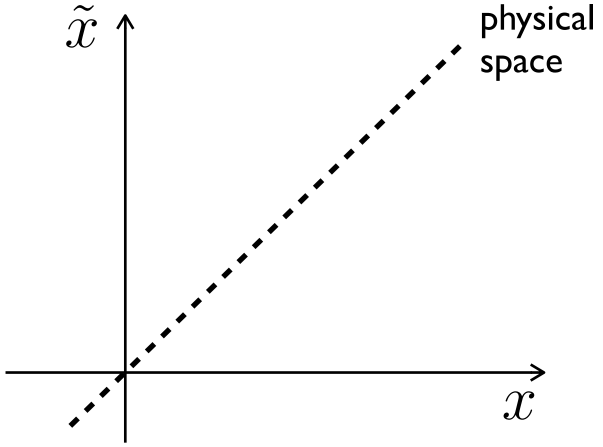

$$x \xleftrightarrow[\text{T-duality}]{} \tilde{x}$$

$$\vec{p} \longleftrightarrow \vec{w}$$

$$E = \frac{n}{R} \qquad\qquad E = m\tilde{R}$$

$$n\,,R \qquad\qquad m_{.},\ \frac{1}{R}$$

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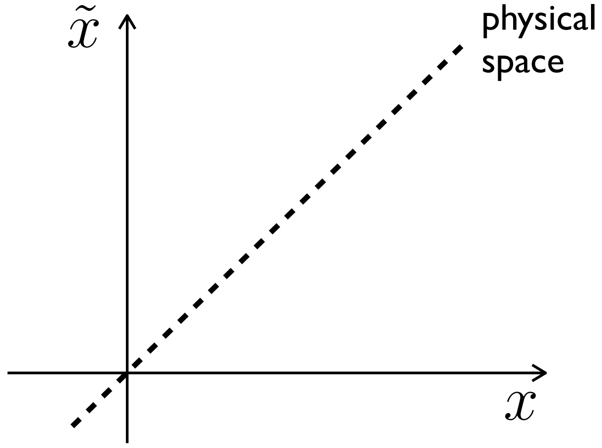

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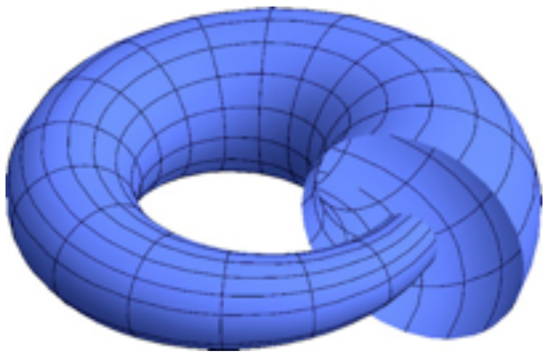
$$n\,, R \qquad m_{.},\ \frac{1}{R}$$

physical space



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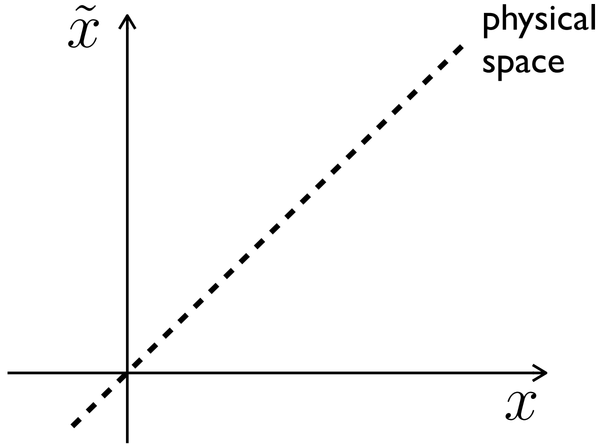
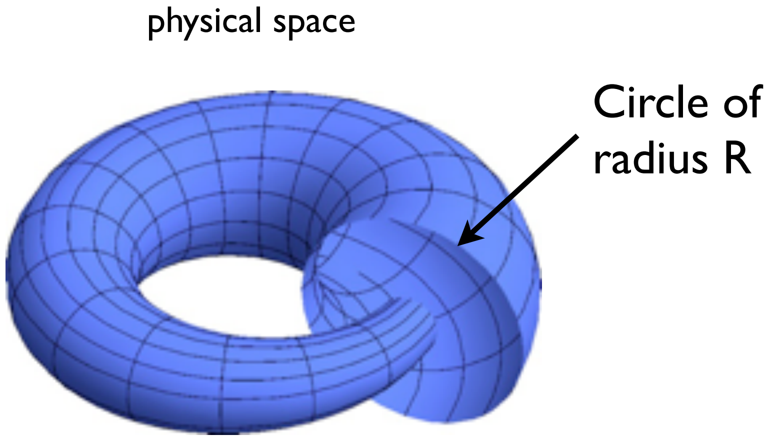
Physical space can vary as we move around

Hull et al 2006





$$x \longleftrightarrow \tilde{x}$$

T-duality

$$\vec{p} \longleftrightarrow \vec{w}$$

$E = \frac{n}{R}$ $E = m\tilde{R}$

n , R $m., \frac{1}{R}$

Generalizing the geometry of space-time: from gravity to supergravity **and beyond...?**

Generalized geometry → doubled tangent space

single (10D) space (coordinates)

Hitchin 2001

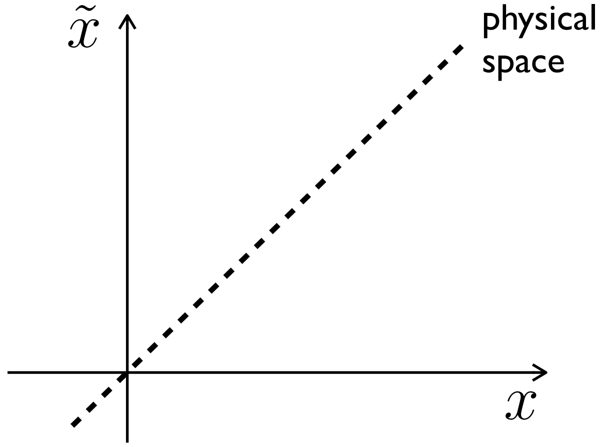
Doubled geometry → doubled space

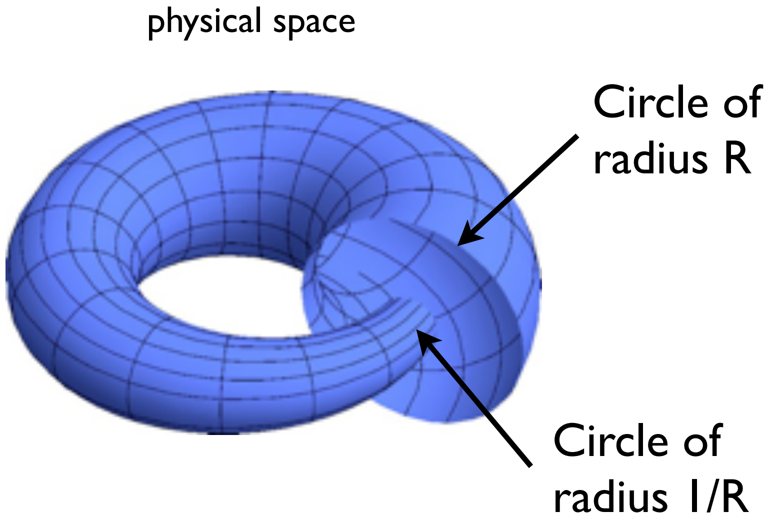
coordinates dual to momentum (x) & winding ($\tilde{x}$)

Hull et al 2006

$\partial_\mu \rightarrow \partial_M$ subject to constraint $\partial_M \, \partial^M = 0$

Physical space can vary as we move around





$$x \longleftrightarrow \tilde{x}$$

T-duality

$$\vec{p} \longleftrightarrow \vec{w}$$

$$E = \frac{n}{R} \qquad E = m\tilde{R}$$

$$n\,,R \qquad m_{.},\ \frac{1}{R}$$

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Generalized geometry → doubled tangent space

single (10D) space (coordinates)

Hitchin 2001

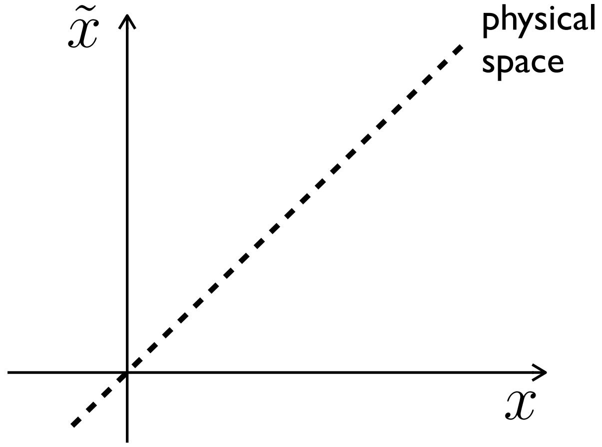
Doubled geometry → doubled space

coordinates dual to momentum (x) & winding ($\tilde{x}$)

Hull et al 2006

$\partial_\mu \rightarrow \partial_M$ subject to constraint $\partial_M \, \partial^M = 0$

Physical space can vary as we move around



$$x \longleftrightarrow \tilde{x}$$

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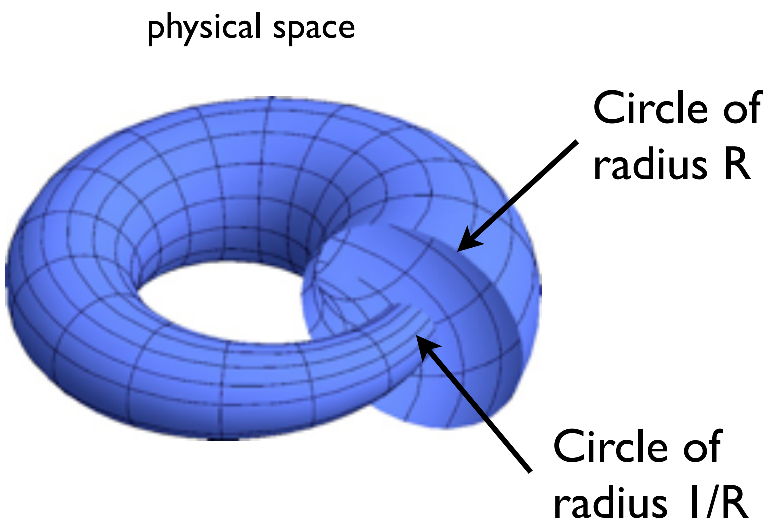

Non-geometric background

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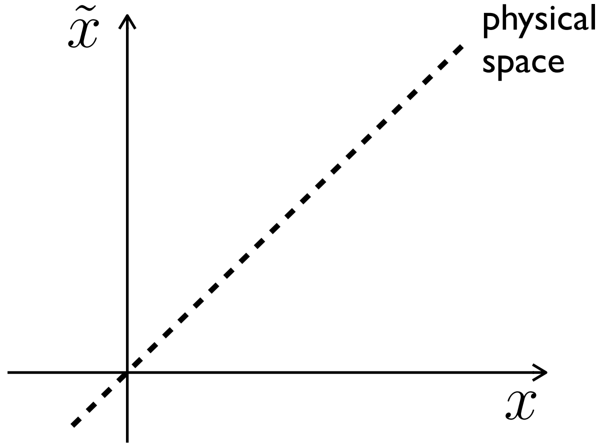
$\partial_\mu \rightarrow \partial_M$ subject to constraint $\partial_M \, \partial^M = 0$

Physical space can vary as we move around



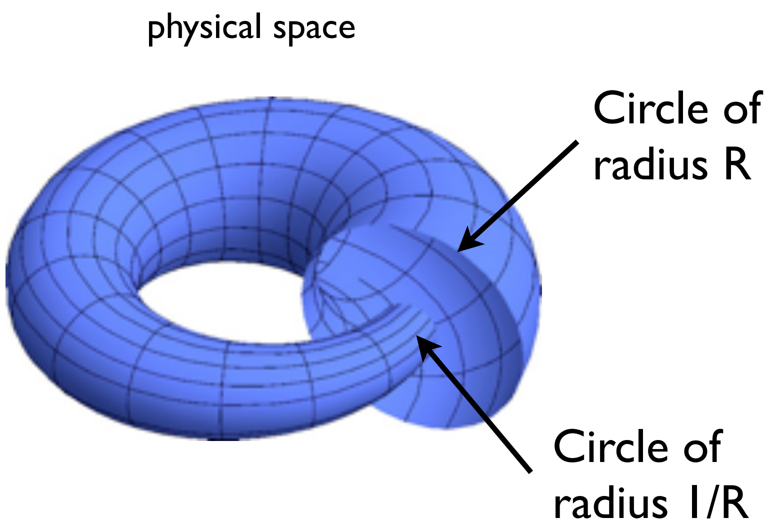
$$x \longleftrightarrow \tilde{x}$$

T-duality

$$\vec{p} \longleftrightarrow \vec{w}$$

$$E = \frac{n}{R} \qquad E = m\tilde{R}$$

$$n\,, R \qquad m., \frac{1}{R}$$



Non-geometric background

T-fold

Conclusions

Conclusions

gravity
=
geometry

Conclusions

gravity
=
geometry

$$R_{\mu\nu} = 0$$

Conclusions

gravity
=
geometry

$$R_{\mu\nu} = 0$$

2-torus

Ricci-flat 2d

Conclusions

gravity
=
geometry

supergravity

$$R_{\mu\nu} = 0$$

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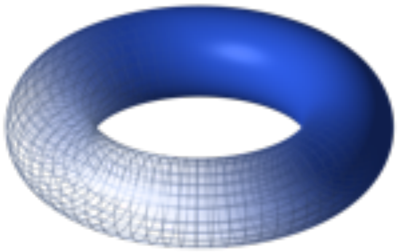

2-torus + B-field

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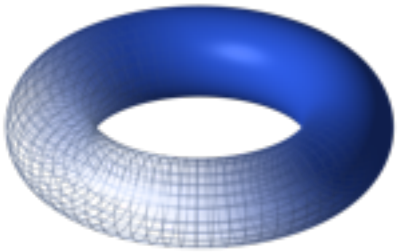
Ricci-flat 2d



2-torus + B-field

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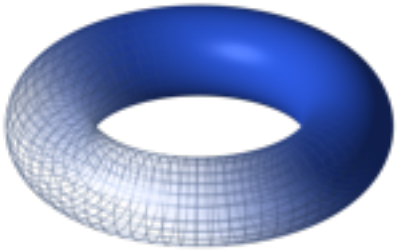
2-torus

Ricci-flat 2d

→



2-torus + B-field

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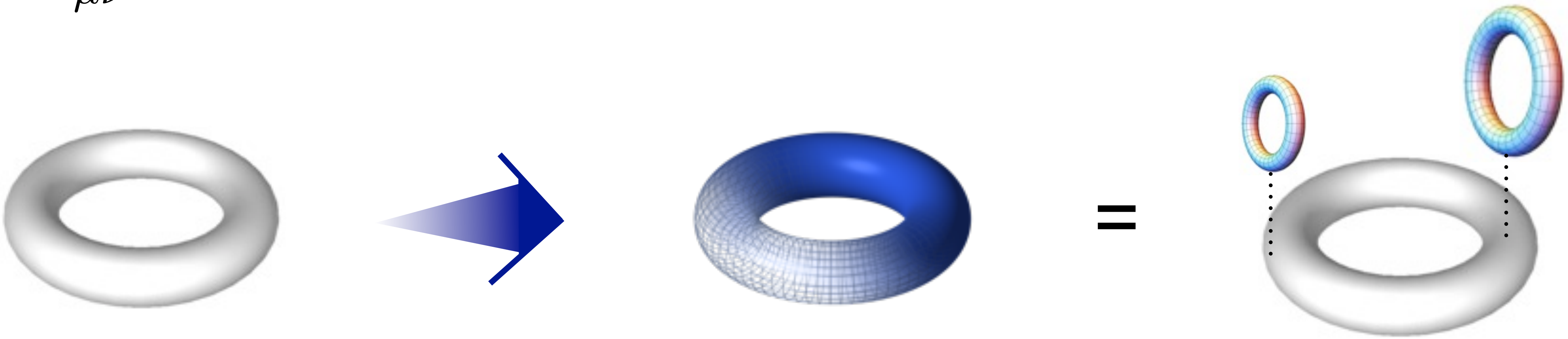

2-torus

Ricci-flat 2d

2-torus + B-field

Ricci-flat 4d

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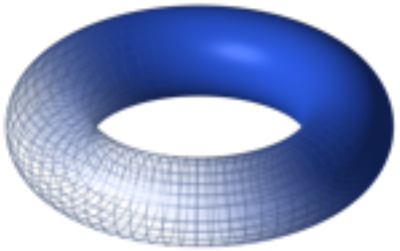
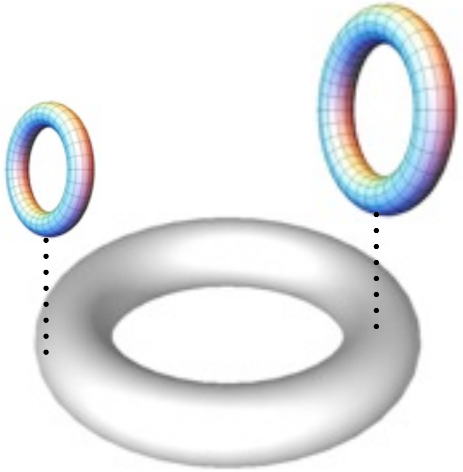


2-torus + B-field

=



Ricci-flat 4d

⋮

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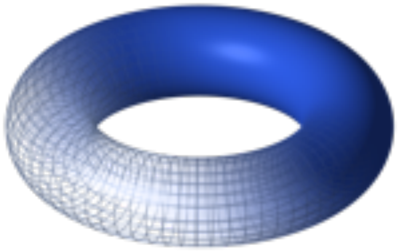

2-torus + B-field

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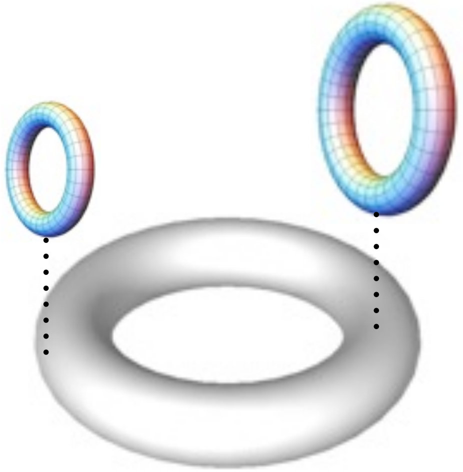
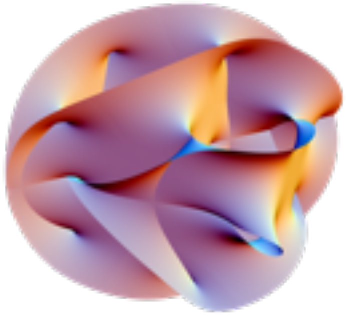


Ricci-flat 4d

⋮



K3

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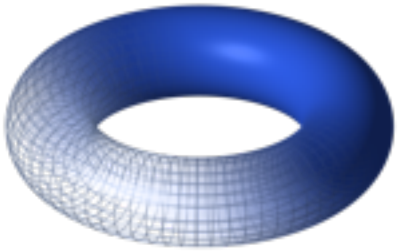
Ricci-flat 2d

→



2-torus + B-field

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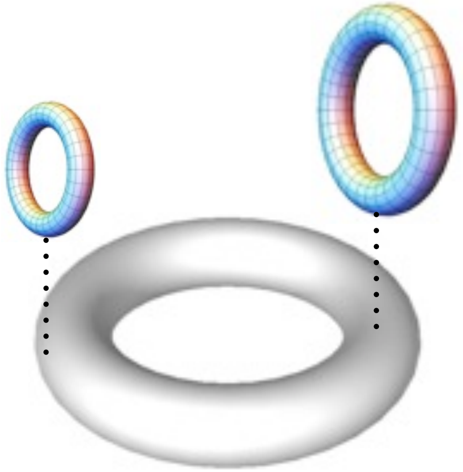
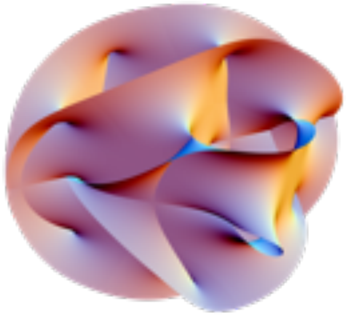


Ricci-flat 4d

beyond supergravity
or geometry...

⋮



K3

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2-torus

Ricci-flat 2d

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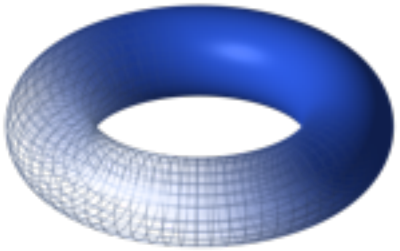

2-torus + B-field

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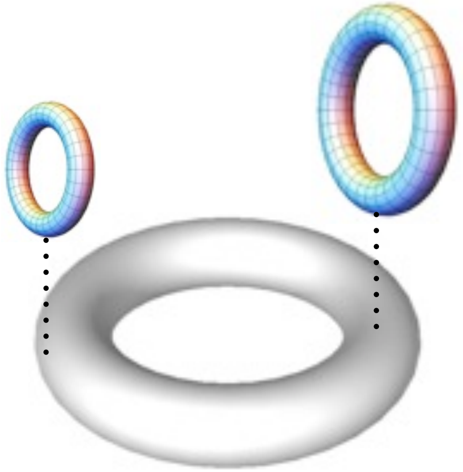

Ricci-flat 4d

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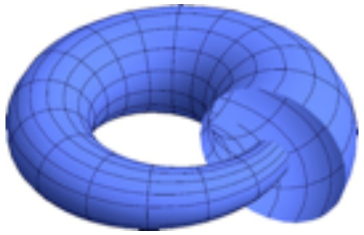

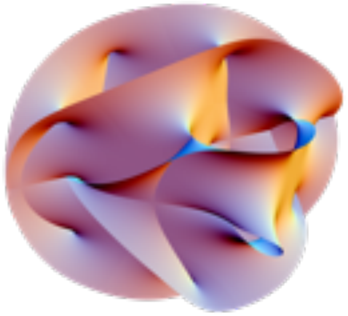

K3

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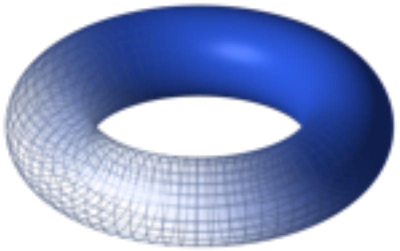

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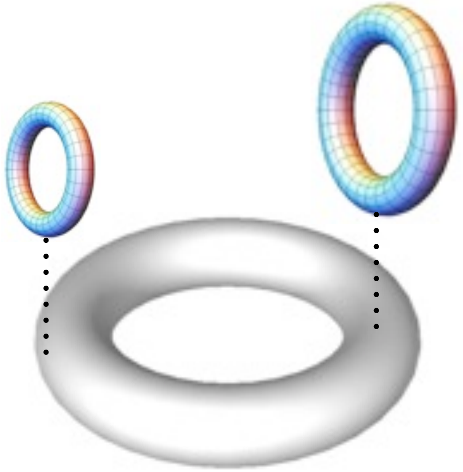

Ricci-flat 4d

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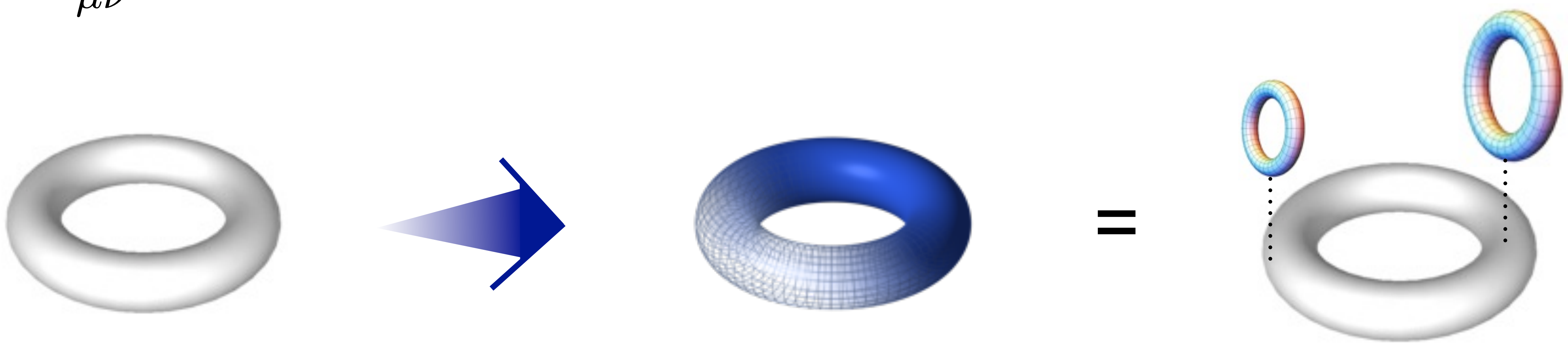

2-torus

Ricci-flat 2d

2-torus + B-field

Ricci-flat 4d

- Purely geometrical description of supergravity

Conclusions

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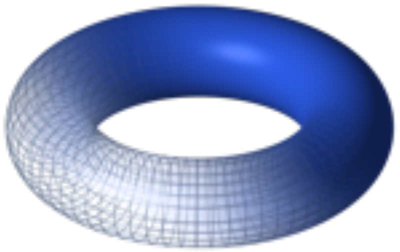
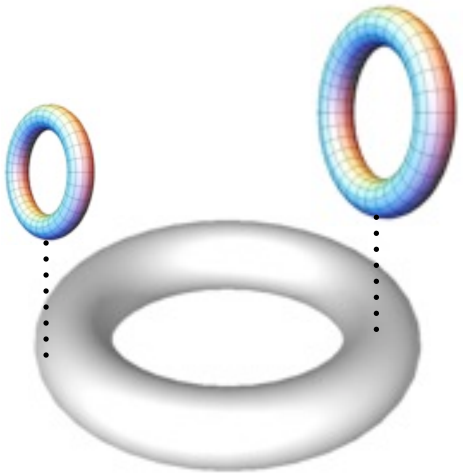
Ricci-flat 2d



2-torus + B-field

=



Ricci-flat 4d

- Purely geometrical description of supergravity
- Purely geometrical description of string theory?