UV Completions of Composite Higgs Models with Partial Compositeness

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Motivation

Address SM Naturalness problem: mechanism to protect Higgs mass.

Features of our Setup

- BSM sector giving rise to a (pseudo)NGB field with the quantum numbers of the Higgs;
- coupling of SM fields to BSM physics through Partial Compositeness;
- purely 4d strongly interacting sector;
- low energy description of strongly coupled physics with the help of supersymmetry, via Seiberg duality for gauge theories.

Higgs as a pNGB

G_f/H_f, $\quad \mathrm{SO}(5)/\mathrm{SO}(4)$ $\quad\Rightarrow\quad$ V(h)=0 at tree level

$\mathrm{SU}(2) \times \mathrm{U}(1) \subseteq G_{SM} \subseteq H_f$

$\Rightarrow\quad$ V(h)≠0

$\mathcal{L} \supseteq \epsilon M \xi, \quad \xi \in \mathrm{SM}$

Partial Compositeness

$$\mathcal{L} = \bar{\xi} i \partial\!\!\!/ \xi + \bar{M}(i\partial\!\!\!/ - m_M)M + \epsilon M \xi + h.c.$$

$$\tan\phi = \frac{\epsilon}{m_M}$$

$$\begin{aligned} \text{light} &= \xi \cos\phi + M \sin\phi \\ \text{heavy} &= -\xi \sin\phi + M \cos\phi \end{aligned}$$

- Flavour hierarchies
- GIM-like mechanism suppressing FCNC and $\not{C}\!\!\not{P}$ processes

The General Setup

$$\mathcal{N}=1 \quad \mathrm{SO}(N) \quad N_f = N$$

$$G_f = \mathrm{SO}(5)\times \mathrm{SU}(N-5) \qquad\qquad W_{el} = m_{ab}Q^aQ^b + \lambda_{IJK}Q^IQ^J\xi^K$$

$$\downarrow$$

$$\mathcal{N}=1 \quad \mathrm{SO}(4)_m \qquad W_{mag} = q_I M^{IJ} q_J - \mu^2 M_{aa} + \epsilon_{IJK}M^{IJ}\xi^K$$

$$\epsilon_{IJK} = \lambda_{IJK}\Lambda, \qquad \mu^2 = -m_Q\Lambda$$

$$F_{M_{ab}} = q_a^n q_b^n - \mu^2 \delta_{ab}$$

$$\langle q_a^n \rangle = \left(\begin{array}{c|c} & 0 \\ \mu \mathbb{1}_4 & 0 \\ & 0 \\ & 0 \end{array} \right) \qquad a = (m, 5),\ m, n = 1, 2, 3, 4$$

$$\mathrm{SO}(4)_m \times \mathrm{SO}(5) \times \mathrm{SU}(N-5) \to \mathrm{SO}(4)_D \times \mathrm{SU}(N-5)$$

6 : along the broken $\mathrm{SO}(4)_m \times \mathrm{SO}(4)$ directions eaten by the magnetic vector bosons;

4 : along the broken $\mathrm{SO}(5)/\mathrm{SO}(4)_D$ directions identified with the Higgs field.

Explicit Soft SUSY Breaking

- Gauginos' masses

  (as in the MSSM)

- SM Sparticles' masses

$$-\mathcal{L}_{\not{SUSY}} = \widetilde{m}_L^2|\widetilde{t}_L|^2 + \widetilde{m}_R^2|\widetilde{t}_R|^2 + \left(\epsilon_L B_L(\xi_L)_{ia} M_{ia} + \epsilon_R B_R(\xi_R)_{ia} M_{ia} + \frac{1}{2}\widetilde{m}_{g,\alpha}\lambda_\alpha\lambda_\alpha + h.c.\right)$$

$\Longrightarrow$ no qualitative change in the spectrum

General breaking:

$$\mathcal{L} \supseteq \widetilde{m}_{1el}^2 Q^{\dagger a} Q^a + \widetilde{m}_{2el}^2 Q^{\dagger i} Q^i + \left(\frac{1}{2}\widetilde{m}_\lambda \lambda^{ab}\lambda^{ab} + h.c.\right)$$

Higgs Potential

$$\sin\frac{h}{f} = s_h$$

$$V = -\gamma s_h^2 + \beta s_h^4 + \delta s_h^4 \log s_h + \mathcal{O}(s_h^6)$$

$$\gamma = \gamma_{tree} + \gamma_g + \gamma_m, \quad \beta = \beta_g + \beta_m, \quad \delta = \delta_g + \delta_m$$

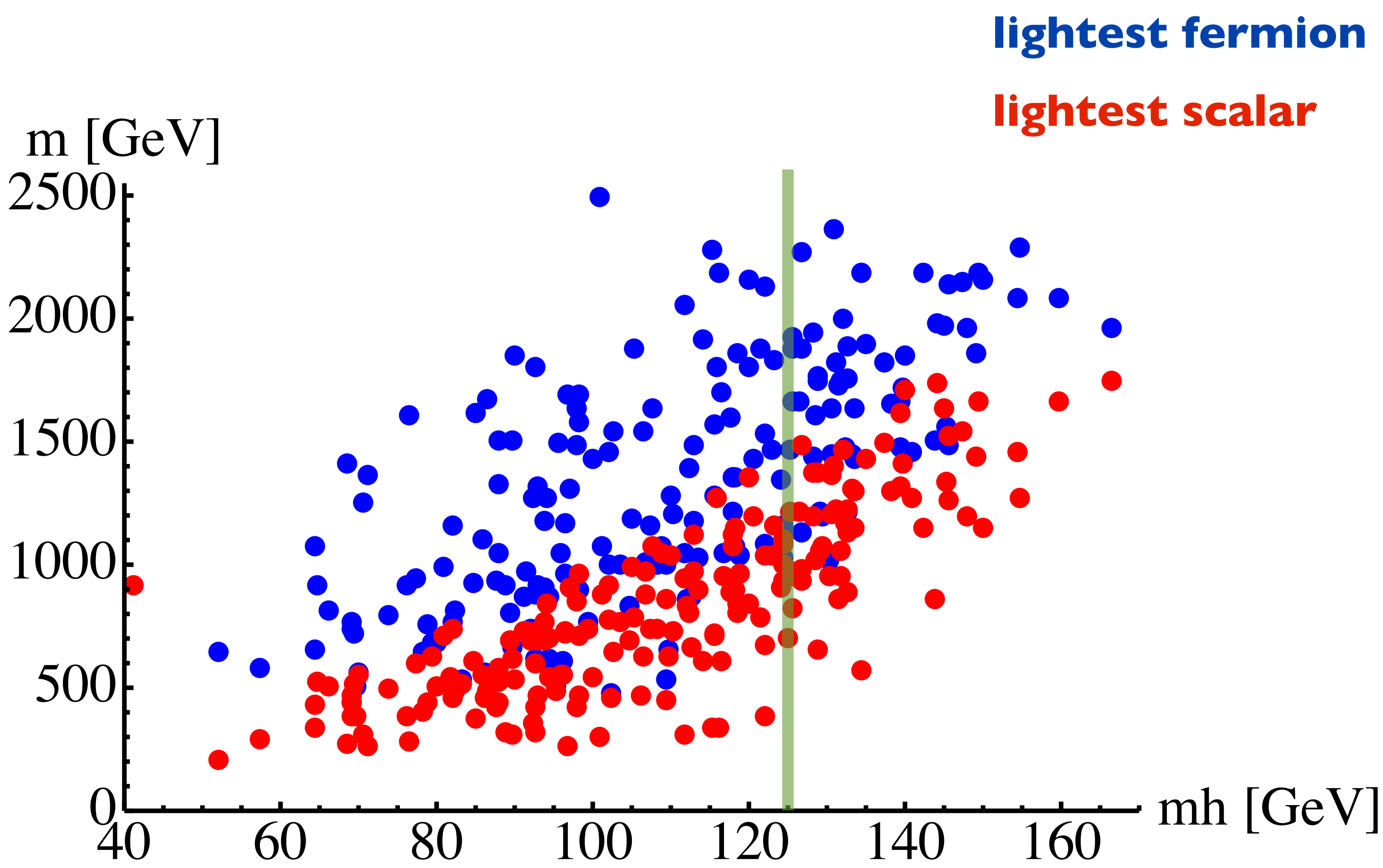
$$\xi = \sin^2\frac{\langle h \rangle}{f} = \frac{\gamma}{2\beta} + O\left(\delta\right)$$

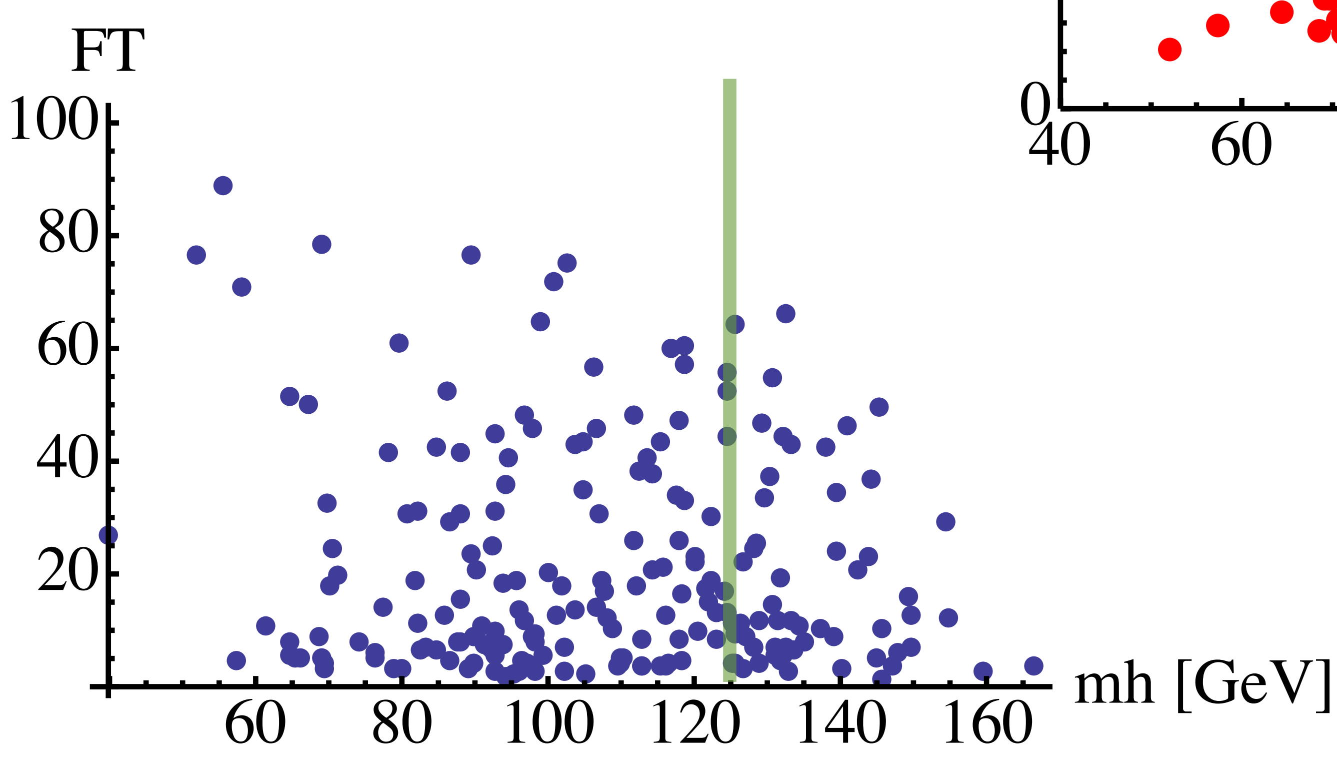
Higgs Potential

Preliminar Results:

$$N = N_f = 11$$

$$G_f = \mathrm{SO}(5) \times \mathrm{SO}(6)$$





Conclusions

- Explicit 4d realization of pNGB Higgs idea
- Partial Compositeness
- SUSY

Possible Future Directions (in progress)

- Higgs Potential
- ~~SUSY~~
- Non top SM fields masses:
  - W deformations
  - K deformations
- Pert. unitar. of $W_L W_L$ scattering (4 x 4 = 1+6+9)

Thank You

Backup Transparencies

Model I

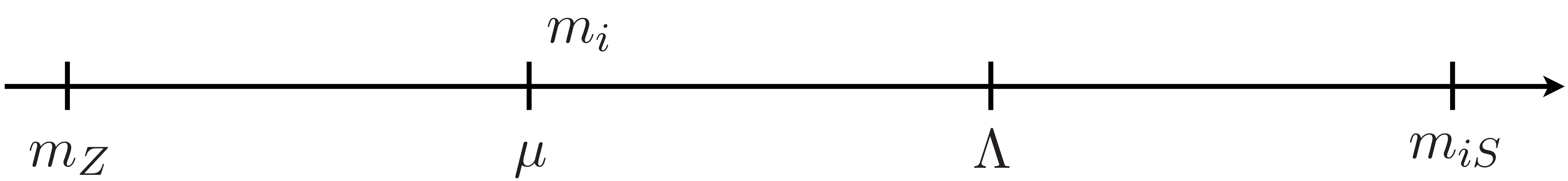
$N = N_f = 11$ $\qquad$ $G_f = \mathrm{SO}(5) \times \mathrm{SO}(6)$

$$W_{el} \supseteq \frac{1}{2} m_{1S} S_{ij}^2 + \lambda_1 Q^i Q^j S_{ij} + \frac{1}{2} m_{2S} S_{ia}^2 + \lambda_2 Q^i Q^a S_{ia}$$

	$\mathrm{SO}(11)_{el}$	$\mathrm{SO}(5)$	$\mathrm{SO}(6)$
Q_i^N	$\mathbf{11}$	$\mathbf{1}$	$\mathbf{6}$
Q_a^N	$\mathbf{11}$	$\mathbf{5}$	$\mathbf{1}$
S_{ij}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{20} \oplus \mathbf{1}$
S_{ia}	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{6}$

	$\mathrm{SO}(4)_{mag}$	$\mathrm{SO}(5)$	$\mathrm{SO}(6)$
q_i^n	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{6}$
q_a^n	$\mathbf{4}$	$\mathbf{5}$	$\mathbf{1}$
M_{ij}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{20} \oplus \mathbf{1}$
M_{ia}	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{6}$
M_{ab}	$\mathbf{1}$	$\mathbf{14} \oplus \mathbf{1}$	$\mathbf{1}$

$$W_{mag} \supset -\frac{1}{2} m_1 M_{ij}^2 - \frac{1}{2} m_2 M_{ia}^2$$



Model II

$N = N_f = 9$

$W_{el} \supseteq \lambda Q^i Q^j S_{ij}$

$G_f = \mathrm{SO}(5) \times \mathrm{SU}(4)$

	$\mathrm{SO}(9)_{el}$	$\mathrm{SO}(5)$	$\mathrm{SU}(4)$
Q_i^N	$\mathbf{9}$	$\mathbf{1}$	$\overline{\mathbf{4}}$
Q_a^N	$\mathbf{9}$	$\mathbf{5}$	$\mathbf{1}$
S_{ij}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$

$W_{mag} \supseteq M M_{ij} S_{ij}$

	$\mathrm{SO}(4)_{mag}$	$\mathrm{SO}(5)$	$\mathrm{SU}(4)$
q_i^n	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{4}$
q_a^n	$\mathbf{4}$	$\mathbf{5}$	$\mathbf{1}$
M_{ia}	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{4}}$
M_{ab}	$\mathbf{1}$	$\mathbf{14} \oplus \mathbf{1}$	$\mathbf{1}$

$\mathrm{SU}(4) \supset \mathrm{SU}(3)_c \times \mathrm{U}(1)_X$

$\mathbf{4} = \mathbf{3}_{2/3} + \mathbf{1}_{-2}$

M_{i5} stays massless:

$M_{\alpha 5},\ \alpha = 6, 7, 8$

M_{95}

RG Flow of Soft Terms

$$\mathcal{L}_{el} = \int d^4\theta \sum_{I=1}^{N_f} Z_I(E) Q_I^\dagger e^{V_{el}} Q_I + \left(\int d^2\theta S(E) W_{el}^\alpha W_{el,\alpha} + h.c. \right)$$

$$Z_I(E) = Z_I^0(E)\Big(1 - \theta^2 B_I(E) - \bar\theta^2 B_I^\dagger(E) - \theta^2\bar\theta^2(\widetilde{m}_I^2(E) - |B_I(E)|^2)\Big)$$

$$S(E) = \frac{1}{g^2(E)} - \frac{i\Theta}{8\pi^2} + \theta^2 \frac{\widetilde{m}_\lambda(E)}{g^2(E)}$$

$$\mathrm{U}(1)^{N_f}$$

$$Q_I \to e^{A_I} Q_I\,, \quad Z_I \to e^{-A_I - A_I^\dagger} Z_I\,, \quad S \to S - \sum_{I=1}^{N_f} \frac{t_I}{8\pi^2} A_I$$

RG Flow of Soft Terms cont'd

$$\Lambda_S = E e^{-\frac{8\pi^2 S(E)}{b}} \,, \quad \hat{Z}_I = Z_I(E) e^{-\int^{R(E)} \frac{\gamma_I(E)}{\beta(R)} dR}$$

$$I = \Lambda_S^\dagger \Big(\prod_{I=1}^{N_f} \hat{Z}_I^{\frac{2t_I}{b}} \Big) \Lambda_S$$

$$\mathcal{L}_{mag} = \int d^4\theta \Big(c_{M_{IJ}} \frac{M_{IJ}^\dagger \hat{Z}_I \hat{Z}_J M_{IJ}}{I} + c_{q_I} q_I^\dagger e^{V_{mag}} \hat{Z}_I^{-1} (\prod_J \hat{Z}_J^{\frac{t_J}{b}}) q_I \Big)$$
$$+ \int d^2\theta \Big(S_m(E) W_m^\alpha W_{m,\alpha} + \frac{q_I M_{IJ} q_J}{\Lambda_S} \Big) + h.c. \,,$$

$$\widetilde{m}^2_{M_{IJ}} = \widetilde{m}^2_I + \widetilde{m}^2_J - \frac{2}{b} \sum_{K=1}^{N_f} \widetilde{m}^2_K \,, \qquad \widetilde{m}^2_{q_I} = -\widetilde{m}^2_I + \frac{1}{b} \sum_{K=1}^{N_f} \widetilde{m}^2_K$$

Road to Higgs Potential

$$M_{ab} \to (UMU^t)_{ab}, \qquad \psi_{M_{ab}} \to (U\psi_M U^t)_{ab}$$

$$M_{ia} \to U_{ab} M_{ib}, \qquad \psi_{M_{ia}} \to U_{ab}\psi_{M_{ib}}$$

1205.0770

$$\mathcal{L}_{f,0} = \bar{q}_L i\not{D} q_L + \bar{t}_R i\not{D} t_R + \sum_{i=1}^{N_S} \bar{S}_i (i\not{\nabla} - m_{iS}) S_i + \sum_{j=1}^{N_Q} \bar{Q}_j (i\not{\nabla} - m_{iQ}) Q_j +$$
$$\sum_{i=1}^{N_S} \left(\frac{\epsilon^i_{tS}}{\sqrt{2}} \bar{\xi}_R P_L U S_i + \epsilon^i_{qS} \bar{\xi}_L P_R U S_i \right) + \sum_{j=1}^{N_Q} \left(\frac{\epsilon^j_{tQ}}{\sqrt{2}} \bar{\xi}_R P_L U Q_i + \epsilon^j_{qQ} \bar{\xi}_L P_R U Q_i \right) + h.c.$$

model I

$$\epsilon_{tS} = \epsilon_R\,, \;\; \epsilon^1_{tQ} = \epsilon_R \cos\omega\,, \;\; \epsilon^2_{tQ} = \epsilon_R \sin\omega\,,$$
$$\epsilon_{qS} = \frac{\epsilon_L}{\sqrt{2}}\,, \;\; \epsilon^1_{qQ} = \frac{\epsilon_L}{\sqrt{2}} \cos\omega\,, \;\; \epsilon^2_{qQ} = \frac{\epsilon_L}{\sqrt{2}} \sin\omega\,.$$

model II

$$\epsilon_{qS} = \epsilon_{qQ} = \epsilon_t$$

Bottom Mass

$$\lambda_{ab}\xi_L Q_a Q_b \xi_R \longrightarrow \epsilon_{ab}\xi_L M_{ab}\xi_R$$

$$\Delta\mathcal{L} \sim \bar{b}_R b_L h \frac{\Lambda}{\Lambda_L}(\langle M_{nn}\rangle - \langle M_{55}\rangle)$$

Vacuum Stability

$$M_{ab} = X\,\delta_{ab}\,, \quad M_{ij} = Y\,\delta_{ij}$$

$$W = 2\Lambda^{-\frac{5}{2}}(\det M)^{\frac{1}{2}} - \mu^2 M_{aa} - \frac{1}{2}m_1 M_{ij}^2 - \frac{1}{2}m_2 M_{ia}^2$$

$$\epsilon = \frac{\mu}{\Lambda}, \quad m_1 = \Lambda\,\epsilon^{\kappa} \qquad\qquad \frac{2}{3} < \kappa \leq 1$$

$$S_b \sim \frac{|X|^4}{V_{Max}} \sim \epsilon^{-\frac{16}{3}+2\kappa} \gtrsim \epsilon^{-\frac{10}{3}}$$

Mixing Terms

$$\lambda_t \xi^{ia} Q_i Q_a + \lambda_\phi \phi^{ia} Q_i Q_a \quad \longrightarrow \quad \epsilon_t \xi^{ia} M_{ia} + \epsilon_\phi \phi^{ia} M_{ia}$$

$$\xi^{ia} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L^1 & b_L^2 & b_L^3 & 0 \\ -ib_L^1 & -ib_L^2 & -ib_L^3 & 0 \\ t_L^1 & t_L^2 & t_L^3 & 0 \\ it_L^1 & it_L^2 & it_L^3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{2/3} , \qquad \phi^{ia} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi^c \end{pmatrix}_{-2}$$

Explicit SUSY Breaking

$$-\mathcal{L}_{\not{SUSY}} = \widetilde{m}_L^2|\widetilde{t}_L|^2 + \widetilde{m}_\psi^2|\widetilde{\psi}|^2 + (\epsilon_L B_L(\xi_L)_{ia} M_{ia} + \frac{1}{2}\widetilde{m}_{g,\alpha}\lambda_\alpha\lambda_\alpha + h.c.)$$
$$+ \widetilde{m}_1^2|M_{ia}|^2 + \widetilde{m}_2^2|M_{ab}|^2 + \widetilde{m}_3^2|q_i|^2 - \widetilde{m}_4^2|q_a|^2 \,,$$

$$\mathcal{L} \supseteq \widetilde{m}_{1el}^2 Q^{\dagger a} Q^a + \widetilde{m}_{2el}^2 Q^{\dagger i} Q^i \qquad \frac{\widetilde{m}_{2el}^2}{\widetilde{m}_{1el}^2} > \frac{8}{5}$$

$$\langle q_m^n \rangle = \delta_m^n \mu \to \delta_m^n \sqrt{\mu^2 + \frac{1}{2}\widetilde{m}_4^2} \equiv \delta_m^n \widetilde{\mu}$$

$$W_{el} \supseteq m_Q Q^a Q^a \qquad m_Q \to m_Q(1 + \theta^2 B_m)$$

$$\langle q_m^n \rangle,\ \langle M_{mn} \rangle,\ \langle M_{55} \rangle \neq 0 \qquad \mathrm{Re} q_5^n, \quad \mathrm{Re} M_{5n}$$

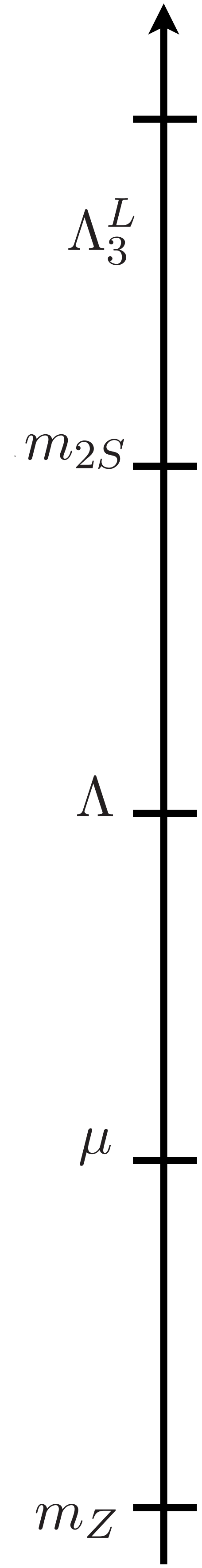
Landau Poles model I

$$\Lambda_3^L = m_{2S}\, \exp\left(\frac{2\pi}{21\alpha_3(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{-\frac{1}{3}}\left(\frac{\mu}{\Lambda}\right)^{\frac{2}{7}}\left(\frac{\Lambda}{m_{2S}}\right)^{\frac{16}{21}},$$

$$\Lambda_2^L = m_{2S}\, \exp\left(\frac{2\pi}{17\alpha_2(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{-\frac{19}{102}}\left(\frac{\mu}{\Lambda}\right)^{\frac{22}{17}}\left(\frac{\Lambda}{m_{2S}}\right)^{\frac{11}{17}},$$

$$\Lambda_1^L = m_{2S}\, \exp\left(\frac{2\pi}{91\alpha_1(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{\frac{41}{546}}\left(\frac{\mu}{\Lambda}\right)^{\frac{336}{546}}\left(\frac{\Lambda}{m_{2S}}\right)^{\frac{215}{273}}.$$

$$\Lambda_3^L \sim 10^2 - 10^3 \text{ TeV}$$



Landau Poles model II

$$\Lambda_3^L = \Lambda \, \exp\left(\frac{\pi}{2\alpha_3(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{-\frac{7}{4}}\left(\frac{\mu}{\Lambda}\right)^{\frac{1}{4}},$$

$$\Lambda_2^L = \Lambda \, \exp\left(\frac{2\pi}{9\alpha_2(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{-\frac{19}{54}}\left(\frac{\mu}{\Lambda}\right)^{2},$$

$$\Lambda_1^L = \Lambda \, \exp\left(\frac{6\pi}{305\alpha_1(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{\frac{41}{610}}\left(\frac{\mu}{\Lambda}\right)^{\frac{236}{305}}$$

$$\Lambda_1^L \sim 10^3 \text{ TeV}$$

$$\mathrm{SU}(4) \supset \mathrm{SU}(3)_c \times \mathrm{U}(1)_X$$

$$\mathbf{4} = \mathbf{3}_{2/3} + \mathbf{1}_{-2}$$

$$\mathbf{10} = \mathbf{1}_2 + \mathbf{3}_{2/3} + \mathbf{6}_{-2/3}$$

Higgs as a pNGB

$$G_f/H_f, \qquad SU(2)\times U(1) \subseteq H_f$$

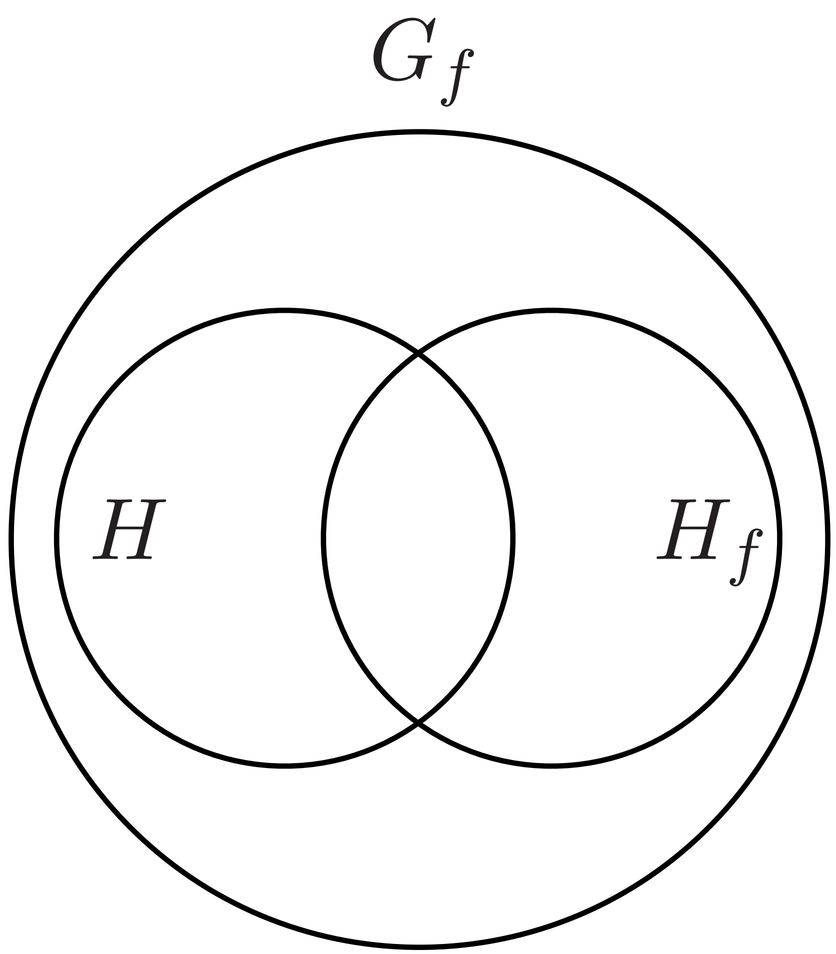
$$G_f = SO(5)\times U(1)_X$$

$$\downarrow$$

$$H_f = SO(4)\times U(1)_X$$

$$Y = T_{3R} + X$$

$$\Lambda_{NP} = \Lambda \approx 4\pi f$$



SM interactions $\Rightarrow$ V(h)

$$\xi \equiv \frac{v^2}{f^2}$$

Partial Compositeness

$$\mathcal{L} = \bar{\psi}_L \, i \, \partial\!\!\!/ \, \psi_L + \bar{\chi} \, (i \, \partial\!\!\!/ - m) \, \chi + \Delta_L \bar{\psi}_L \chi_R + h.c.$$

$$\tan \varphi_L = \frac{\Delta_L}{m}$$

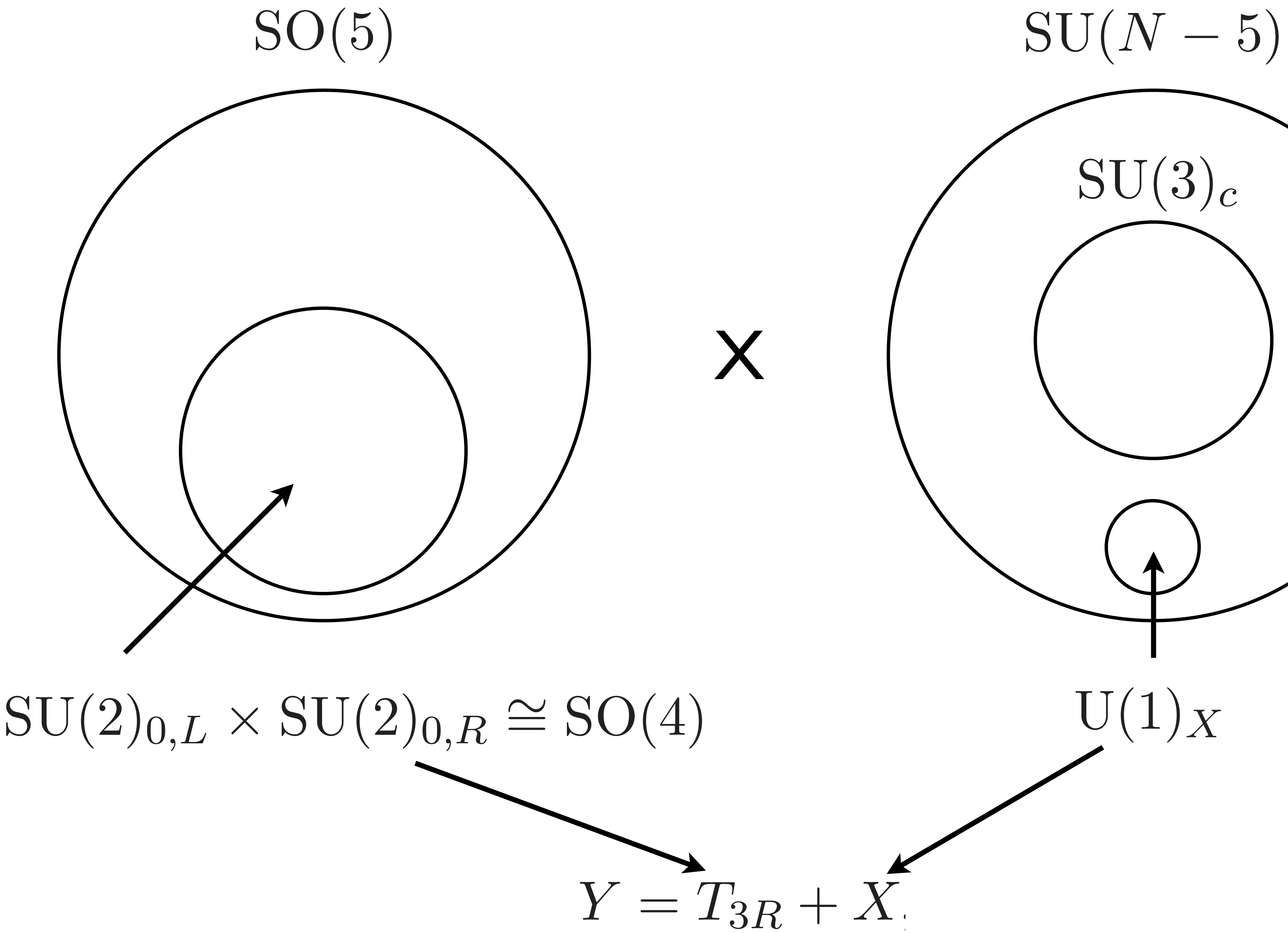
$$|\text{light}\rangle = \cos \varphi_L |\psi\rangle + \sin \varphi_L |\chi\rangle$$

$$|\text{heavy}\rangle = - \sin \varphi_L |\psi\rangle + \cos \varphi_L |\chi\rangle$$

$$\mathcal{L} \supseteq \bar{\chi} Y_* H \tilde{\chi} + h.c. \qquad \Rightarrow \qquad y = Y_* \sin \varphi_L \sin \varphi_R$$

- Flavour hierarchies
- GIM-like mechanism suppressing FCNC and $\not{CP}$ processes

SM Gauge Group



Higgs Potential

Preliminar Results:

plot??

```
ListPlot[{Data[[All, {imh, imLMF}]], Data[[All, {imh, imLMS}]]}, AxesLabel -> {"mh [GeV]", "m [GeV]"}, PlotRange -> {{40, 170}, {0, 2550}}, AxesStyle -> Thick,
LabelStyle -> "Large",PlotStyle -> {Directive[PointSize[0.02], Blue], Directive[PointSize[0.02], Red]}, ImageSize -> 600, PlotLegends -> SwatchLegend[{Style
["Lightest Fermion", Blue, Large], Style["Lightest Scalar", Red, Large]}, LegendMarkers -> "Bubble"]]
```

```
Data = ToExpression[Import["/Users/albertoparolini/Dropbox/Higgs potential in susy\compositeness/susy chm/data/DataAllRangeXi01blind.dat", "Table"]];
```

plot??

```
ListPlot[Data[[All, {imh, iFT}]], AxesLabel -> {"mh [GeV]", "FT"}, AxesStyle -> Thick, LabelStyle -> "Large", PlotStyle -> PointSize[0.02],ImageSize -> 500]
```

Outline

- Introduction
- The General Setup
- Explicit Realizations
- Comparison with Bottom-up Approaches
- Higgs Potential
- Conclusions

Motivation

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em} \qquad H(x) = \frac{1}{\sqrt{2}} e^{i\sigma^a \chi^a(x)/v} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\delta m^2 \sim \frac{\#}{16\pi^2} \Lambda_{NP}^2, \qquad \Lambda_{NP} \sim M_{Pl}$$

UV free gauge theory $\longrightarrow$ Dimensional Transmutation $\longrightarrow$ $\Lambda_{NP} \ll M_{Pl}$ naturally

Seiberg Duality for $\mathcal{N} = 1$ SO(N) SQCD

	$SO(N)_g$	$SU(N_f)$	$U(1)_R$
Q_I^N	$\mathbf{N}$	$\mathbf{N_f}$	$\frac{(N_f - N + 2)}{N_f}$

$$b = 3(N-2) - N_f$$

$$\Lambda_{el} = E \exp\left(-\frac{8\pi^2}{b\, g_{el}^2(E)}\right)$$

$$(N-2) < N_f < 3(N-2)$$

	$SO(N_f - N + 4)_g$	$SU(N_f)$	$U(1)_R$
q_I^n	$\mathbf{N_f - N + 4}$	$\overline{\mathbf{N_f}}$	$\frac{N-2}{N_f}$
M_{IJ}	$\mathbf{1}$	$\frac{1}{2}\mathbf{N_f(N_f+1)}$	$\frac{2(N_f - N + 2)}{N_f}$

$$M_{IJ} \sim Q_I^N Q_J^N$$

$$W_{mag} \propto \frac{1}{\mu} q_I^n M^{IJ} q_J^n$$

$$\Lambda_{el}^{3(N-2)-N_f} \Lambda_{mag}^{3(N_f - N + 2) - N_f} \propto (-1)^{N_f - N} \mu^{N_f}$$

$$(N-2) < N_f \leq \frac{3}{2}(N-2) \quad \Rightarrow \quad g_{mag} \xrightarrow{IR} 0$$

$$\mathcal{N} = 1 \text{ SUSY SO}(N) \qquad N_f = N$$

$$\downarrow$$

$$N \leq 3(N-2)/2 \quad \Rightarrow \quad N \geq 6$$

$$\text{SO}(N_f - N + 4)_m = \text{SO}(4)_m$$

$$Q_I^N = \begin{pmatrix} Q_1^N \\ \vdots \\ Q_5^N \\ \hline Q_6^N \\ \vdots \\ Q_{N_f}^N \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} Q \\ \vdots \\ Q \end{matrix}}\right\} Q_a^N \\ \left.\vphantom{\begin{matrix} Q \\ \vdots \\ Q \end{matrix}}\right\} Q_i^N \end{matrix}$$

$$W_{mag} = q_I M^{IJ} q_J - \mu^2 M_{aa} + \epsilon_{IJK} M^{IJ} \xi^K$$

$$\epsilon_{IJK} = \lambda_{IJK}\Lambda, \qquad \mu^2 = -m_Q \Lambda$$

Model I

$N = N_f = 11$ $G_f = \mathrm{SO}(5) \times \mathrm{SO}(6)$

$\Lambda_3^L \sim 10^2 - 10^3\ \mathrm{TeV}$

Model II

$N = N_f = 9$ $G_f = \mathrm{SO}(5) \times \mathrm{SU}(4)$

$\Lambda_1^L \sim 10^3\ \mathrm{TeV}$

Main difference: $t_R \in M_{ia}$ fully composite

Top Quark Partial Compositeness (model I)

$$W_{el} \supseteq \lambda_L(\xi_L)^{ia} Q_i Q_a + \lambda_R(\xi_R)^{ia} Q_i Q_a$$

$$\downarrow$$

$$W_{mag} \supseteq \epsilon_L(\xi_L)^{ia} M_{ia} + \epsilon_R(\xi_R)^{ia} M_{ia}$$

$$(\xi_L)^{ia} = \begin{pmatrix} b^1 & -ib^1 & t^1 & it^1 & 0 \\ -ib^1 & -b^1 & -it^1 & t^1 & 0 \\ b^2 & -ib^2 & t^2 & it^2 & 0 \\ -ib^2 & -b^2 & -it^2 & t^2 & 0 \\ b^3 & -ib^3 & t^3 & it^3 & 0 \\ -ib^3 & -b^3 & -it^3 & t^3 & 0 \end{pmatrix}_{2/3} , \quad (\xi_R)^{ia} = \begin{pmatrix} 0 & 0 & 0 & 0 & (t^c)^1 \\ 0 & 0 & 0 & 0 & i(t^c)^1 \\ 0 & 0 & 0 & 0 & (t^c)^2 \\ 0 & 0 & 0 & 0 & i(t^c)^2 \\ 0 & 0 & 0 & 0 & (t^c)^3 \\ 0 & 0 & 0 & 0 & i(t^c)^3 \end{pmatrix}_{-2/3} ,$$

Comparison with Bottom-up Approaches

$$\mathrm{SO}(5) \times \mathrm{SO}(4) \to \mathrm{SO}(4)_D$$

$$q_b^n = \exp\left(\frac{i\sqrt{2}}{f} h^{\hat{a}} T_{\hat{a}} + \frac{i}{2f} \pi^a T_a\right)_{bc} \widetilde{q}_c^m \, \exp\left(\frac{i}{2f}\pi^a T_a\right)_{mn}$$

effective $\mathrm{SO}(5)/\mathrm{SO}(4)$

$$U = \exp\left(i \frac{\sqrt{2}}{f} h^{\hat{a}} T_{\hat{a}} \right), \qquad U \to g\, U\, h^\dagger, \qquad f = \sqrt{2}\mu$$

$$m_W = \frac{gf}{2} \sin\frac{\langle h \rangle}{f} \equiv \frac{gv}{2}, \qquad m_Z = \frac{m_W}{\cos\theta_W}$$

Higgs Potential

$$V^{(0)} = m_1^2|q_5^n|^2 + m_2^2|q_m^n|^2 + \sum_{i=1}^{5}|h_i|^2|F_{ab}^{M(i)}|^2$$

$$W_{mag} = \sum_{i=1}^{5} h_i(q_a M^{ab} q_b)^{(i)} - \mu^2 M^{aa}$$

$$(\mathbf{1}_0\cdot\mathbf{1}_0\cdot\mathbf{1}_0),\ \ (\mathbf{1}_0\cdot\mathbf{2}_{\pm1/2}\cdot\mathbf{2}_{\mp1/2}),\ \ (\mathbf{2}_{\pm1/2}\cdot\mathbf{3}_{\mp1}\cdot\mathbf{2}_{\pm1/2}),\ \ (\mathbf{2}_{\mp1/2}\cdot\mathbf{3}_0\cdot\mathbf{2}_{\pm1/2}),\ \ (\mathbf{2}_{\mp1/2}\cdot\mathbf{1}'_0\cdot\mathbf{2}_{\pm1/2})$$

$$V^{(1)} = \frac{1}{16\pi^2}\sum_n \frac{(-1)^{2s_n}}{4}(2s_n+1)m_n^4\left(\log\frac{m_n^2}{Q^2} - \frac{3}{2}\right) = \frac{1}{64\pi^2}\mathrm{STr}\left[M^4\left(\log\frac{M^2}{Q^2} - \frac{3}{2}\right)\right]$$