# Planck inflation and the Kähler potential in supergravity and string theory


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with


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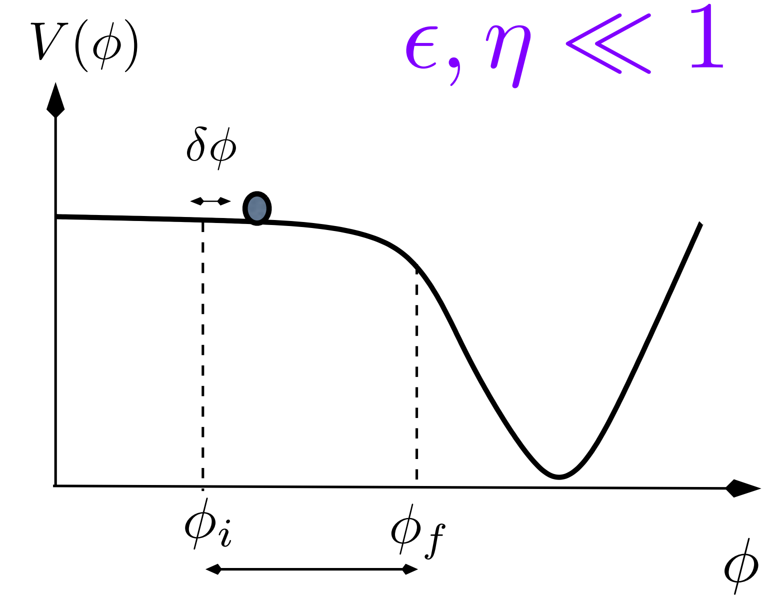
# Vanilla Inflation

Inflation is a period of *accelerated* expansion driven by a *single scalar* field with very *flat potential*

**slow roll inflation**

$\epsilon, \eta \ll 1$




$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta = M_P^2 \frac{V''}{V}$$

slow-roll parameters

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Generic predictions on the properties of the scalar density perturbations:

- They are approximately *scale invariant* $n_s \sim 1$
- They approximately *Gaussian* $f_{NL} \sim 0$

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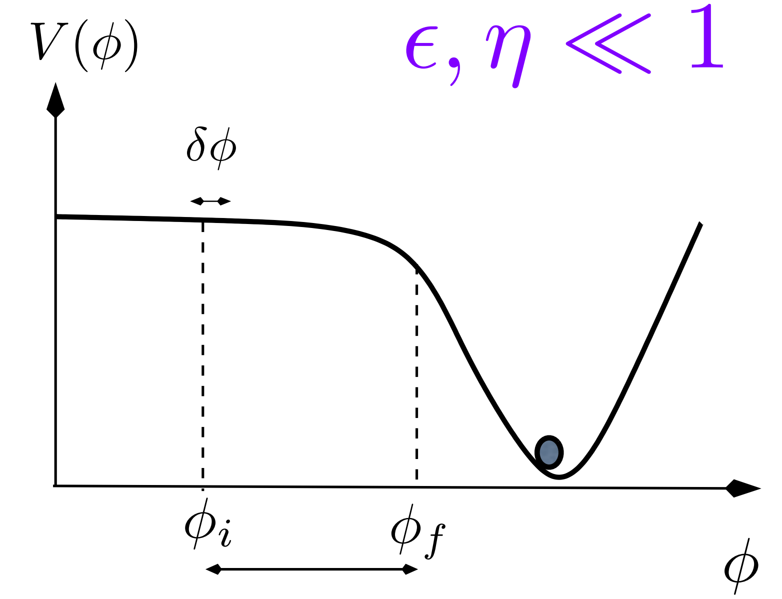


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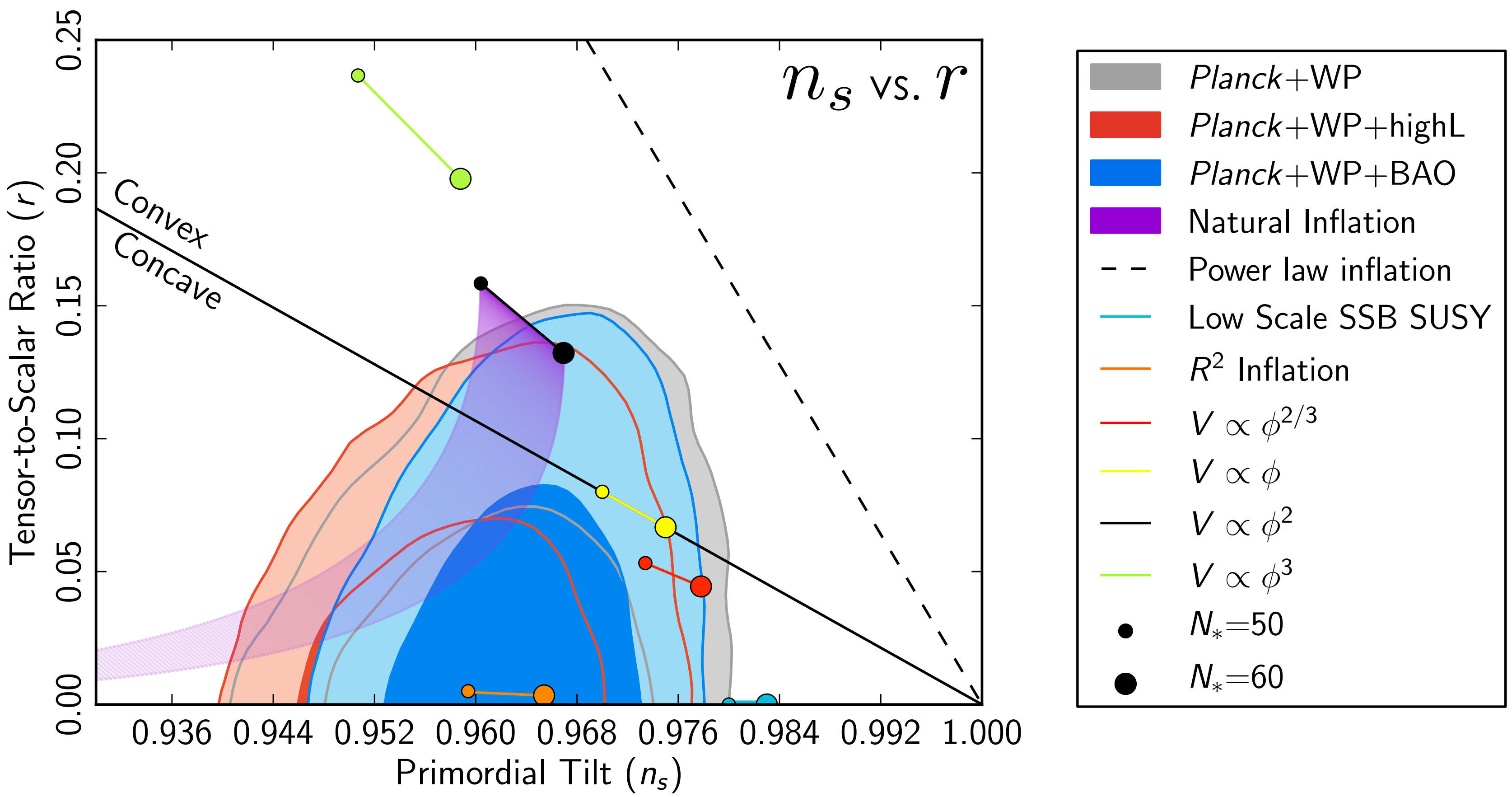
# Planck Inflation 2013




$n_s$ = spectral index

$r$ = tensor 2 scalar ratio (small field)

## Non-Gaussianity

$f_{NL} = 2.7 \pm 5.8$ *local*

$f_{NL} = -42 \pm 75$ *equilateral*

$f_{NL} = -25 \pm 39$ *orthogonal*

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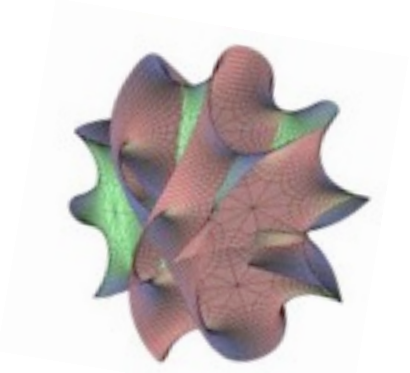
- Can we embed *Planck inflation* *(slow roll single small field)* models into sugra and string theory frameworks?

# $\mathcal{N} = 1$ supergravity

- Matter content
  - gravity multiplet: $g_{MN}, \quad \psi_\mu$
  - n-chiral multiplets: $\chi_i, \quad \Phi_i$

    $i = 1, \ldots, n$



$\partial_i \partial_{\bar{j}} K = K_{i\bar{j}}$

$K_{i\bar{j}} \partial \Phi^i \partial \bar{\Phi}^{\bar{j}}$

scalars organise themselves into a complex manifold

$\Phi_i, \; K_{i\bar{j}}$

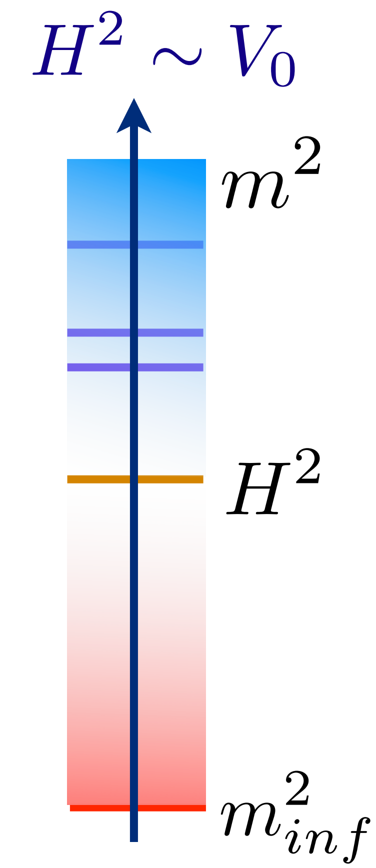
- Theory is fully specified by
  - *Kähler potential* $K(\Phi, \bar{\Phi})$
  - Holomorphic *superpotential,* $W(\Phi)$
- The *scalar potential* is thus given by:

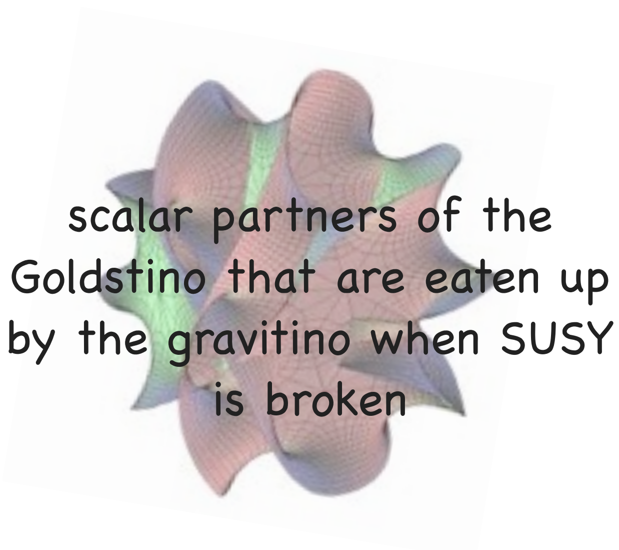
$$V = e^K \left[ K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3 W \overline{W} \right]$$

$D_i W = \partial_i W + \partial_i K \, W$

# A geometric bound on F-term inflation

- During inflation ~~SUSY~~ $D_a W \neq 0$. A *spectrum of scalar masses* below and above Hubble scale




- *sGoldstini directions* in moduli space *are* singled out as ~~SUSY~~ directions. (Useful to determine scalar instabilities.)

[Gómez-Reino et al. '06-'08]

- In F-term sugra (vector fields subdominant), under assumptions:

[Borghese, Roest, IZ, '12]

i) gravitino mass well below inflationary scale $(m^2_{3/2} \sim 1\text{TeV})$

ii) non-negligible overlap between inflaton and sGoldstini directions

only two conditions can be satisfied:

- ✱ *Single field inflation*
- ✱ *Slow roll inflation*
- ✱ *Small field*

# The $\eta$–problem

Consider the F-term scalar potential in sugra:

$$V = e^K \left[ K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3 W \overline{W} \right] \qquad \eta = \frac{V''}{V}$$

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$$\rightarrow \eta \sim \mathcal{O}(1)$$

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- Symmetry protected Kähler: *shift symmetry* [Kawasaki et al. '00]

$$\Phi = \mathrm{Re}(\Phi) + i\,\mathrm{Im}(\Phi)$$

Inflaton:

☑ $K = -\frac{1}{2}\left(\Phi - \bar{\Phi}\right)^2 \quad \rightarrow \quad \mathrm{Re}(\Phi)$

$$(\mathrm{Im}(\Phi) = 0)$$

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Kähler transformations

$$V \rightarrow V$$
$$K \rightarrow K + g(\Phi) + g(\bar{\Phi})$$
$$W \rightarrow e^{-g(\Phi)} W$$

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- *Logarithmic* Kähler potential: [Roest, Scalisi, IZ '13]

☑ $K = -\alpha \ln(\Phi + \bar{\Phi}) \quad \rightarrow$

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Arises in string theory for geometric moduli

# sGoldstino inflation

[Alvarez-Gaumé et al. '10-'11]
[Achúcarro et al. '12]

For a single superfield: inflaton ⇔ sGoldstino

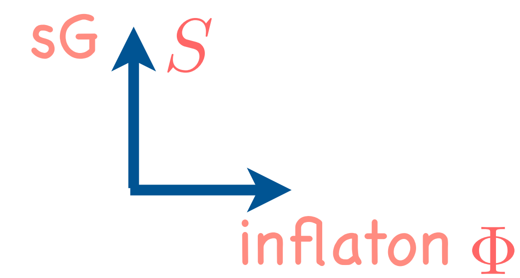
- Geometric bound applies
- Taking $$K = -\frac{1}{2}\left(\Phi - \bar{\Phi}\right)^2 , \quad W = f(\Phi)$$

The potential becomes

$$V = -3f(\phi)^2 + f'(\phi)^2 \qquad \mathrm{Re}(\Phi) = \phi$$

*Small single field slow roll inflation severely constrained*

# Orthogonal inflation




- To overcome *geometric* bound, introduce a second superfield, orthogonal to sG: inflaton

$$\Phi, S$$

- Single field inflation with an *arbitrary scalar potential* can be implemented un sugra under assumptions:

Kähler and superpotential are of the form [Kallosh-Linde-Rube '10]

$$K = K\left((\Phi - \Phi)^2, S\bar{S}, S^2, \bar{S}^2\right) \ , \quad W = S f(\Phi)$$

shift symmetry ⇔ inflaton direction

Inflaton potential

$$V(\phi) = f(\phi)^2$$

$$\mathrm{Re}\,\Phi = \phi$$

Inflationary trajectory

$$\begin{cases} \mathrm{Im}\,\Phi = 0 \\ S = 0 \end{cases}$$

# Orthogonal inflation in sugra and string theory

[Roest, Scalisi, IZ '13]

*Relax* $\mathbb{Z}_2$ *symmetry* in Kähler potential, with same W

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In *string theory* a combination of S, Φ which appears in several models is:

$$X = \Phi + \bar{\Phi} - S\bar{S}$$

$\Phi \Leftrightarrow$ geometric modulus, $S \Leftrightarrow$ matter field

Heisenberg symmetry

$$\Phi \to \Phi + ia + \bar{b}S + \frac{1}{2}|b|^2$$

$$S \to S + b$$

$$a \in \mathbb{R}\,, \quad b \in \mathbb{C}$$

Focus on interesting Kähler potential:

$$K = -\alpha \ln(X)$$

the general scalar potential becomes (@ $\mathrm{Im}\,\Phi = 0$, $S = 0$)

$$V = \frac{X^{1-\alpha} f(\phi)^2}{\alpha}$$

# Two interesting models

For a linear superpotential of the form: [Cecotti, '87]

$$W = 3MS(\Phi - 1)$$

☑ $\alpha = 3,$ corresponds to *Starobinsky's* model!

[Kallosh-Linde, '13]
[Buchmüller et al., '13]
[Farakos et al., '13]

but mass spectrum: $m^2 = (0, 4H^2, -2H^2)$

*need* to add S-stabilising terms to K(S)

$$H^2 = V/3$$

☑ $\alpha = 1,$ now mass spectrum:

[Ellis, Nanopoulos, Olive, '13]
[Roest, Scalisi, IZ '13]

$$m^2 = (0, 24H^2, 6H^2)$$

*no need* to add S-stabilising terms to K!

Inflationary predictions:

$$N = 50: \quad n_s = 0.961\,, \quad r = 0.0015$$
$$N = 60: \quad n_s = 0.967\,, \quad r = 0.0011$$

# Summary

- Discussed Kähler potentials which allow truncation to a *single* scalar, identified with the inflaton, in F-term sugra
- To evade $\eta$-problem *shift symmetry* or *logarithm* function can be used.
- To circumvent geometric bound, a second field needs to be introduced, *orthogonal* to sGoldstino: inflation
- For *Heisenberg invariant K, general potential can be generated in sugra and string theory*
- *For linear W, two choices of* $\alpha$ *allow for small single field slow roll inflation, compatible with Planck*