An alternative IIB embedding of F(4) gauged supergravity

Özgür Kelekçi

Middle East Technical University & Siirt University

based on work with J. Jeong & E. Ó Colgáin, JHEP 1305 (2013) 079

SUSY 2013, Trieste, ITALY

The Abdus Salam International Centre for Theoretical Physics

METU

Motivation

Dualities

- In physics, dualities, equivalent descriptions of the same system, provide valuable insights into non-perturbative regimes.

- In fact, in Stat. Mech. one can determine the exact temperature of the phase transition in 2D Ising Model. Kramers-Wannier (KW) (1941).

- All well-understood dualities are Abelian and can be traced to KW.
  Non-Abelian well motivated, e.g. Heisenberg ferromagnets, but troublesome.

- Key to our understanding of String theory are various dualities, e.g. T-duality, AdS/CFT correspondence.

- Can String theory tell us anything about non-Abelian dualities?

Dualities in String Theory

- T-duality relates theories which are compactified on circles.

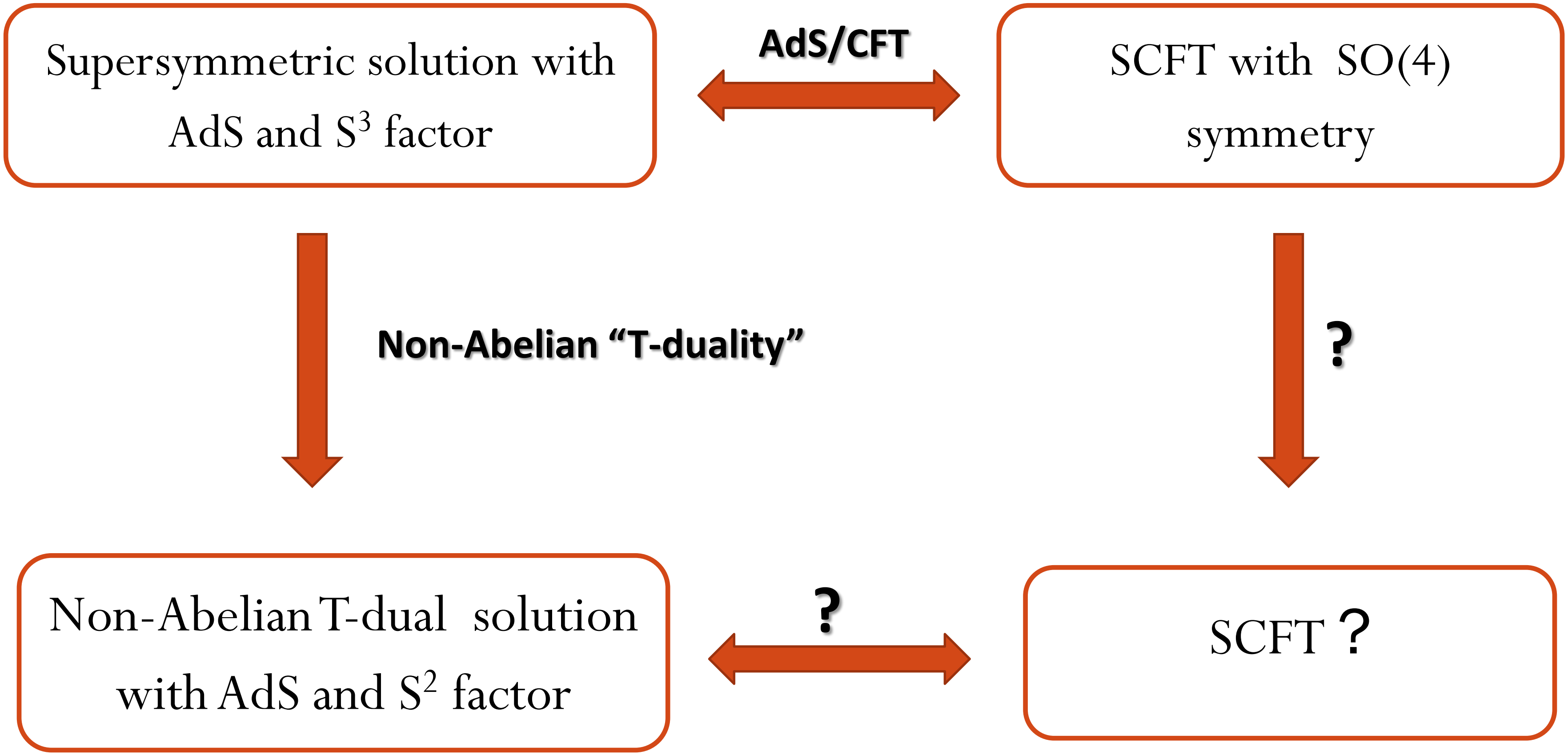
Type IIA String Theory ⟷ (T-duality) ⟷ Type IIB String Theory

- The AdS/CFT correspondence is a conjectured equivalence between a quantum field theory in *d* spacetime dimensions with conformal symmetry and a quantum theory of gravity in (*d* + *1*)-dimensional Anti-de Sitter space. Maldacena (1997)

Type IIB String Theory on AdS_5 x S^5 ⟷ (AdS/CFT) ⟷ 4-dim. $\mathcal{N}=4$ Super Yang-Mills (SCFT)

METU

Natural Question



Abelian T-duality & Buscher Rules

Nonlinear sigma model

$$S = \int d^2z(g_{mn}(x) + b_{mn}(x))\partial x^m \bar{\partial} x^n$$

U(1) isometry, $x^1 \to x^1 + c, \quad x^{\hat{m}} \to x^{\hat{m}}$

replace the derivatives of x^1 by a vector field $(A, \bar{A})$, add a lagrange multiplier field $\tilde{x}^1$

$$S = \int d^2z[g_{11}A\bar{A} + l_{1\hat{m}}A\bar{\partial}x^{\hat{m}} + l_{\hat{m}1}\partial x^{\hat{m}}\bar{A} + l_{\hat{m}\hat{n}}\partial x^{\hat{m}}\bar{\partial}x^{\hat{n}} + \tilde{x}^1(\partial\bar{A} - \bar{\partial}A)] \qquad l_{mn} = g_{mn} + b_{mn}$$

i) Integrate out the $\tilde{x}^1$ → $\partial\bar{A} - \bar{\partial}A = 0$

Can be solved by $A = \partial x^1$ *and* $\bar{A} = \bar{\partial}x^1$ → back to original model

ii) Integrate out the vector field and obtain T-dualized action

Compare sigma model with T-dualized one

$$g'_{11} = 1/g_{11}, \quad g'_{1\hat{m}} = b_{1\hat{m}}/g_{11}, \quad b'_{1\hat{m}} = g_{1\hat{m}}/g_{11}$$

$$\phi' = \phi - \frac{1}{2}\log g_{11}$$

Nonabelian T-duality

The Buscher procedure can be naturally generalized to either the Fermionic or non-Abelian isometries. Fermionic T-duality: [Berkovits & Maldacena, Beisert etal.]

Gauge isometry $\quad \partial x^m \to D x^m = \partial x^m + A^\alpha (T_\alpha)^m{}_n x^n$

The flatness of the gauge fields is imposed by the term

$$\int Tr(\chi F) \quad , \text{ here } \quad F = \partial \bar{A} - \bar{\partial} A + [A, \bar{A}].$$

Buscher procedure gives us the metric, B-field and dilaton.
Isometry (and potentially supersymmetry) is partially destroyed.

Transforming RR-fluxes: Hassan (1999), Sfetsos & Thompson (2010)

$$\hat{P} = P\Omega^{-1} \qquad \Omega^{-1}\Gamma^a\Omega = \Lambda^a{}_b\Gamma^b$$

$$P_{IIB} = \frac{e^\Phi}{2}\sum_{n=0}^{4}\frac{1}{(2n+1)!}\not{F}_{2n+1}, \quad P_{IIA} = \frac{e^\Phi}{2}\sum_{n=0}^{5}\frac{1}{(2n)!}\not{F}_{2n}$$

Bi-spinors for type-IIB and type-IIA

In principle completely systematic, no guess-work

METU

F(4) gauged supergravity

In1978 Nahm classified all simple superalgebras; it was noted that there existed supergravity theories with vacua invariant under symmetries of superalgebras.

There was no supergravity corresponding to F(4) superalgebra, bosonic symmetries SO(5,2) x SU(2). Found in L.J. Romans (1985)

$$e^{-1}\,\mathcal{L}_6 = -\tfrac{1}{4}R + \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{-\sqrt{2}\phi}\left((\mathcal{F}+mB)^2 + (F^i)^2\right) + \tfrac{1}{12}e^{2\sqrt{2}\phi}G^2 + \tfrac{1}{8}\left(g^2 e^{\sqrt{2}\phi} + 4gme^{-\sqrt{2}\phi} - m^2 e^{-3\sqrt{2}\phi}\right) + \cdots$$

Theory consists of a graviton $e^{\alpha}{}_{\mu}$, three SU(2) gauge potentials $A^{i}{}_{\mu}$, an Abelian potential A_{μ}, a two-index tensor gauge field $B_{\mu\nu}$, a scalar ϕ, fermions .

Post Maldacena's conjecture (AdS/CFT), Brandhuber-Oz found a solution to type II supergravity which is warped AdS_6 x S^4 (near horizon D4-D8). Brandhuber-Oz (1999)

Cvetic, Lu, Pope constructed consistent KK reduction from massive IIA and got Romans' F(4) gauged supergravity. Cvetic, Lu, Pope (1999)

METU

New Supersymmetric AdS_6 solution

Brandhuber-Oz solution is warped, so isometry is not SO(5), but only SO(4)

$$ds^2 = \frac{1}{4}(m\cos\theta)^{-1/3}[9ds^2(AdS_6) + 4(d\theta^2 + \sin^2\theta ds^2(S^3)]$$

SO(4) isomorphic to $SU(2)_G$ x $SU(2)_R$

Can write S3 as Hopf-fibre [U(1) x SU(2)] , see that Killing spinors independent of U(1) fibre, so one can do Abelian T-duality and preserve supersymmetry. (Ó Colgáin, unpublished work)

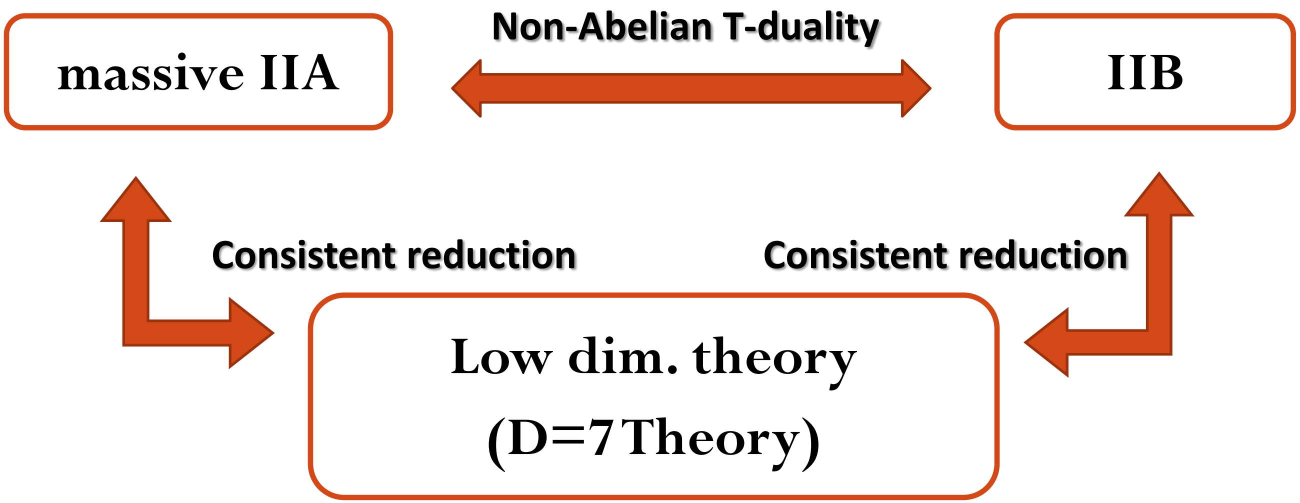
Novel observation: Can also perform SU(2) transformation

Note work of Passias, JHEP 01 (2013) , supersymmetric AdS_6 solution unique in massive IIA, but in IIB there are at least two! Lozano etal. (2013)

Abelian vs non-Abelian

Type IIA or type IIB theory → circle reduction to $\mathcal{N}=2$ 9-dimensional supergravity. Match the fields using the fact that nine dimensional supergravity is unique. Bergshoeff, Hull & Ortin (1995)

Can we use this idea of Bergshoeff etal. for non-Abelian T duality?



This has been checked to work for spacetimes with SO(4) isometry, but it is expected to work more generally. Itsios etal. (2012)

KK reduction on S^3

Natural ansatz with SO(4) symmetry

$$ds^2 = ds^2(M_7) + e^{2A} ds^2(S^3)$$

RR fluxes

$$F_0 = m\,, \quad F_2 = G_2\,, \; F_4 = G_4 + G_1 \wedge \mathrm{Vol}(S^3)$$

Apart from SU(2) gauge fields we have an embedding of F(4) gauged supergravity in type IIB.

$$d\hat{s}^2_{10} = (\sin\xi)^{1/12} X^{1/8}\Big[\Delta^{3/8}\, ds_6^2 + 2g^{-2}\,\Delta^{3/8} X^2\, d\xi^2 + \frac{1}{2} g^{-2}\,\Delta^{-5/8} X^{-1}\cos^2\xi \sum_{i=1}^{3}(\sigma^i - gA^i_{(1)})^2\Big]$$

Key point: with SU(2) gauge fields KK reduction ansatz has SU(2) symmetry, without these fields it has SO(4) symmetry.

Can we find the full embedding of F(4) gauged supergravity ? Jeong, OK, Ó Colgáin (2013)

Option-1: Use trial and error and fact that R symmetry is gauged. [Gauntlett &Varela, 2007]

Option-2: Use non-Abelian T-duality for general spacetimes with SU(2) isometry. results published in arXiv:1301.6755. (Our project was well under way before this work appeared)

Conjecture

Conjecture of Gauntlett & Varela, Phys.Rev.D76,2007

*"For any supersymmetric solution of D =10 or D = 11 supergravity that consists of a warped product of d + 1-dimensional anti-de-Sitter space with a Riemannian manifold M , $AdS_{d+1} \times_{\omega} M$, there is a consistent Kaluza-Klein truncation on M to a gauged supergravity theory in d + 1- dimensions for which the fields are dual to those in the superconformal current multiplet of the d-dimensional dual SCFT. "**

We can do a consistent KK reduction by gauging the R-symmetry.

*For some cases it has been proven.

Reduction from IIA

The massive IIA reduction on S^4 can be decomposed into a reduction on S^3 to $D = 7$ followed by a further reduction on the remaining angular coordinate of the S^4.

Goal: To construct the alternative reduction from type IIB to $D = 7$ leading to an embedding of Romans' theory in type IIB.

Full reduction to D=6 is messy. Better to work with the EOMs of an intermediate D=7 theory. We match the EOMs at this level.

$$ds^2 = ds^2(M_7) + e^{2A}\sum_{i=1}^{3}(\sigma^i - A^i)^2 \qquad F_0 = m \ , \ F_2 = G_2$$

$$F_4 = G_4 + G_1 \wedge h_1 \wedge h_2 \wedge h_3 + h_i \wedge H_3^i + \tfrac{1}{2}\epsilon_{ijk} H_2^i \wedge h_j \wedge h_k$$

Cvetic, Lu, Pope reduction ansatz fits into this class.

This allows us to identify the correct D=7 EOMs that become the D=6 EOMs of Romans' theory. For example dilaton equation is:

$$0 = \bar{R} + \tfrac{3}{2}e^{-2A} - 6\nabla^2 A - 12(\partial A)^2 + 12\partial A \cdot \partial\Phi + 4\nabla^2\Phi - 4(\partial\Phi)^2 - \tfrac{1}{12}H^2 - \tfrac{1}{4}e^{2A}F^i_{\mu\nu}F^{i\mu\nu}$$

This just involves the D=10 NS sector.

Reduction from IIB

Non abelian T-dual has a residual S^2 corresponding to R symmetry.

$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2$$

Parameterization of S^2

$$ds^2(S^2) = d\mu^i d\mu^i, \quad \mu^1 = \sin\theta\sin\phi, \quad \mu^2 = \sin\theta\cos\phi, \quad \mu^3 = -\cos\theta.$$

Then, in the spirit of Gauntlett, Varela (2007), we simply take residual S^2 (R-symmetry) and gauge it in the natural way.

$$ds^2 = ds^2(M_7) + e^{-2A}dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}} D\mu^i D\mu^i, \qquad D\mu^i = d\mu^i - \epsilon_{ijk}\mu^j A^k$$

$$\tilde{B} = B - \frac{r^3}{r^2 + e^{4A}} \tfrac{1}{2}\epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k + A^i \wedge d(r\mu^i) + r\tfrac{1}{2}\epsilon_{ijk}\mu^i A^j \wedge A^k,$$

$$e^{-2\tilde{\Phi}} = e^{-2\Phi} e^{2A}(r^2 + e^{4A})$$

This gives NS sector and we have one equation (dilaton equation) that is the same before and after the T-duality. So we are on the correct path.

RR Fluxes

Now that we have the correct NS sector, we can reconstruct the RR sector via trial and error.

$$F_1 = -G_1 + mrdr.$$ one-form flux

$$F_3 = e^{3A} *_7 G_4 + rdr \wedge G_2 + \frac{r^2}{r^2 + e^{4A}} \left[rG_1 + me^{4A} dr \right] \wedge \mathrm{vol}(\tilde{S}^2) - r\mu^i H_3^i - (rD\mu^i + \mu^i dr) \wedge H_2^i$$ three-form flux

$$F_5 = \frac{r^2 e^{3A}}{r^2 + e^{4A}} \left(-r *_7 G_4 + e^A dr \wedge G_2 \right) \wedge \mathrm{vol}(\tilde{S}^2) - e^{3A} *_7 G_2 + rdr \wedge G_4 - (rD\mu^i + \mu^i dr) \wedge e^A *_7 H_3^i - r\mu^i e^{-A} *_7 H_2^i - rH_3^i \wedge dr \wedge \epsilon_{ijk} \mu^j D\mu^k + \frac{r^3}{r^2 + e^{4A}} \mu^i H_2^i \wedge dr \wedge \mathrm{vol}(\tilde{S}^2) - \mu^i \frac{r^2 e^{4A}}{(r^2 + e^{4A})} H_3^i \wedge \mathrm{vol}(\tilde{S}^2)$$ self-dual five-form flux

We uplifted the following solutions from 6 dimensions:

1. Supersymmetric domain wall
2. Supersymmetric magnetovac
3. Lifshitz solution
4. Black hole solution

METU

Summary and Future directions

- We obtained new full embedding of Romans' F(4) gauged supergravity in type IIB.
- Any solution to Romans' theory can now be uplifted not just to massive IIA, but also to type IIB.
- This is another example of the conjecture of Gauntlett, Varela (2007)
- This is the most general check that non-Abelian T-duality satisfies the EOMS of type II supergravity.
- We have an AdS_6 vacuum that is supersymmetric. Is there a dual CFT? Can one work out the spectrum for the fluctuations?
- Are there more than 2 supersymmetric AdS_6 solutions for type IIB?
- What is the connection between non-Abelian T-duality and "Born Reciprocity in String Theory and the Nature of Spacetime" ? L. Freidel etal. [arXiv:1307.7080] Can (non) abelian T-duality be viewed as a Fourier Transform?

METU

SIIRT UNIVERSITY 2007