R-Symmetries from Heterotic Orbifolds

Damián K. Mayorga Peña

Based on arXiv: 1301.2322, 1308.5669 in collaboration with N. Cabo Bizet, T. Kobayashi, S. Parameswaran, M. Schmitz and I. Zavala

universität bonn

bctp Bethe Center for Theoretical Physics

Trieste, August 27, 2013

Plan

- Motivation
- Orbifolds and their CFT
- R-Symmetries from Orbifold Isometries
- Universal R-Anomalies
- Conclusions and Prospects

Motivation

- Many MSSM like models can be obtained from the Heterotic orbifolds such as $\mathbb{Z}_{6\mathrm{II}}$, $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$,...

  see e.g. talks by N. Cabo Bizet, S Ramos Sánchez

- Their success is mostly due to the many orbifold symmetries present in the effective field theory
  - non-R-Symmetries:
    Remnants of $U(1)$-symmetries, permutations of fixed points,...
    $\rightarrow$ Flavor symmetries, etc
  - R-Symmetries:
    Surviving components of the Lorentz group in compact space.
    $\rightarrow$ Well motivated in SUSY-pheno

    see talk by P. Pais

    $\rightarrow$ Stringy R-symmetries could solve the μ-problem

    Casas, Munoz'93; Lebedev *et. al.*'08, Kappl *et. al.*'09

Orbifolds CFTs

- Assume the target space of the heterotic string to be of the form

$$\mathcal{M}_{10} = \mathcal{M}_{3,1} \times \frac{T^6}{P} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{P \ltimes \Gamma_6} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{S}$$

P is an isometry of Γ_6 which we take as $\mathbb{Z}_N$, with

$$\theta = (e^{2\pi i v^1}, e^{2\pi i v^2}, e^{2\pi i v^3})$$

its generating element.

- String boundary conditions:

$$Z(\sigma + \pi, \tau) = \theta^k Z(\sigma, \tau) + \lambda \qquad (\theta^k, \lambda) \in S$$

Orbifolds CFTs

- Assume the target space of the heterotic string to be of the form

$$\mathcal{M}_{10} = \mathcal{M}_{3,1} \times \frac{T^6}{P} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{P \ltimes \Gamma_6} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{S}$$

  P is an isometry of Γ_6 which we take as $\mathbb{Z}_N$, with

$$\theta = (e^{2\pi \mathrm{i} v^1}, e^{2\pi \mathrm{i} v^2}, e^{2\pi \mathrm{i} v^3})$$

  its generating element.

- P has some **fixed points** z_f, which fall into **conjugacy classes**

$$[z_\mathrm{f}] = \{z'_\mathrm{f} \,|\, z'_\mathrm{f} = h z_\mathrm{f} \text{ for some } h \in S\}\,,$$

  supporting **twisted string states**.

Orbifolds CFTs

- Assume the target space of the heterotic string to be of the form

$$\mathcal{M}_{10} = \mathcal{M}_{3,1} \times \frac{T^6}{P} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{P \ltimes \Gamma_6} = \mathcal{M}_{3,1} \times \frac{\mathbb{C}^3}{S}$$

  P is an isometry of Γ_6 which we take as $\mathbb{Z}_N$, with

$$\theta = (e^{2\pi i v^1}, e^{2\pi i v^2}, e^{2\pi i v^3})$$

  its generating element.
- The space group also acts one the gauge coords.

$$g = (\theta^k, n_\alpha e_\alpha) \hookrightarrow V_g = kV + n_\alpha W_\alpha$$

  the shift V and the Wilson lines W_α are fractional vectors from the $E_8 \times E_8$ lattice.

Orbifolds CFTs

Building Blocks for Massless States

Dixon, Friedan, Marinec, Shenker'87

- Internal space coords.

  $\rightarrow$ Oscillators ∂Z^i, $\partial \bar{Z}$

- Gauge coords.

  $\rightarrow$ Gauge weights $e^{2\pi \mathrm{i} p^I_{sh} X^I}$, $I = 1, ..., 16$

- Right moving fermions

  $\rightarrow$ Reps. under the little group $e^{2\pi \mathrm{i} q^{b,f}_{sh} \cdot H}$

$$q^b_{sh} = q^f_{sh} + (\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2})$$

- Space group

  $\rightarrow$ **Twist fields** creating vacua from the untwisted ones:

$$\sigma = \sum_{z'_{\mathrm{f}} \in [z_{\mathrm{f}}]} e^{-2\pi \mathrm{i} r \tilde{\gamma}(z'_{\mathrm{f}})} \sigma_{z'_{\mathrm{f}}} ,$$

Orbifolds CFTs

Physical States

The physical vertices take the form:

$$V_{b,f} = e^{-a\phi}\left(\prod_{i=1}^{3}(\partial X^i)^{\mathcal{N}_\mathrm{L}^i}\ (\partial\bar{X}^i)^{\bar{\mathcal{N}}_\mathrm{L}^i}\right)\ e^{\mathrm{i}q_{sh}^{b,f}\cdot H^m}\ e^{\mathrm{i}p_{sh}^I X^I}\,\sigma$$

Hamidi, Vafa'87; Font, Ibanez, Nilles, Quevedo'88

- Any state must be invariant under the full space group. Take a generic $h = (\theta^m, \mu)$

$$\partial Z^i \xrightarrow{h} e^{2\pi \mathrm{i} v_h^i}\partial Z^i \quad X^I \xrightarrow{h} X^I + 2\pi V_h^I\,, \quad H^i \to H^i - v_h^i\,.$$

- What about the twist fields? ...auxiliary pieces

$$\sigma_{z'_\mathrm{f}} \xrightarrow{h} e^{2\pi\mathrm{i}\Phi(h,z'_\mathrm{f})}\sigma_{hz'_\mathrm{f}}\,.$$

$\Phi(h, z'_\mathrm{f})$ is a vacuum phase fixed by modular invariance.

Orbifolds CFTs

Physical States

The physical vertices take the form:

$$V_{b,f} = e^{-a\phi} \left(\prod_{i=1}^{3} (\partial X^i)^{\mathcal{N}_L^i} \ (\partial \bar{X}^i)^{\bar{\mathcal{N}}_L^i} \right) \ e^{\mathrm{i} q_{sh}^{b,f} \cdot H^m} \ e^{\mathrm{i} p_{sh}^I X^I} \ \sigma$$

Hamidi, Vafa'87; Font, Ibanez, Nilles, Quevedo'88

- Any state must be invariant under the full space group. Take a generic $h = (\theta^m, \mu)$

$$\partial Z^i \xrightarrow{h} e^{2\pi \mathrm{i} v_h^i} \partial Z^i \qquad X^I \xrightarrow{h} X^I + 2\pi V_h^I \,, \qquad H^i \to H^i - v_h^i \,.$$

- Hence

$$\sigma \xrightarrow{h} e^{2\pi \mathrm{i} [\gamma(h) + \Phi(h, z_\mathrm{f}')]} \sigma \,.$$

where $\gamma(h) = \tilde{\gamma}(h z_\mathrm{f}') - \tilde{\gamma}(z_\mathrm{f}')$

Orbifolds CFTs

Physical States

Putting all pieces together we get the following

- If $hz'_{\mathrm{f}} = z'_{\mathrm{f}} \Rightarrow \gamma(h) = 0$. Thus

$$p_{sh} \cdot V_h - v_h^i (q_{sh}^{b,f\ i} - \mathcal{N}_L^i + \bar{\mathcal{N}}_L^i) + \Phi(h, z'_{\mathrm{f}}) \stackrel{!}{=} 0$$

$\Rightarrow$ these conds. project out some states

- ...otherwise

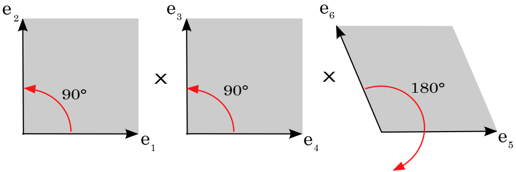
$$\gamma(h) = -p_{sh} \cdot V_h + v_h^i (q_{sh}^{b,f\ i} - \mathcal{N}_L^i + \bar{\mathcal{N}}_L^i) - \Phi(h, z'_{\mathrm{f}})$$

the values of $\gamma(h)$ fix all $\tilde{\gamma}(hz'_{\mathrm{f}})$ in σ up to an overall phase.

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

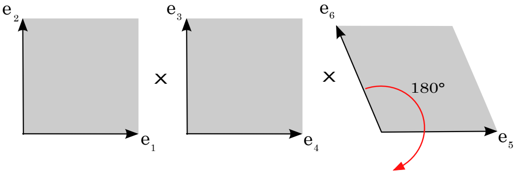
- For Example: $\mathbb{Z}_4$



R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$

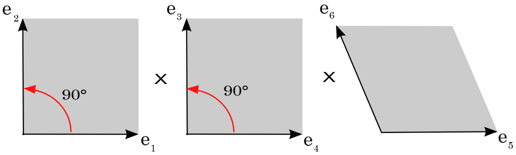


- ✓ rotation of $180°$ in the third plane

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$

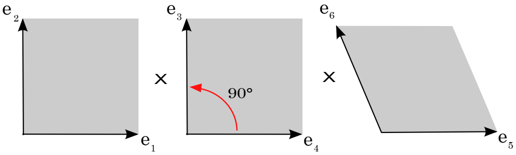


- ✓ rotation of $180°$ in the third plane
- ✓ simultaneous rotations by $90°$ in the first two planes

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$

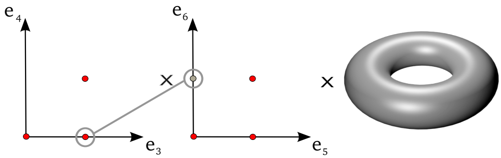


- ✓ rotation of 180° in the third plane
- ✓ simultaneous rotations by 90° in the first two planes

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$ The θ^2 sector

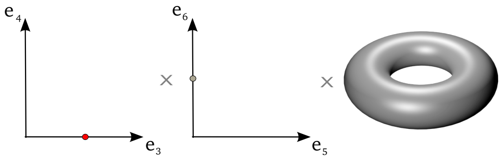


- ✓ rotation of 180° in the third plane
- ✓ simultaneous rotations by 90° in the first two planes

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
→ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$ The θ^2 sector

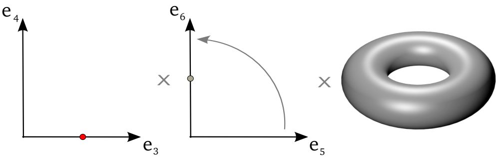


- ✓ rotation of $180°$ in the third plane
- ✓ simultaneous rotations by $90°$ in the first two planes

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$ The θ^2 sector

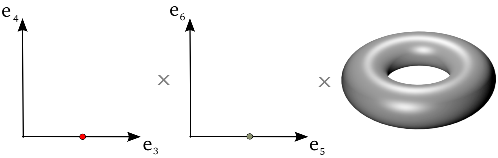


- ✓ rotation of 180° in the third plane
- ✓ simultaneous rotations by 90° in the first two planes

R-Symmetries from Orbifold Isometries

Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
$\rightarrow$ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_4$ The θ^2 sector

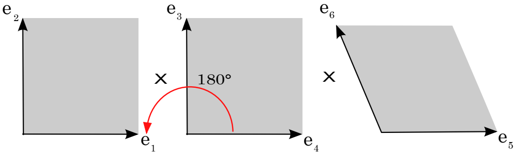


- ✓ rotation of 180° in the third plane
- ✓ simultaneous rotations by 90° in the first two planes
- ✗ rotation by 90° in any of the first two planes

R-Symmetries from Orbifold Isometries

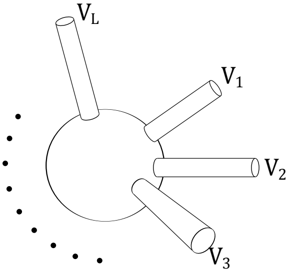
Orbifold Isometries: Lattice automorphisms which survive the orbifolding.
→ these are rotations which leave fixed points invariant

- For Example: $\mathbb{Z}_{\mathbf{4}}$



- ✓ rotation of 180° in the third plane
- ✓ simultaneous rotations by 90° in the first two planes
- ✗ rotation by 90° in any of the first two planes
- ✓ rotations by 180° in any of the first two planes

R-Symmetries from the CFT



- L-point coupling $\psi\psi\Phi^{L-3} \subset \int \mathrm{d}\theta^2 \mathcal{W}$ given by the correlator of the emission vertices

$$\mathcal{F} = \langle V_1 V_2 \ldots V_L \rangle$$

- Given an *orbifold isometry*

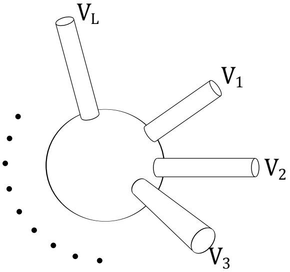
$$\varrho = (e^{2\pi \mathrm{i} \xi^1}, e^{2\pi \mathrm{i} \xi^2}, e^{2\pi \mathrm{i} \xi^3})$$

$\mathcal{F}$ vanishes unless $\sum_{\alpha=1}^{N} r_\alpha = R \mod M$, where

$$r_\alpha = \sum_{i=1}^{3} M\xi^i \left(q_\alpha - \mathcal{N}_\mathrm{L}^i + \bar{\mathcal{N}}_\mathrm{L}^i\right) - M\gamma(h_\varrho)$$

M is the smallest integer s.t. $-M \sum_{i=1}^{3} \xi^i = R \in \mathbb{Z}$.

R-Symmetries from the CFT



- L-point coupling $\psi\psi\Phi^{L-3} \subset \int \mathrm{d}\theta^2 \mathcal{W}$ given by the correlator of the emission vertices

$$\mathcal{F} = \langle V_1 V_2 \dots V_L \rangle$$

- Given an *orbifold isometry*

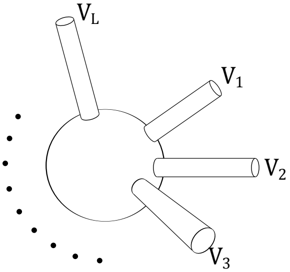
$$\varrho = (e^{2\pi \mathrm{i}\xi^1}, e^{2\pi \mathrm{i}\xi^2}, e^{2\pi \mathrm{i}\xi^3})$$

$\mathcal{F}$ vanishes unless $\sum_{\alpha=1}^{N} r_\alpha = R \bmod M$, where

$$r_\alpha = \sum_{i=1}^{3} M\xi^i \left(q_\alpha - \mathcal{N}_\mathrm{L}^i + \bar{\mathcal{N}}_\mathrm{L}^i \right) - M\gamma(h_\varrho)$$

h_ϱ is a space group element s.t. $h_\varrho z'_\mathrm{f} = \varrho z'_\mathrm{f}$.

R-Symmetries from the CFT



- L-point coupling $\psi\psi\Phi^{L-3} \subset \int \mathrm{d}\theta^2 \mathcal{W}$ given by the correlator of the emission vertices

$$\mathcal{F} = \langle V_1 V_2 \ldots V_L \rangle$$

- Given an *orbifold isometry*

$$\varrho = (e^{2\pi \mathrm{i}\xi^1}, e^{2\pi \mathrm{i}\xi^2}, e^{2\pi \mathrm{i}\xi^3})$$

$\mathcal{F}$ vanishes unless $\sum_{\alpha=1}^{N} r_\alpha = R \mod M$, where

$$r_\alpha = \sum_{i=1}^{3} M\xi^i \left(q_\alpha - \mathcal{N}_\mathrm{L}^i + \bar{\mathcal{N}}_\mathrm{L}^i\right) - M\gamma(h_\varrho)$$

Now we have a $\mathbb{Z}_M^R$ symmetry in the field theory.

Universal R-Anomalies

- Heterotic Orbifold model possess only one axion field.

  All anomalies must be cancelled by the same axion shift *(anomaly universality).*

- For the $\mathbb{Z}_M^R$

  The Anomaly coefficients read

$$A_{G_a^2-\mathbb{Z}_M^R} = C_2(G_a)\frac{R}{2} + \sum_\alpha \left(r_\alpha - \frac{R}{2}\right) T(\mathbf{R}_a^\alpha)\,,$$

$$A_{\text{grav.}^2-\mathbb{Z}_M^R} = \left(-21 - 1 - N_T - N_U + \sum_a \mathsf{dim}\{\mathsf{adj}(G_a)\}\right)\frac{R}{2} + \sum_\alpha \left(r_\alpha - \frac{R}{2}\right)\cdot \mathsf{dim}\{\mathbf{R}^\alpha\}\,,$$

for $G_a \subset E_8 \times E_8$ a gauge factor and α running over all matter reps.

Universal R-Anomalies

- Heterotic Orbifold model possess only one axion field.

  All anomalies must be cancelled by the same axion shift *(anomaly universality).*

- For the $\mathbb{Z}_M^R$

  Anomaly universality implies

$$\boxed{A_{G_a^2-\mathbb{Z}_M^R} \text{ mod } MT(\mathbf{N}_a) = A_{G_b^2-\mathbb{Z}_M^R} \text{ mod } MT(\mathbf{N}_b)\,,}$$

$$\boxed{A_{G_a^2-\mathbb{Z}_M^R} \text{ mod } MT(\mathbf{N}_a) = \frac{1}{24}\left(A_{\text{grav.}^2-\mathbb{Z}_M^R} \text{ mod } \frac{M}{2}\right)\,,}$$

for any pair of non-Abelian gauge factors G_a and G_b.

Universal R-Anomalies

- **Results**

orbifold	lattice	twist	ϱ ξ_1	ξ_2	ξ_3	R	M
$\mathbb{Z}_4$	$SO(4)^2 \times SU(2)^2$	$(\frac{1}{4}, \frac{1}{4}, -\frac{2}{4})$	$1/4$	$1/4$	0	-1	2
			$1/2$	0	0	-1	2
			0	0	$1/2$	$+1$	2
$\mathbb{Z}_{6I}$	$G_2 \times G_2 \times SU(3)$	$(\frac{1}{6}, \frac{1}{6}, -\frac{2}{6})$	$1/6$	$1/6$	0	-1	3
			0	0	$-1/3$	$+1$	3
$\mathbb{Z}_{6II}$	$G_2 \times SU(3) \times SU(2)^2$	$(\frac{1}{6}, \frac{2}{6}, -\frac{3}{6})$	$1/6$	0	0	-1	6
			0	$1/3$	0	-1	3
			0	$-1/2$	0	$+1$	2
$\mathbb{Z}_{8I}$	$SO(9) \times SO(5)$	$(\frac{1}{8}, -\frac{3}{8}, \frac{2}{8})$	$1/4$	$-3/4$	0	-1	2
			0	0	$1/2$	-1	2

...

Universal R-Anomalies

- **Results**

orbifold	lattice	twist	ϱ			R	M
			ξ_1	ξ_2	ξ_3		
$\mathbb{Z}_4$	$SO(4)^2 \times SU(2)^2$	$(\frac{1}{4}, \frac{1}{4}, -\frac{2}{4})$	$1/4$	$1/4$	0	-1	2
			$1/2$	0	0	-1	2
			0	0	$1/2$	$+1$	2
$\mathbb{Z}_{6I}$	$G_2 \times G_2 \times SU(3)$	$(\frac{1}{6}, \frac{1}{6}, -\frac{2}{6})$	$1/6$	$1/6$	0	-1	3
			0	0	$-1/3$	$+1$	3
$\mathbb{Z}_{6II}$	$G_2 \times SU(3) \times SU(2)^2$	$(\frac{1}{6}, \frac{2}{6}, -\frac{3}{6})$	$1/6$	0	0	-1	6
			0	$1/3$	0	-1	3
			0	$-1/2$	0	$+1$	2
$\mathbb{Z}_{8I}$	$SO(9) \times SO(5)$	$(\frac{1}{8}, -\frac{3}{8}, \frac{2}{8})$	$1/4$	$-3/4$	0	-1	2
			0	0	$1/2$	-1	2

...

- We constructed the models for all inequivalent shift embeddings, the symmetries of interest were found to give universal anomalies.

Universal R-Anomalies

- **Results**

orbifold	lattice	twist	ϱ			R	M
			ξ_1	ξ_2	ξ_3		
$\mathbb{Z}_4$	$SO(4)^2 \times SU(2)^2$	$(\frac{1}{4}, \frac{1}{4}, -\frac{2}{4})$	$1/4$	$1/4$	0	-1	2
			$1/2$	0	0	-1	2
			0	0	$1/2$	$+1$	2
$\mathbb{Z}_{6I}$	$G_2 \times G_2 \times SU(3)$	$(\frac{1}{6}, \frac{1}{6}, -\frac{2}{6})$	$1/6$	$1/6$	0	-1	3
			0	0	$-1/3$	$+1$	3
$\mathbb{Z}_{6II}$	$G_2 \times SU(3) \times SU(2)^2$	$(\frac{1}{6}, \frac{2}{6}, -\frac{3}{6})$	$1/6$	0	0	-1	6
			0	$1/3$	0	-1	3
			0	$-1/2$	0	$+1$	2
$\mathbb{Z}_{8I}$	$SO(9) \times SO(5)$	$(\frac{1}{8}, -\frac{3}{8}, \frac{2}{8})$	$1/4$	$-3/4$	0	-1	2
			0	0	$1/2$	-1	2

...

- For each shift embedding 10.000 Wilson line configurations were randomly generated, again we see universal anomaly coefficients.

bctp

Conclusions and Outlook

- Orbifold isometries were identified as symmetries of the worldsheet instantons, and thus explicit symmetries of the three point correlators.
- The universality of the anomalies found for these new R-charges in all models explored gives confidence on the robustness of these results.
- The new R-charges contain now gauge information, it is worth studying what is the counterpart of this phenomenon in the context of smooth CY manifolds, or the orbifold regime in GLSMs.
- From the Orbifold point there is a long way down to the (...N)MSSM. Phenomenologically viable R-symmetries from stringy models are to be found.

Conclusions and Outlook

- Orbifold isometries were identified as symmetries of the worldsheet instantons, and thus explicit symmetries of the three point correlators.
- The universality of the anomalies found for these new R-charges in all models explored gives confidence on the robustness of these results.
- The new R-charges contain now gauge information, it is worth studying what is the counterpart of this phenomenon in the context of smooth CY manifolds, or the orbifold regime in GLSMs.
- From the Orbifold point there is a long way down to the (...N)MSSM. Phenomenologically viable R-symmetries from stringy models are to be found.

Molte Grazie