

A Zip-code for Quarks, Leptons and Higgs Bosons



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In collaboration with D. K. Mayorga and H. P. Nilles
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The \mathbb{Z}_{6-III} Mini-Landscape

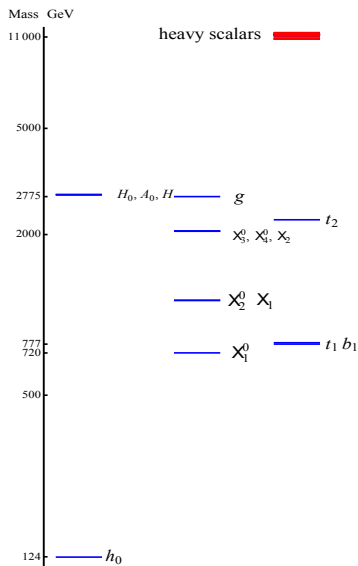
~ 200 realistic MSSM models based on the \mathbb{Z}_{6-III} orbifold geometry have been constructed in computer based searches

Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter '06-08

Main features

- 1 The Higgs system
 - Higgs doublets are **untwisted** (Gauge-Higgs-Unification) Fairlie, et. al.'79
 - Shift symmetry in Kahler potential solves vacuum instability Hebecker, et. al.'12
 - μ -term forbidden by R-Symmetry Lebedev et al., 2008; Kappl et al., 2009
- 2 The Top-Quark
 - Top-Quark is **untwisted** with tree level Higgs Yukawa coupling
 \leftrightarrow Gauge-Top-Unification Hosteins, Kappl, Ratz, Schmidt-Hoberger et. al.'07
 - Third family is a **patchwork**, completed by fields from different locations.
- 3 The light families
 - **Twisted** fields localized at fixed points
 - Form a complete **16** of $SO(10)$

SUSY Breaking Pattern



Dilaton stabilization needs:

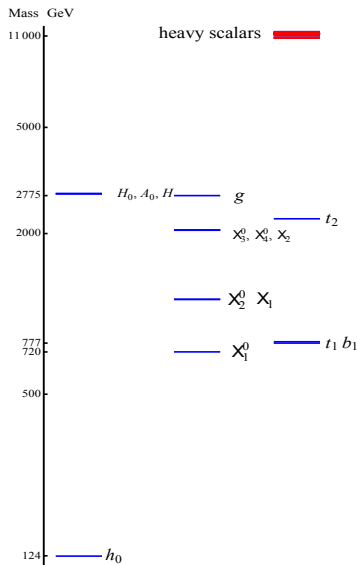
① Hidden sector gaugino condensation, favored

[Lebedev et. al.'07]

② Down-lifting of vacuum energy

$\Rightarrow m_{3/2}$ in multi-TeV range

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- Fields at **Fixed Points** feel only $\mathcal{N} = 1$ SUSY
 \rightarrow Scalar masses $\sim m_{3/2}$
- Fields in **Bulk** and **Fixed Tori** feel remnants of $\mathcal{N} = 4, 2$ SUSY
 \rightarrow Superpartner masses suppressed by $\log(M_{\text{Pl}}/m_{3/2})$

\rightarrow **Natural SUSY**

[Krippendorf et. al.'12]

Motivation

- How **general** are these results?
- What can be expected in **different geometries**?

A Zip-code for Quarks, Leptons and Higgs Bosons

Outline:

- 1 $\mathbb{Z}_2 \times \mathbb{Z}_4$ Orbifold Geometry
- 2 $\mathbb{Z}_2 \times \mathbb{Z}_4$ Phenomenology and Examples
- 3 Conclusion

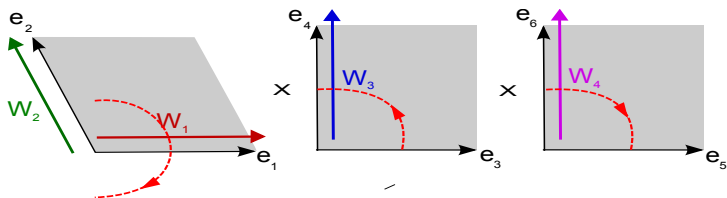
$\mathbb{Z}_2 \times \mathbb{Z}_4$ Geometry

Orbifold identification under twist $\theta(k_1, k_2)$

$$\theta(k_1, k_2) = \text{Diag}(e^{2\pi i(k_1 v_2^i + k_2 w^i)})$$

Geometric Definitions

- Choose factorizable lattice $SU(2)^2 \times SO(4) \times SO(4)$
- Choose the shifts $v_2 = (0, \frac{1}{2}, -\frac{1}{2}, 0)$ $v_4 = (0, 0, \frac{1}{4}, -\frac{1}{4})$
 - 8 sectors: $T_{(k_1, k_2)}$ $k_1 = 0, 1$ and $k_2 = 0, , 3$
 - 4 Wilson lines of order two: W_1, W_2, W_3, W_4



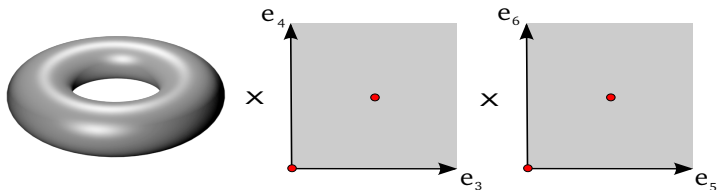
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Twisted Sectors

- $T_{(0,1)}/T_{(0,3)}$ Twisted sector
 - 4 Fixed Tori



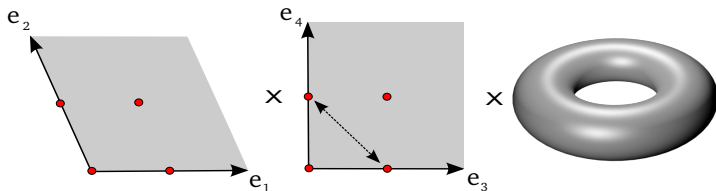
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Twisted Sectors

- $T_{(1,0)}$ Twisted Sector
 - $\boxed{12} = 8 + 4$ Fixed Tori
 - \mathbb{Z}_4 identification: Gauge enhancement at special fixed tori
 - Broken degeneracy without Wilson lines



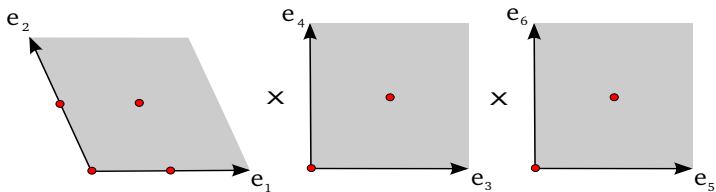
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Twisted Sectors

- $T_{(1,1)}/T_{(1,3)}$ Twisted Sector
 - 16 Fixed Points
 - Fixed points, high flexibility in breaking the degeneracy via Wilson lines



The Gauge Embedding

Modular invariance of the one-loop partition function

↔ Embedding of space group twist as translation in gauge lattice $\Lambda_{E_8 \times E_8}$

Construction of inequivalent Embeddings

Quotienting the automorphism group out of $\Lambda_{E_8 \times E_8}$, we classified all

144 (61 in \mathbb{Z}_{6-II})

inequivalent models and their brother model

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Gauge Spectrum

- 35 embeddings with $SO(10)$ gauge factors (13 in \mathbb{Z}_{6-II})
- 26 embeddings with E_6 gauge factors (16 in \mathbb{Z}_{6-II})
- 25 embeddings with $SU(5)$ gauge factors (4 in \mathbb{Z}_{6-II})
 ↪ Seems more fertile for Model building

Phenomenological Input and Constraints

Goal: Want to judge the fertility of this geometry and find preferred field localizations

Focus on $SO(10)$ breaking via Wilson lines to SM

- 1 Renormalizable Top-Yukawa from $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$
- 2 Achieve doublet-triplet splitting via Wilson lines
- 3 Break the fixed point degeneracy via Wilson lines
↪ Get three families as complete as possible

Which models are compatible with those constraints?

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Constraints Higgs location to be Wilson line affected
- ③ Break the fixed point degeneracy via Wilson lines
↔ Get three families as complete as possible
Fixes Wilson configuration and family locations

Which models are compatible with those constraints?

Example Model

Given by shifts

$$V_2 = (1, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0) (\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{1}{4})$$

$$V_4 = (\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0) (\frac{5}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2})$$

Leading to $SO(10) \times SU(2) \times SU(2) \times SU(8) \times U(1)^2$ with spectrum

U	$1 (16, 2, 1, 1)_{0,-1}$
	$1 (16, 1, 2, 1)_{0,1}$
	$1 (10, 2, 2, 1)_{0,0}$
	$1 (1, 1, 1, 1)_{-12,0}$
	$1 (1, 1, 1, 1)_{12,0}$
	$1 (1, 1, 1, 28)_{6,0}$
	$1 (1, 1, 1, 28)_{6,0}$

$T(0, 1)$	$4 (1, 1, 1, 8)_{6,1}$
	$4 (1, 1, 1, 8)_{0,1}$
$T(0, 2)$	$10 (1, 2, 2, 1)_{6,0}$
	$10 (10, 1, 1, 1)_{-6,0}$
	$6 (1, 1, 1, 1)_{-6,-2}$
	$6 (1, 1, 1, 1)_{-6,2}$

$T(0, 3)$	$4 (1, 1, 1, 8)_{6,-1}$
	$4 (1, 1, 1, 8)_{0,-1}$
$T(1, 0)$	$4 (1, 1, 2, 8)_{-3,0}$
$T(1, 1)$	Empty
$T(1, 2)$	$4 (1, 2, 1, 8)_{-3,0}$
$T(1, 3)$	$16(16, 1, 1, 1)_{3,0}$
	$16 (1, 2, 1, 1)_{3,1}$
	$16 (1, 1, 2, 1)_{3,-1}$

Example Model

Given by shifts

$$V_2 = (1, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0)(\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{1}{4})$$

$$V_4 = (\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0)(\frac{5}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2})$$

and Wilson lines

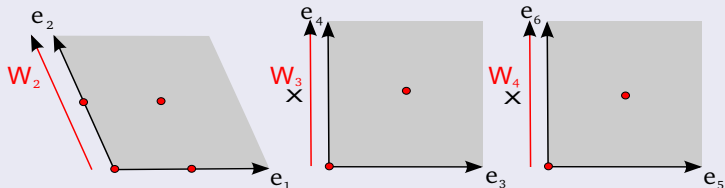
$$W_2 = (-\frac{1}{2}, \frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}, 0, -1, -1, 2)(-\frac{3}{4}, -\frac{7}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{7}{4}, \frac{3}{4}, \frac{3}{4})$$

$$W_3 = (-1, \frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, 0, 2)(-\frac{1}{2}, 1, -2, 0, \frac{3}{2}, -1, -\frac{3}{2}, -\frac{3}{2})$$

$$W_4 = (-\frac{5}{4}, \frac{5}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{5}{4}, \frac{9}{4})(0, 1, 1, 2, -1, -\frac{1}{2}, 2, \frac{3}{2})$$

Wilson line breaking

$$SO(10) \times SU(2) \times SU(2) \times SU(8) \times U(1)^2 \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3) \times SU(2) \times U(1)^9$$



Example Model

Given by shifts

$$V_2 = (1, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0)(\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}, \frac{1}{4})$$

$$V_4 = (\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0)(\frac{5}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2})$$

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Break the degeneracy

Lower the degeneracy by factor of 8

Two local **16-plets** unaffected

→ Two light families

$T(0, 3)$	4 $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \bar{\mathbf{8}})_{6, -1}$ 4 $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{8})_{0, -1}$
$T(1, 0)$	4 $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{8})_{-3, 0}$
$T(1, 1)$	Empty
$T(1, 2)$	4 $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \bar{\mathbf{8}})_{-3, 0}$
$T(1, 3)$	16 $(\mathbf{16}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{3, 0}$ 16 $(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{3, 1}$ 16 $(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})_{3, -1}$

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Project out unwanted triplets

U	1 $(16, 2, 1, 1)_{0,-1}$
	1 $(16, 1, 2, 1)_{0,1}$
	1 $(10, 2, 2, 1)_{0,0}$
	1 $(1, 1, 1, 1)_{-12,0}$
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	1 $(1, 1, 1, \mathbf{28})_{6,0}$
	1 $(1, 1, 1, \mathbf{\bar{28}})_{6,0}$

$$(16, 2, 1, 1)_{0,-1} \rightarrow (1, 1, 1, 1)_{-1, \dots} + \underbrace{(\bar{3}, 1, 1, 1)_{1/3, \dots}}_{\bar{u}}$$

$$(16, 1, 2, 1)_{0,1} \rightarrow \underbrace{(3, 2, 1, 1)_{-1/6, \dots}}_Q$$

$$(10, 2, 2, 1)_{0,0} \rightarrow \underbrace{(1, 2, 1, 1)_{-1/2, \dots}}_{H_u} + \underbrace{(1, 2, 1, 1)_{1/2, \dots}}_{H_d}$$

Example Model

Yukawa Couplings

U	1 (16, 2, 1, 1) _{0,-1}
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Trilinear Top-Yukawa:

$$\leftrightarrow (10, 2, 2, 1) \cdot (16, 1, 2, 1) \cdot (16, 2, 1, 1) \xrightarrow{W_2, W_3, W_4} H_u Q \bar{U}$$

Vector-like Higgs pair protected by R-Symmetry:

$$\leftrightarrow (10, 2, 2, 1) \cdot (10, 2, 2, 1) \xrightarrow{W_2, W_3, W_4} H_u H_d$$

Example Model

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Specific Model Details

- Net MSSM spectrum
- Non-Anomalous Hypercharge
- One unique pair of Higgs doublets
- All MSSM Exotics are vector-like
- Hidden Sector must be completely broken
- VEV configuration breaks all symmetries, preventing μ -term

The General Story

The Higgs System

- 1 Untwisted Higgs seems favored due to high flexibility to couple trilinearly
- 2 Same solution for μ -problem as in the \mathbb{Z}_{6-II} orbifold due to \mathbb{Z}_2 twist

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The Three Families

- 1 Models with three all local **16**-plets, **NOT** incompatible with heavy top
- 2 Models with 2 local **16**-plets favored
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Properties of Untwisted Sector crucial ingredient

Untwisted sectors with these features are **frequent: $\sim 75\%$ of all embeddings**

Conclusion

We investigated the MSSM matter properties found in the \mathbb{Z}_{6-II} and extended them to $\mathbb{Z}_2 \times \mathbb{Z}_4$

What we did

- Considered the factorisable $\mathbb{Z}_2 \times \mathbb{Z}_4$ geometry
- Constructed all gauge twist embeddings
- Setting a Strategy to find and then explored promising models

Our findings

The $\mathbb{Z}_2 \times \mathbb{Z}_4$ is an **extension** of the \mathbb{Z}_{6-II} Mini-Landscape with **similar** lessons:

- Higgs and Top Quark are favorably **untwisted** fields
- \mathbb{Z}_2 twist does **two** things at a time:
 - **Induces** a vector-like Higgs pair
 - **Forbids** μ -term by its R-Symmetry
- Light families originating from **localized 16** plots

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Thank you!

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Example Model: Full Spectrum

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times \underbrace{SU(3) \times SU(2) \times U(1)}_{\text{Hidden}}^9$$

#	Rep.	label	#	Rep.	label
3	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})_{\frac{2}{3}}$	\bar{u}	69	$(\mathbf{1}, \mathbf{1}, \mathbf{1})_0$	n
3	$(\mathbf{1}, \mathbf{1}, \mathbf{1})_{-1}$	\bar{e}	32	$(\mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	r
3	$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{-\frac{1}{6}}$	q	4	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	b
4	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	l	30	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	\bar{r}
1	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	\bar{l}	4	$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_0$	s
9	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{1}{3}}$	\bar{d}	10	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_0$	\check{v}
6	$(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{\frac{1}{3}}$	d	8	$(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})_0$	\bar{s}
6	$(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{1}{6}}$	f	2	$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{2}}$	χ
8	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})_0$	ν	5	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	\bar{b}
1	$(\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{-\frac{1}{6}}$	m	2	$(\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})_{-\frac{1}{2}}$	$\check{\chi}$
8	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{\frac{1}{6}}$	\bar{f}			