

# Holographic R-symmetric flows and the $\tau_U$ -conjecture

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SISSA

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# Credits

Based on:

M. Bertolini, L. Di Pietro and FP

- [arXiv:1304.1481](https://arxiv.org/abs/1304.1481)

# Intro & Motivations

- **Buican's conjecture:** in a 4d R-symmetric QFT one can define a quantity,  $\tau_U$ , which *decreases along the RG-flow*. This puts a bound on the amount of accidental symmetries. [Buican '11]
- Via holography monotonic quantities are expected to correspond to monotonic functions of the extra coordinate in 'domain wall' geometries.
- Our aim is to explore the existence of a *monotonically decreasing function* in the context of 5d SUGRA, both to test the conjecture and to refine the holographic dictionary outside the conformal regime.

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$\mathcal{N} = 1$  4d QFT preserving  $U(1)_R$ 

In such theories one can always define an *R-multiplet* and often a *Ferrara-Zumino multiplet*

## R-multiplet

$$\begin{aligned}\bar{D}^{\dot{\alpha}} \mathcal{R}_{\alpha\dot{\alpha}} &= \chi_{\alpha} \\ \bar{D}_{\dot{\alpha}} \chi_{\alpha} &= D^{\alpha} \chi_{\alpha} - \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = 0\end{aligned}$$

## Ferrara-Zumino

$$\begin{aligned}\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} &= D_{\alpha} X \\ \bar{D}_{\dot{\alpha}} X &= 0\end{aligned}$$

## R-multiplet (II)

$$\begin{aligned}\bar{D}^{\dot{\alpha}} \mathcal{R}_{\alpha\dot{\alpha}} &= \bar{D}^2 D_{\alpha} U \\ U^{\dagger} &= U\end{aligned}$$

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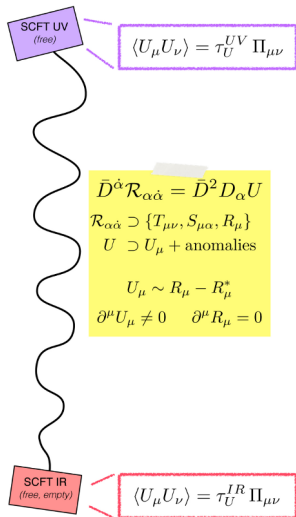
# Superconformal limit

## When the theory is superconformal

- There is a special conserved R-current  $R_\mu^*$  that can be found by a-maximization.
- $U$  becomes linear  $D^2 U = \bar{D}^2 U = 0$ : contains a conserved flavor current  $U_\mu$ .
- $U_\mu = R_\mu - R_\mu^*$
- 2-point fns of conserved currents are fixed by CI up to a positive definite hermitian matrix

$$\langle J_\mu^I J_\nu^J \rangle = \tau^{IJ} \Pi_{\mu\nu}$$

## The QFT conjecture



- **SCFT UV:**  $U = R - R_{UV}^*$  is conserved.
- **UV + deformation:**  $R_{UV}^*$  and  $U$  are broken. One picks a particular  $R$  performing a-maximization within the subset of *preserved* R-symmetries.
- **SCFT IR:**  $U = R - R_{IR}^*$  conserved. There can be accidental symmetries, if there are none  $U = R - R_{IR}^* = 0$ .

## Dictionary

Boundary:  $\mathcal{N} = 1$  4d QFTBulk:  $\mathcal{N} = 2$  5d SUGRA $\mathcal{R}^*_{\alpha\dot{\alpha}}$  $\{g, \Psi, A^*\}$  gravity multiplet

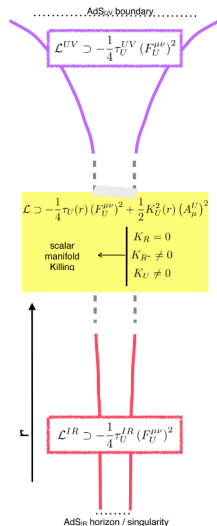
U

 $\{q, \zeta\} + \{A, \lambda, \rho\}$  hyper+vectorcurrents  $J_\mu$ gauge fields  $A_\mu$  $\tau_{IJ}$ 

gauge kinetic terms @ AdS cp's

## Holographic picture

- **near boundary:**  $A^*$ ,  $A^U$  massless gauge fields.
- **bulk:**  $A^U$  gets an  $r$ -dependent mass. Gauge kinetics terms also acquire a dependence on the extra coordinate.
- **near horizon:**  $A^U$  becomes again massless.



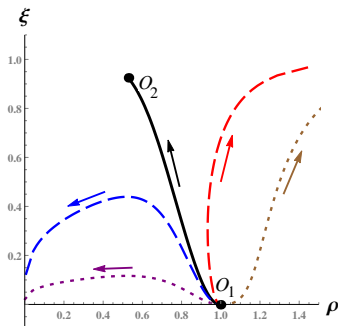
# $\mathcal{N} = 2$ 5d SUGRA with $U(1) \times U(1)_R$ gauging

## The model

[Ceresole et al. '01]

- $\mathcal{N} = 2$  SUGRA coupled to 1 hyper + 1 vector multiplets.
- Admits a two-parameter family of smooth AdS-to-AdS solns in addition to a large class of singular solns.
- All solutions have the same near boundary geometry, two scalars running, one massive vector in the bulk.
- singular solns of 2 kinds:
  - dual to confining gauge theories with mass gap
  - run into bad singularity

[Gubser '00]



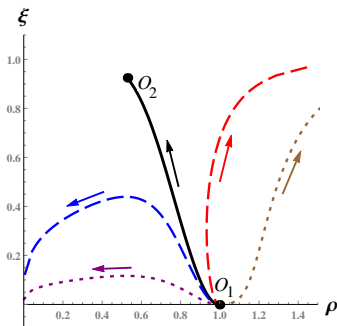
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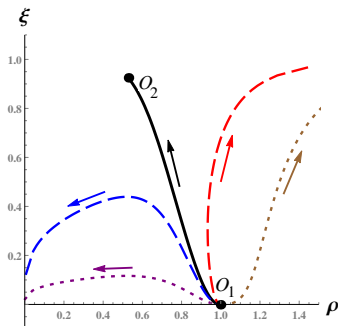
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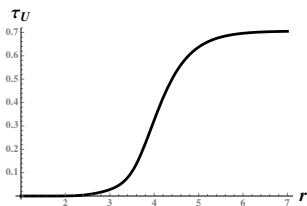
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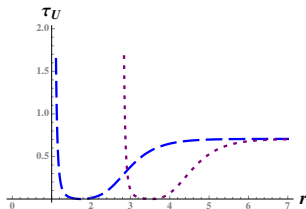


## Results



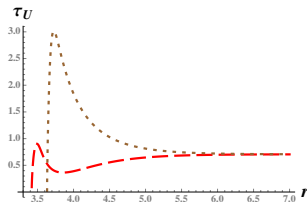
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$\tau_U$  decreases monotonically to zero. As expected for the dual of RG-flow without emergent symmetries.



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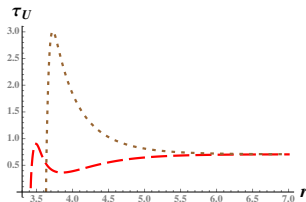
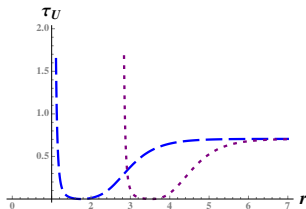
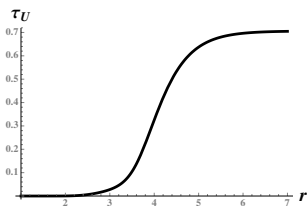
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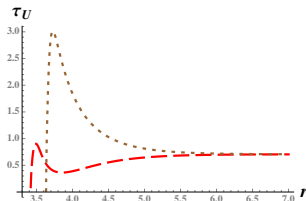
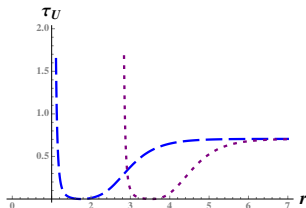
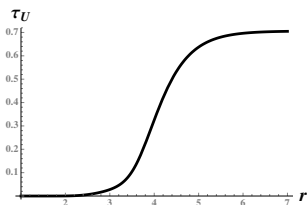
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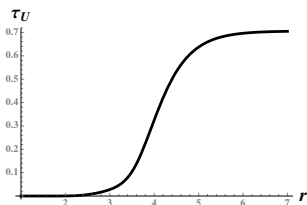
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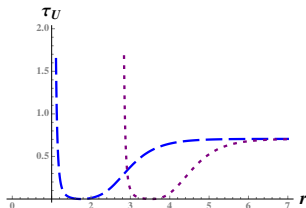
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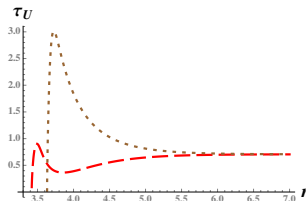
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Thank you!