Exact Results in Supersymmetric Field Theories And Applications

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Introduction

- Wealth of new exact results in supersymmetric gauge theories:
  - Exact computation of novel observables
  - New insights into the non-perturbative dynamics
  - Surprising connections with other areas of physics and maths

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Theme: Localization of SUSYc QFT's in non-trivial background geometries

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Theme: Localization of SUSYc QFT's in non-trivial background geometries

- Classic example is the Witten index:

$$Z(T^D) = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{-\beta H}$$

  - Order Parameter for SUSY breaking in strongly coupled gauge theories
  - Can be computed exactly. It captures $N^B_{E=0} - N^F_{E=0}$

- Gauge theories in curved spacetime yield more refined observables:
  - Probe more deeply the structure of the theory
  - Rich, non-trivial functions of parameters

Examples

- Partition function on $S^1 \times S^{D-1}$ Romelsberger
  - Measures short representations of SUSY, not just the vacuum states
  - Non-trivial function of chemical potentials for charges
  - If theory flows in IR to a SCFT: computes the superconformal index

- Partition function on S^D
  - In $4d$ $\mathcal{N} = 2$ gauge theories it is a non-trivial function of g^2 Pestun
  - $Z(S^D)$ is a fundamental object in QFT:
    - Unified description of c-theorem, a-theorem, F-theorem...

$$Z(S^D) = e^{-F} \qquad F_{UV} > F_{IR}$$

    - Entanglement entropy of CFT with spherical entangling surface in $R^{1,D-1}$

- Wealth of new insights into dynamics:
  - S-duality. Mapping of operators under dualities:

    $$\langle \text{Wilson loops} \rangle \longleftrightarrow \langle \text{'t Hooft loops} \rangle$$ J.G, Okuda, Pestun

  - Dynamics of strongly coupled, IR fixed points of renormalization group :
    - Infrared dualities (Seiberg duality, mirror symm,..) Kapustin, Willet, Yaakov
    - Conformal dimensions of operators at IR fixed points Jafferis
  - Surprising connections:

    gauge theories ↔ 2d nonrational CFT AGT

  2d gauge theories ↔ exact Kähler potential Calabi-Yau compactifications

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Report on recent advances in exact results in the last topic

String Theory Compactifications on Calabi-Yau Manifolds

- Phenomenologically appealing, embedded in a UV complete theory of gravity
- Calabi-Yau NLSM is a 2d $\mathcal{N}=(2,2)$ SCFT on worldsheet
- Rich interplay between spacetime and worldsheet dynamics

massless fields $\longleftrightarrow$ exactly marginal operators in SCFT

spacetime couplings $\longleftrightarrow$ correlators in SCFT

- Two classes of metric moduli and marginal operators in the SCFT

complex moduli $\longleftrightarrow$ chiral ring : $\mathcal{O}_i$ $\qquad i=1,\ldots,h^{1,2}$

Kähler moduli $\longleftrightarrow$ twisted chiral ring : $\mathcal{O}_a$ $\qquad a=1,\ldots,h^{1,1}$

Annihilated by supercharges and R-charges in $\mathcal{N}=(2,2)$ SCA

$$[q_A,\mathcal{O}_i]=[R,\mathcal{O}_i]=0$$

$$[q_B,\mathcal{O}_a]=[\mathcal{A},\mathcal{O}_a]=0$$

- Kinetic energy of moduli captured by the Zamolodchikov metric of SCFT

$$G^K_{i\bar{j}} = \langle \mathcal{O}_i(N)\,\overline{\mathcal{O}}_{\bar{j}}(S)\rangle_{S^2} \qquad G^C_{a\bar{b}} = \langle \mathcal{O}_a(N)\overline{\mathcal{O}}_{\bar{b}}(S)\rangle_{S^2}$$

Captured by the Kähler potential in Kähler and complex moduli space

$$G^K_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}^K \qquad\qquad G^C_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathcal{K}^C$$

- $\mathcal{K}^K$ has nonperturbative, worldsheet instanton corrections. Integrate over

$$\bar{\partial}\varphi = 0$$

Worldsheet instanton generating function

$$F = \sum_{\beta \in H_2(X)} N_\beta \, e^{2\pi i \int_\beta t}$$

N_β : Gromov-Witten invariants

- Yukawa couplings also corrected by worldsheet instantons

Strategy

- Realize Calabi-Yau NLSM as gauged linear sigma model(GLSM)

$$\text{GLSM} \xrightarrow{\text{IR}} \text{NLSM}$$

- Represent $G^K_{i\bar{j}}$ and $G^C_{a\bar{b}}$ as supersymmetric correlators in the GLSM

- Compute exactly by supersymmetric localization of functional integrals

$\mathcal{N} = (2,2)$ Supersymmetric Field Theories on S^2

- In the UV, can realize at most an $SU(2|1)$ subalgebra of $\mathcal{N} = (2,2)$ SCA

$$\{\bar{Q}_\alpha, Q_\beta\} = \gamma^m_{\alpha\beta} J_m - \frac{1}{2}\varepsilon_{\alpha\beta} T$$

J_m: $SU(2)$ isometry generators T: R-charge

- There are two inequivalent $SU(2|1)$ subalgebras on the two-sphere:

$SU(2\|1)_A$		$SU(2\|1)_B$
Q_A		Q_B
S_A	$\overset{\text{Mirror Automorphism}}{\longleftrightarrow}$	S_B
R		$\mathcal{A}$
J_m		J_m

- $\exists$ suitable supercharge $\mathcal{Q}_A$ and $\mathcal{Q}_B$ in $SU(2|1)_A$ and $SU(2|1)_B$

$$\langle \mathcal{O}_i \, \overline{\mathcal{O}}_{\bar{j}} \rangle \text{ is } \mathcal{Q}_A\text{-invariant}$$

$$\langle \mathcal{O}_a \, \overline{\mathcal{O}}_{\bar{b}} \rangle \text{ is } \mathcal{Q}_B\text{-invariant}$$

$\mathcal{N} = (2,2)$ Supersymmetric Field Theories on S^2

- Find a representation of $SU(2|1)_A$ and $SU(2|1)_B$ on supermultiplets

  vector multiplet: V chiral multiplet: $\overline{D}_+\Phi = \overline{D}_-\Phi = 0$

  Additional multiplets are available in $d = 2$. In particular

  twisted chiral multiplet: $\overline{D}_+ Y = D_- Y = 0$

- Lagrangian is a deformation of covariantized flat space one

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{r}\mathcal{L}_1 + \frac{1}{r^2}\mathcal{L}_2$$

  Comments:

  - Field content depends on:

    G: gauge group for vector multiplet

    $\mathbf{R}$: representation of G for matter multiplets

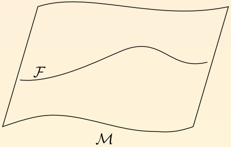
  - Lagrangian depends on two holomorphic functions:

    $\mathcal{W}(\Phi)$: superpotential $W(Y)$: twisted superpotential

Supersymetric Localization

Witten

- Functional integral localizes:



  - deform by a $\mathcal{Q}$-exact term $\mathcal{L} \to \mathcal{L} + t\,\mathcal{Q} \cdot V$
  - path integral independent of t, one-loop exact wrt $\hbar_{\text{eff}} \equiv 1/t$
  - but exact with respect to original parameters!!

- Correlation function of $\mathcal{Q}$-invariant operators given by:

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \int_{\mathcal{F}} \mathcal{O}_1 \ldots \mathcal{O}_n|_{\mathcal{F}}\ Z_{\text{1-loop}}|_{\mathcal{F}}\, e^{-S|_{\mathcal{F}}}$$

$\mathcal{F} = \{$Saddle points of $\mathcal{Q} \cdot V$: Solution to PDE's$\}$

Gauge Theory and Calabi-Yau Kähler potential

- $\exists$ supercharge $\mathcal{Q}_A$ and $\mathcal{Q}_B$ in $SU(2|1)_A$ and $SU(2|1)_B$ obeying

$$\mathcal{Q}_A^2 = J + \frac{R}{2} \qquad \mathcal{Q}_B^2 = J + \frac{\mathcal{A}}{2}$$

  such that

$$\langle \mathcal{O}_i \, \overline{\mathcal{O}}_{\bar{j}} \rangle \text{ is } \mathcal{Q}_A\text{-invariant}$$

$$\langle \mathcal{O}_a \, \overline{\mathcal{O}}_{\bar{b}} \rangle \text{ is } \mathcal{Q}_B\text{-invariant}$$

- Correlators in GLSM are independent of gauge couplings
- Gauge theory computes the theory in extreme IR: Calabi-Yau NLSM
- Calabi-Yau moduli appear as parameters in the gauge theory

  complex moduli: superpotential $\mathcal{W}$

  Kähler moduli: twisted superpotential W

$$Z_A = e^{-\mathcal{K}^K}, \qquad Z_B = e^{-\mathcal{K}^C}$$

Field Theory Approach to Kähler potential for Kähler moduli

- Field Theory realizes $SU(2|1)_A$ symmetry

1) Landau-Ginzburg model with W. The exact S^2 partition function is:

J.G, Lee

$$Z_A = \int dY d\overline{Y} e^{-4\pi i r W(Y) - 4\pi i r \overline{W}(\overline{Y})}$$

2) Gauge Theory can be localized in two complementary ways:

Doroud, Le Floch, J.G, Lee & Benini, Cremonesi

Coulomb phase:

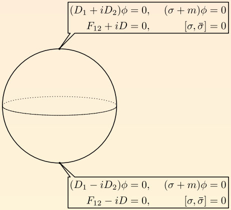
$$Z_{\rm C} = \sum_B \int_{\mathfrak{t}} da\, e^{-4\pi i r \xi a + i\vartheta B} \prod_{\alpha\in\Delta} \left[\left(\frac{\alpha\cdot B}{2r}\right)^2 + (\alpha\cdot a)^2 \right] \prod_{w\in\mathbf{R}} \frac{\Gamma\left(-ir(w\cdot a) - \frac{w\cdot B}{2}\right)}{\Gamma\left(1 + ir(w\cdot a) - \frac{w\cdot B}{2}\right)}$$

Comments:

- Mellin-Barnes representation
- Z factorizes into sum of more fundamental building blocks
- Z annihilated by system of differential operators

Higgs phase:

- $\mathcal{F}_{\text{Higgs}}$ are localized vortices and anti-vortices at the poles



- Putting vortices and anti-vortices together we obtain

$$Z_{\text{H}} = \sum_{v \in \text{vacua}} e^{-S_0}\Big|_{B=0} \text{res}_{a=v} \left[Z_{\text{one-loop}}(a, 0, m)\right] \left|Z_{\text{vortex}}(v, m, e^{2\pi i \tau})\right|^2$$

Comments:

$$Z_A = e^{-\mathcal{K}^K}$$

- Physics approach to Gromov-Witten invariants
  Jockers, Kumar, Lapan, Morrison, Romo & J.G, Lee
- Applies to Calabi-Yau's for which no other methods are available
- Does so without using mirror symmetry

Field Theory Approach to Kähler potential for complex moduli

- Field Theory realizes $SU(2|1)_B$ symmetry
- Partition function can be computed by supersymmetric localization
- Gauge theory realization of Kähler potential in complex structure moduli space of a Calabi-Yau M

Doroud, J.G

$$Z_B = e^{-\mathcal{K}^C} = \int_M \Omega \wedge \overline{\Omega}$$

Conclusions

- Our understanding of the dynamics in supersymmetric gauge theories has enjoyed significant advances in recent years
    - Realization of ubiquity of new observables in gauge theories
    - Exact computations as a tool into the dynamics
    - Deepened our understanding of dualities
    - Applications to other areas of inquiry
- Other complementary approaches to supersymmetric gauge dynamics
    - Integrability
    - Gauge/Gravity Duality