Two-Loop Corrections to Higgs Boson Masses in the Complex MSSM

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Higgs bosons in the MSSM

two complex $SU(2)$-Higgs doublets necessary,

$$h_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_1 + \phi_1^0 - \mathrm{i}\gamma_1^0\right) \\ -\phi_1^- \end{pmatrix} \quad \text{and} \quad h_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \phi_2^0 - \mathrm{i}\gamma_2^0\right) \end{pmatrix},$$

$\Rightarrow$ eight bosonic degrees of freedom:
3 Goldstone bosons, 5 physical Higgs bosons,

real parameters:

Goldstone bosons	$G^0, G^\pm$,
physical *CP* even bosons	h^0, H^0,
physical *CP* odd bosons	A^0,
physical charged Higgs bosons	$H^\pm$.

Higgs potential

- Higgs potential fixed by Lagrangian density:

$$V_{\text{Higgs}} = m_1^2 h_1^\dagger h_1 + m_2^2 h_2^\dagger h_2 - m_{12}^2 \left(h_1 \cdot h_2 + h_1^\dagger \cdot h_2^\dagger \right) + \frac{1}{8} \left(g_1^2 + g_2^2 \right) \left(h_2^\dagger h_2 - h_1^\dagger h_1 \right)^2 + \frac{1}{2} g_2^2 h_1^\dagger h_1 h_2^\dagger h_2,$$

- tree-level masses correlated:

$$m_{H^0,h^0}^2 = \frac{1}{2} \left(m_{A^0}^2 + m_Z^2 \pm \sqrt{\left(m_{A^0}^2 + m_Z^2 \right)^2 - (2 m_Z m_{A^0} \cos 2\beta)^2} \right),$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2,$$

- two free parameters: conventionally $\tan\beta = \frac{v_2}{v_1}$, m_{A^0},
- theoretical upper bound: $m_{h^0}^2 \leq (m_Z \cos 2\beta)^2$.

Higgs boson masses in higher orders

real parameters:

$$\mathcal{M} = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0 h^0} & \\ & m_{H^0}^2 - \hat{\Sigma}_{H^0 H^0} \end{pmatrix}, \quad \hat{\Sigma}_{A^0 A^0} := 0,$$

Higgs boson masses in higher orders

real parameters:

$$\mathcal{M} = \begin{pmatrix} m_{h^0}^2 - \hat{\Sigma}_{h^0 h^0} & -\hat{\Sigma}_{h^0 H^0} \\ -\hat{\Sigma}_{H^0 h^0} & m_{H^0}^2 - \hat{\Sigma}_{H^0 H^0} \end{pmatrix}, \quad \hat{\Sigma}_{A^0 A^0} := 0,$$

Higgs boson masses in higher orders

complex parameters $(\phi_{A_t}, \phi_\mu, \dots)$:

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(most general case also includes longitudinal G^0 and Z),

Higgs boson masses in higher orders

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(most general case also includes longitudinal G^0 and Z),

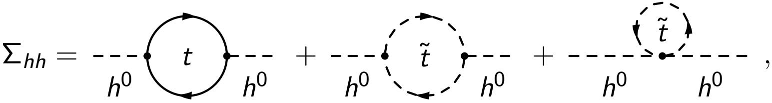
meaning of $\hat{\Sigma}$:

$$\hat{\Sigma} = \Sigma\left(p^2\right) - \delta m^2 = \hat{\Sigma}\left(p^2\right),$$

$$\hat{\Sigma}\left(p^2\right) = \hat{\Sigma}^{\text{one loop}}\left(p^2\right) + \hat{\Sigma}^{\text{two loop}}\left(p^2\right) + \ldots$$

one-loop corrections

- main contributions come from t and $\tilde{t}$ loops;
  order α_t, but proportional to m_t^4:



- additional parameters: $m_{\tilde{q}_\mathrm{L}}, m_{\tilde{t}_\mathrm{R}}, A_t, \mu$,
- mass contribution: ca. 40% of tree-level result,
- uncertainty of the calculation still too big
  (experimental accuracy $\approx \pm 0.6$GeV).

two-loop corrections

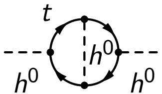
most important parts:
corrections to m_t-enhanced one-loop contributions
in a gauge-less limit,

- corrections by gluons and gluinos already known,
  order $\alpha_t \alpha_s$ in an on-shell scheme,

  [Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/0705.0746, 2007],
- corrections by Higgs and Higgsinos already known
  in the real MSSM in the effective potential approach,
  order α_t^2 in a $\overline{\text{DR}}$ scheme,

  [Brignole, Degrassi, Slavich, Zwirner, arXiv:hep-ph/0112177, 2002],
- corrections by Higgs and Higgsinos
  in the case of the complex MSSM: in process.

- again: enhancement by additional m_t^2,
- Feynman-diagrammatic approach:



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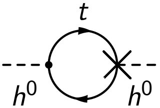

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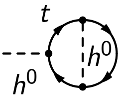
(two-loop) $+(\text{one-loop})^2$, (one-loop) $\cdot$ $(\delta^{(1)})$, $(\delta^{(2)}) + (\delta^{(1)})^2$,

- $(\delta^{(2)}) + (\delta^{(1)})^2$ acquired by renormalizing the Higgs potential, calculation of tadpole diagrams necessary:



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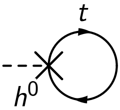

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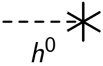

procedure of calculation

- creation of Feynman-diagrams and amplitudes with FeynArts,
  [Hahn, arXiv:hep-ph/0012260, 2001],
- applying approximations,
- reducing one-loop diagrams to master integrals with FormCalc,
  [Hahn, arXiv:hep-ph/0901.1528, 2009],
- reducing two-loop diagrams to master integrals with TwoCalc,
  [Weiglein, Scharf, Böhm, arXiv:hep-ph/9310358, 1993],
- creating counterterms from the Higgs potential,
- applying renormalisation scheme,
- evaluating renormalisation constants with FeynArts and FormCalc,
- expanding master integrals, simplifying result.

applied approximations

(similar as for $\alpha_t \alpha_s$ corrections)

1. gauge-less limit: $g_1 = 0$, $g_2 = 0$,
    - only Yukawa-couplings left,
    - $m_W = 0$, $m_Z = 0$,
    - $m_{h^0} = 0$, $m_{G^0} = 0$, $m_{G^\pm} = 0$, $m_{H^0} = m_{H^\pm}$, $m_{A^0} = m_{H^\pm}$,
    - $m_{\tilde{\chi}^0_{3,4}} = \mu$, $m_{\tilde{\chi}^\pm_2} = \mu$,
    - other Charginos and Neutralinos decouple,
    - Higgs mixing angle $\alpha = \beta - \frac{\pi}{2} < 0$,
2. external momentum equal to zero,
    - only two-loop vacuum diagrams; known analytically,
    - renormalisation constants for genuine two-loop counterterms calculated at zero momentum,
3. bottom mass equal to zero,
    - no mixing in sbottom sector,
    - one sbottom (w. l. o. g. $\tilde{b}_2$) decouples,
    - $m^2_{\tilde{b}_1} = m^2_{\tilde{t}_1} - m^2_t$.

renormalisation scheme

required renormalisation constants:

1. at one-loop:
   - δm_t, $\delta m_{\tilde{t}_1}$ and $\delta m_{\tilde{t}_2}$ fixed by on-shell condition,
   - $\delta m_{\tilde{b}_1}$ dependent on top-stop-sector,
   - δA_t fixed by on-shell condition for mixing of stops,
   - $\delta\mu$ fixed by on-shell condition for $\tilde{\chi}_2^{\pm}$ or defined in $\overline{\text{DR}}$ scheme,
   - Higgs field renormalisation constants $\delta Z_{H_1}\big|_{\text{div.}}$ and $\delta Z_{H_2}\big|_{\text{div.}}$ defined in $\overline{\text{DR}}$ scheme,
   - $\delta \tan\beta = \frac{\tan\beta}{2}(\delta Z_{H_2} - \delta Z_{H_1})\big|_{\text{div.}}$ defined in $\overline{\text{DR}}$ scheme,
   - δM_W, δM_Z (m_t^2-dependent part of $\frac{\delta M_W}{M_W}$ and $\frac{\delta M_Z}{M_Z}$),
2. at one-loop and two-loop:
   - tadpoles $\delta t_{h^0}, \delta t_{H^0}, \delta t_{A^0}$ fixed by on-shell conditions,
   - $\delta m_{H^\pm}$ fixed by on-shell condition,
   - $\delta m_{h^0}, \delta m_{H^0}, \delta m_{A^0}$ and $\delta m_{h^0 H^0}, \delta m_{h^0 A^0}, \delta m_{H^0 A^0}$ dependent on tadpoles, $\delta m_{H^\pm}$ and $\delta\tan\beta$, e. g.
     $$\delta^{(2)} m_{h^0}^2 = -\frac{e}{2 M_W s_W}\left[\delta^{(2)} t_{h^0} + c_\beta^2 \delta\tan\beta \cdot \delta t_{H^0} - \delta t_{h^0}\left(\frac{\delta M_W}{M_W} + \frac{\delta s_W}{s_W}\right)\right] + \dots$$

status and outlook

current status:

- all Feynman diagrams are generated and calculated,
- renormalisation constants for complex parameters determined,
- all divergencies cancel,
- first numerical checks:
  $\mathcal{O}(1\text{GeV})$ corrections by two-loop diagrams

outlook:

- inclusion into FeynHiggs,
  [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, arXiv:hep-ph/1007.0956, 2010],
- investigate influence of external momentum.