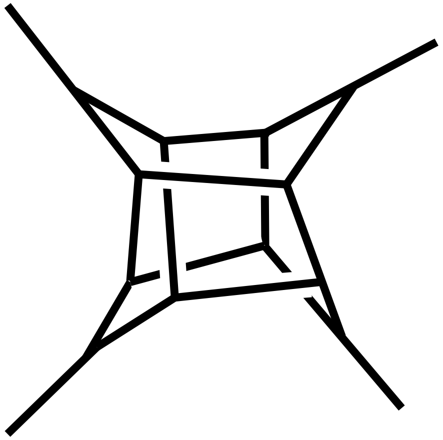
Towards Determining the UV Behavior of Maximal Supergravity



Henrik Johansson

CERN

Aug 29, 2013

SUSY 2013,

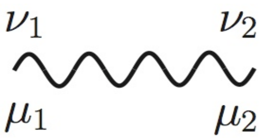
ICTP Trieste

Based on work with: Zvi Bern, John Joseph Carrasco,
Lance Dixon, Radu Roiban

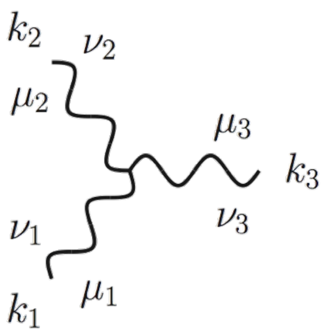
Text-book: perturbative gravity is complicated !

de Donder gauge:

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



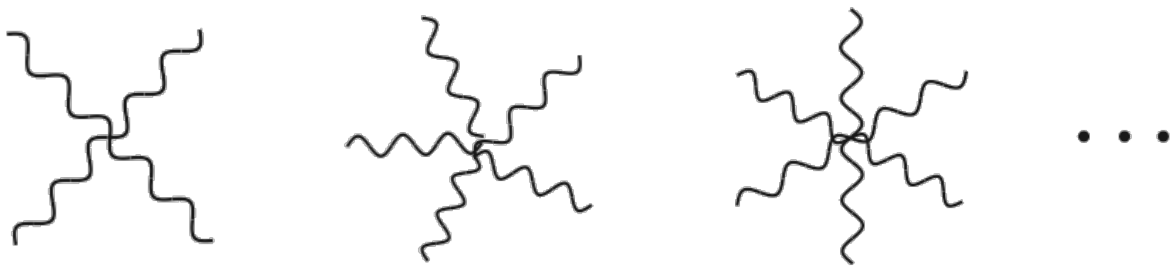
$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2+i\epsilon}$$



$$\begin{aligned} = \mathrm{sym}[&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3}) \\ &+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3}) \\ &+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3}) \\ &+2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]\end{aligned}$$

After symmetrization
~ 100 terms !

higher order vertices…



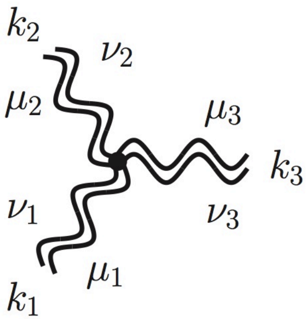
~10^3 terms

On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

Yang-Mills polarization

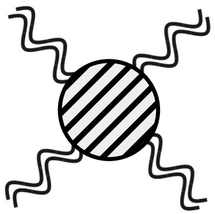
On-shell 3-graviton vertex:



$$= i\kappa\Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3} + \text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:



Yang-Mills amplitude

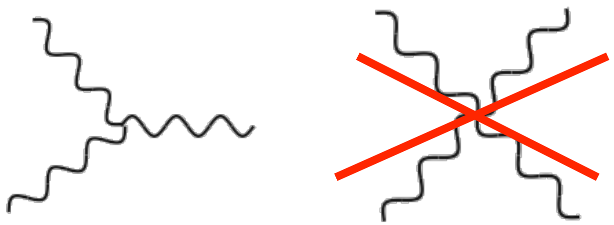
$$M_{\text{tree}}^{\text{GR}}(1,2,3,4) = \frac{st}{u} A_{\text{tree}}^{\text{YM}}(1,2,3,4) \otimes A_{\text{tree}}^{\text{YM}}(1,2,3,4)$$

Gravity processes = squares of gauge theory ones - entire S-matrix

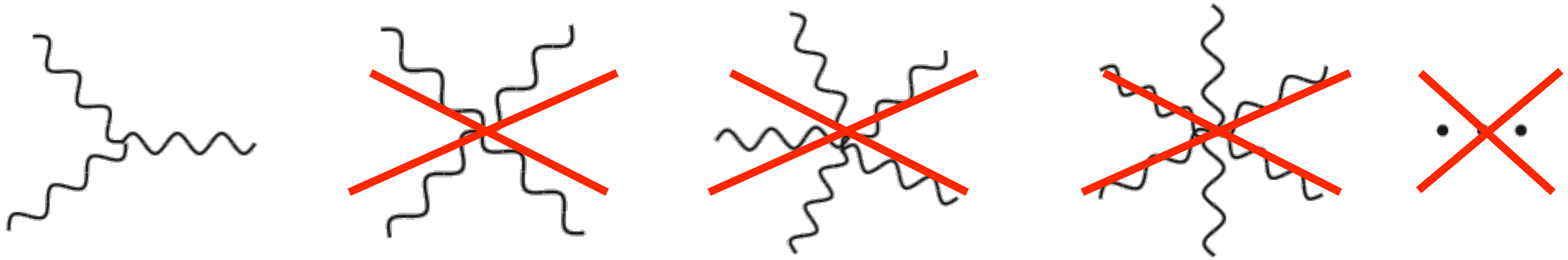
Bern, Carrasco, HJ [BCJ]

Gravity should be simple off shell

Yang-Mills → cubic very schematically: $\mathcal{L}_{\text{YM}} \sim A\Box A + \partial A^3$



Einstein gravity → cubic very schematically: $\mathcal{L}_{\text{G}} \sim h\Box h + \partial^2 h^3$



And gravity should be a double copy of a YM theory: Bern, Carrasco, HJ

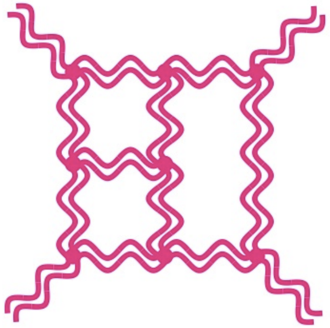
$$h^{\mu\nu} \sim A^\mu A^\nu$$

$$V_{\text{G}}(k_1, k_2, k_3) = V_{\text{YM}}(k_1, k_2, k_3) V_{\text{YM}}(k_1, k_2, k_3)$$

UV problem = basic power counting

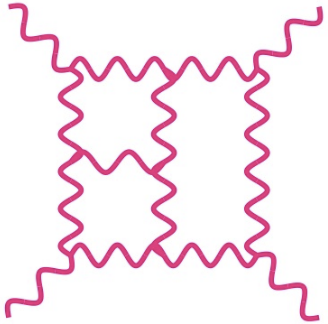
Naively expect gravity to behave worse than Yang-Mills

Gravity: non-renormalizable
dimensionful coupling



$$\sim \int d^{4L}p \frac{\dots(\kappa p^\mu p^\nu)\dots}{p_1^2 p_2^2 p_3^2 \dots p_n^2}$$

Yang-Mills: renormalizable
dimensionless coupling



$$\sim \int d^{4L}p \frac{\dots(g p^\mu)\dots}{p_1^2 p_2^2 p_3^2 \dots p_n^2}$$

For finite gravity → vast cancellations needed $\sim \quad (p^\mu)^{2L} \to (k^\mu)^{2L}$
seems implausible, but exists for *N*=8 SG in all known ampl's.

external momenta

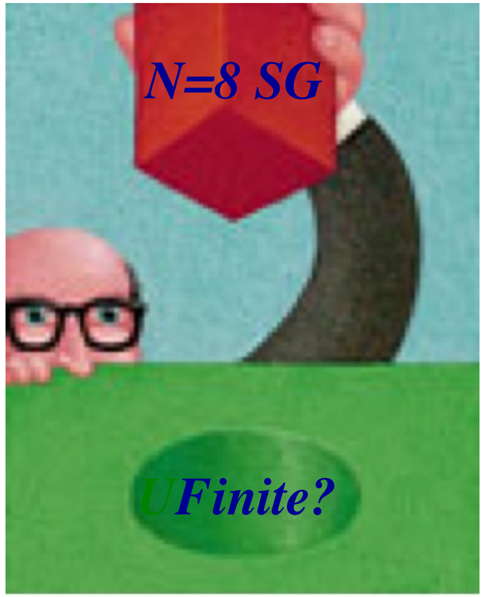
Outline

- UV status of *N*=8 SUGRA
- Duality between color and kinematics
  - Double-copy structure of gravity
  - Ability to calculate
- Amplitude UV behavior from duality
- Current 5-loop SUGRA progress
- Conclusion

SUGRA status on one page

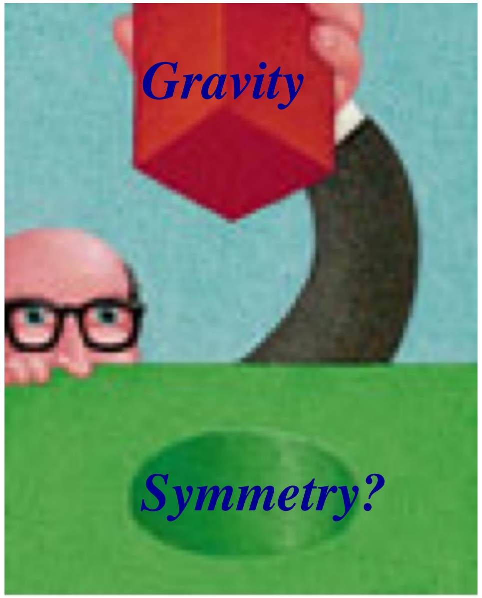
Facts:

- No *D*=4 divergence of pure SG has been found to date.
- Susy forbids 1,2 loop div. ~~R^2, R^3~~ Ferrara, Zumino, Deser, Kay, Stelle, Howe, Lindström, Green, Schwarz, Brink, Marcus, Sagnotti
- Pure gravity 1-loop finite, 2-loop divergent Goroff & Sagnotti
- With matter: 1-loop divergent 't Hooft & Veltman
- Naively susy allows 3-loop div. R^4
- $\mathcal{N}$=8 SG and $\mathcal{N}$=4 SG 3-loop finite!
  Bern, Carrasco, Dixon, HJ, Kosower, Roiban, Davies, Dennen, Huang
- $\mathcal{N}$=8 SG: no divergence before 7 loops
- *D*>4 divergences obey: $D_c = 4 + \frac{6}{L} \quad (L > 1)$
  Marcus, Sagnotti, Bern, Dixon, Dunbar,
  Perelstein, Rozowsky, Carrasco, HJ, Kosower, Roiban
- 7-loop div. in *D*=4 implies a 5-loop div.
  in *D*=24/5 – calculation in progress!



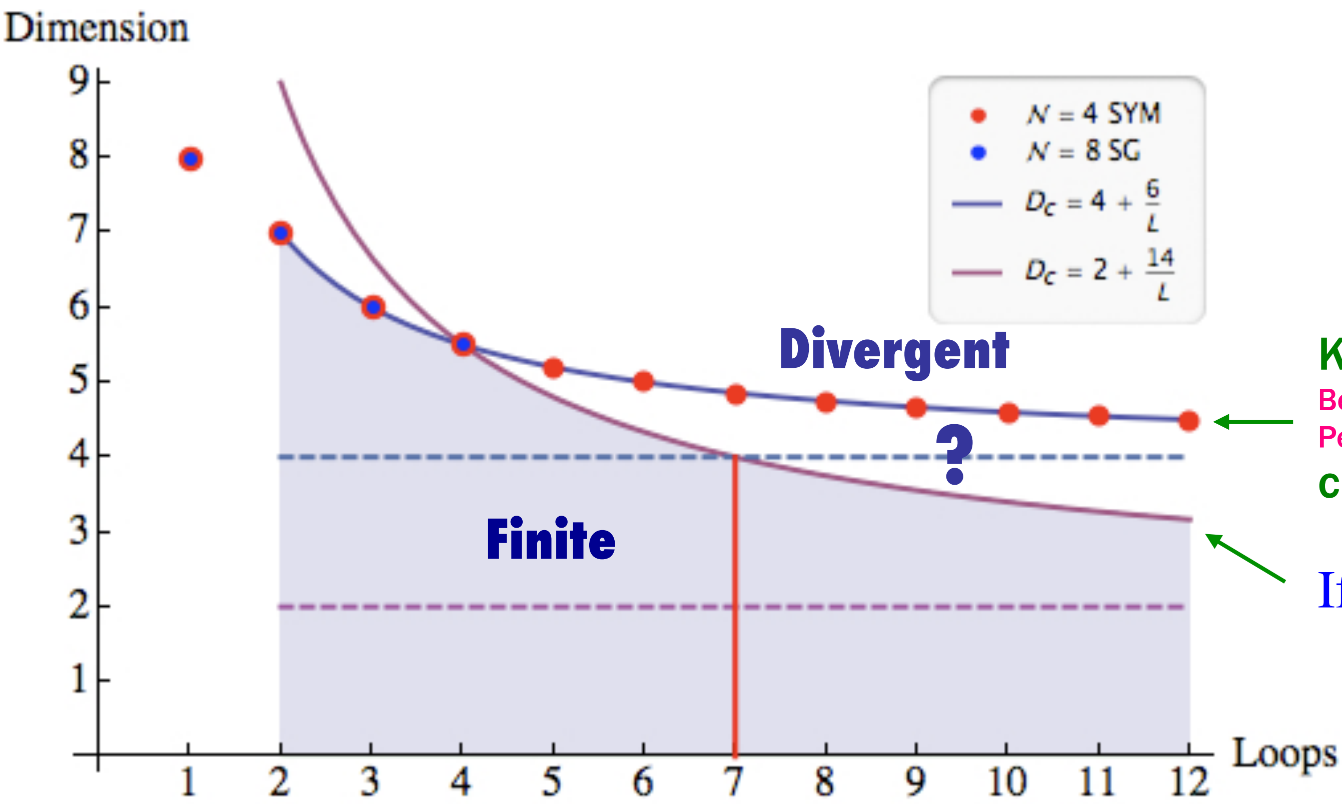
Why is it interesting ?

- If $\mathcal{N}$=8 SG is perturbatively finite, why is it interesting ?
- It better be finite for a good reason!
  - Hidden new symmetry, for example
  - Understanding the mechanism might open a host of possibilities
- Any indication of hidden structures yet?
  - Gravity is a double copy of gauge theories
  - Color-Kinematics: kinematics = Lie algebra
    Bern, Carrasco, HJ
  - Constraints from E-M duality ? Kallosh,....
  - Hidden superconformal *N*=4 SUGRA ?
    Ferrara, Kallosh, Van Proeyen
  - Extended *N*=4 superspace ? Bossard, Howe, Stelle



Known UV divergences in D>4

Plot of critical dimensions of $\mathcal{N}$= 8 SUGRA and $\mathcal{N}$= 4 SYM



Known bound for $\mathcal{N}$= 4

Bern, Dixon, Dunbar, Rozowsky, Perelstein; Howe, Stelle

current trend for $\mathcal{N}$= 8

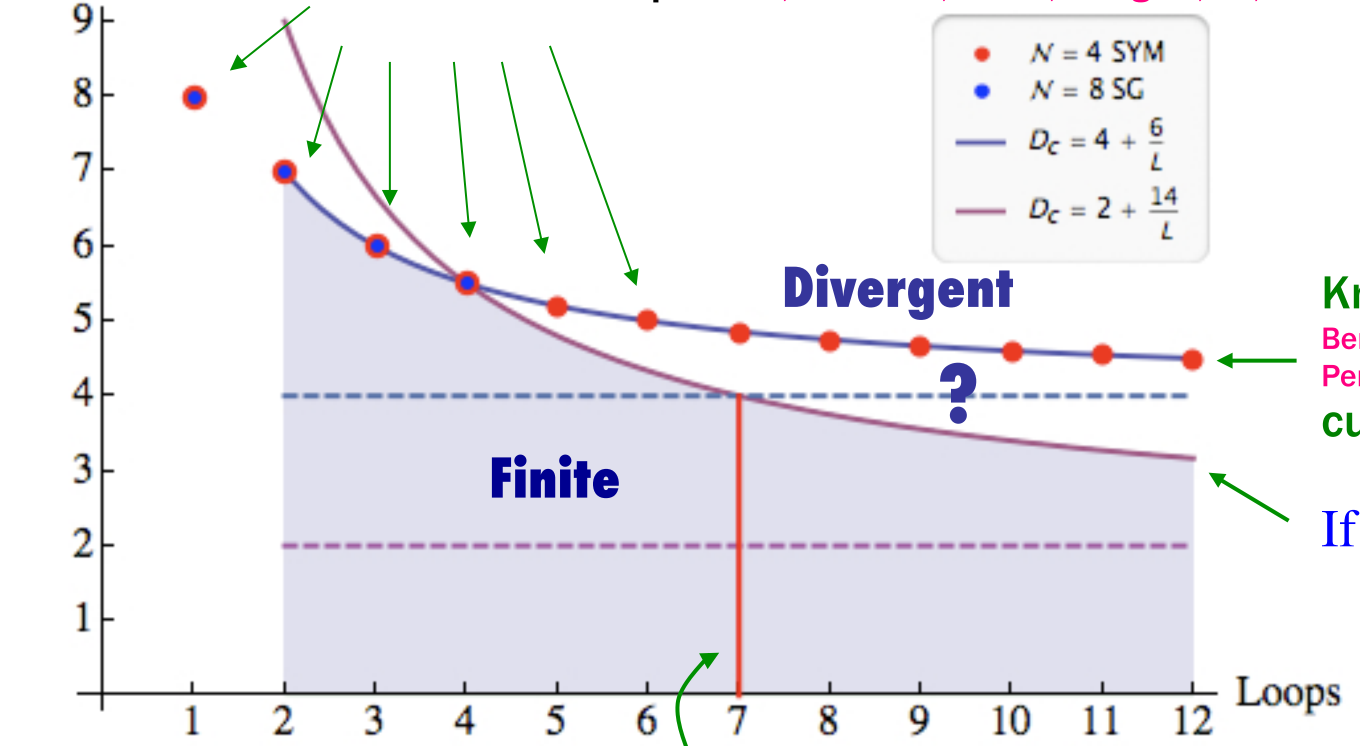
If $\mathcal{N}$= 8 div. at L=7

Known UV divergences in *D*>4

Plot of critical dimensions of $\mathcal{N}$= 8 SUGRA and $\mathcal{N}$= 4 SYM

calculations:

1-2 loops: Green, Schwarz, Brink; Marcus and Sagnotti
3-5 loops: Bern, Carrasco, Dixon, HJ, Kosower, Roiban
6 loops: Bern, Carrasco, Dixon, Douglas, HJ, von Hippel



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If $\mathcal{N}$= 8 div. at L=7

L = 7 lowest loop order for possible D = 4 divergence

Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger;
Björnsson, Green, Bossard, Howe, Stelle, Vanhove Kallosh, Ramond, Lindström, Berkovits, Grisaru, Siegel, Russo, Cederwall, Karlsson, and more….

Known UV divergences in *D*>4

Plot of critical dimensions of $\mathcal{N}$= 8 SUGRA and $\mathcal{N}$= 4 SYM

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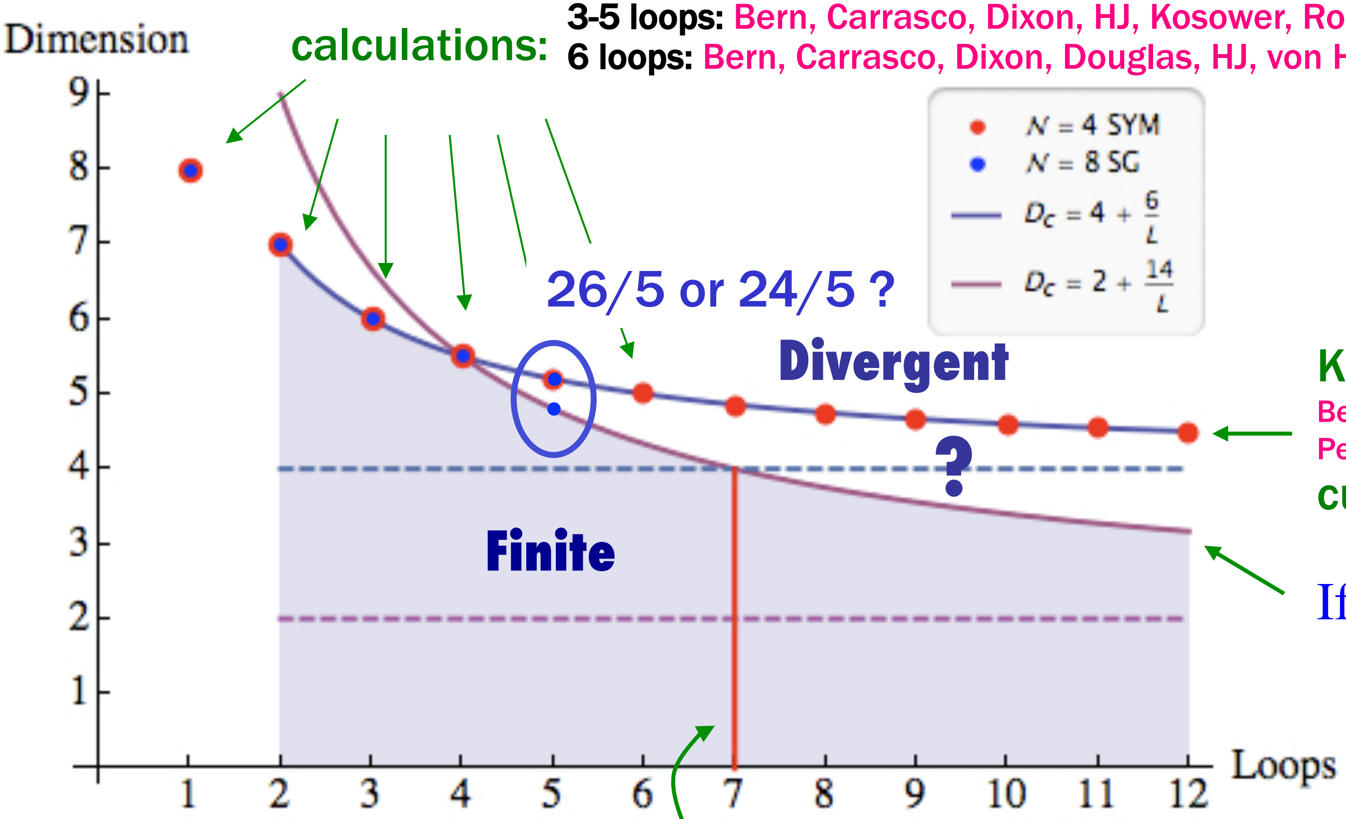

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Historical record – where is the $\mathcal{N} = 8$ div. ?

3 loops	Conventional superspace power counting	**Green, Schwarz, Brink (1982)** **Howe and Stelle (1989)** **Marcus and Sagnotti (1985)**
5 loops	Partial analysis of unitarity cuts; *If* $\mathcal{N} = 6$ harmonic superspace exists; algebraic renormalisation	**Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998)** **Howe and Stelle (2003,2009)**
6 loops	*If* $\mathcal{N} = 7$ harmonic superspace exists	**Howe and Stelle (2003)**
7 loops	*If* $\mathcal{N} = 8$ harmonic superspace exists; string theory U-duality analysis; lightcone gauge locality arguments; $E_{7(7)}$ analysis, unique 1/8 BPS candidate	**Grisaru and Siegel (1982);** **Green, Russo, Vanhove; Kallosh;** **Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger;** **Bossard, Howe, Stelle, Vanhove**
8 loops	Explicit identification of potential susy invariant counterterm with full non-linear susy	**Howe and Lindström;** **Kallosh (1981)**
9 loops	Assume Berkovits' superstring non-renormalization theorems can be carried over to $\mathcal{N} = 8$ supergravity	**Green, Russo, Vanhove (2006)**
Finite	Identified cancellations in multiloop amplitudes; lightcone gauge locality and $E_{7(7)}$, inherited from hidden N=4 SC gravity	**Bern, Dixon, Roiban (2006),** **Kallosh (2009–12),** **Ferrara, Kallosh, Van Proeyen (2012)**

note: above arguments/proofs/speculation are only lower bounds

→ only an explicit calculation can prove the existence of a divergence!

Color-Kinematics Duality

Color-Kinematics Duality

Yang-Mills theories are controlled by a kinematic Lie algebra

- Amplitude represented by cubic graphs:

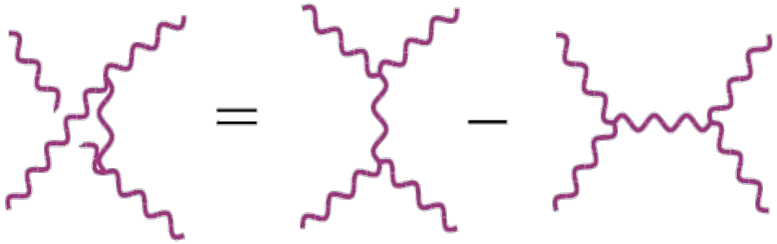
$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

numerators

color factors

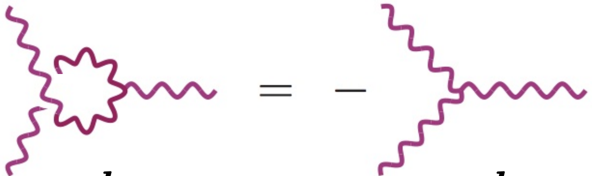
propagators

Color & kinematic numerators satisfy same relations:



$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$

Jacobi identity



$$f^{bac} = - f^{abc}$$

antisymmetry

Duality: color ↔ kinematics

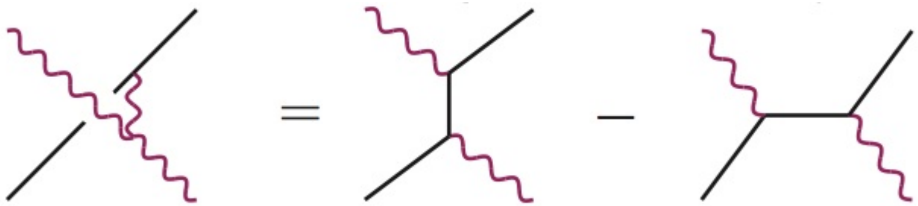
Bern, Carrasco, HJ

Some details of color-kinematics duality

Bern, Carrasco, HJ

can be checked for 4pt on-shell ampl. using Feynman rules

Example with two quarks:



$$\varepsilon_2 \cdot (\overline{u}_1 V\!\!\!/\, u_3) \cdot \varepsilon_4 = \overline{u}_1 \not{\varepsilon}_4 \not{p}_t \not{\varepsilon}_2 u_3 - \overline{u}_1 \not{\varepsilon}_2 \not{p}_s \not{\varepsilon}_4 u_3$$

$$f^{cba} T^c_{ik} = T^b_{ij} T^a_{jk} - T^a_{ij} T^b_{jk}$$

1. $(A^\mu)^4$ contact interactions absorbed into cubic graphs
   - by hand 1=*s*/*s*
   - or by auxiliary field $B \sim (A^\mu)^2$
2. Beyond 4-pts duality not automatic ➔ Lagrangian reorganization
3. Known to work at tree level: all-*n* example Kiermaier; Bjerrum-Bohr et al.
4. Enforces (BCJ) relations on partial amplitudes ➔ (*n*-3)! basis
5. Same/similar relations control string theory S-matrix

Bjerrum-Bohr, Damgaard, Vanhove; Stieberger

Gravity is a double copy

- Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i \in \Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

$$\mathcal{M}_m^{(L)} = \sum_{i \in \Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

BCJ

- The two numerators can belong to different theories:

n_i	$\tilde{n}_i$		
($\mathcal{N}$=4)	× ($\mathcal{N}$=4)	→	$\mathcal{N}$=8 sugra
($\mathcal{N}$=4)	× ($\mathcal{N}$=2)	→	$\mathcal{N}$=6 sugra
($\mathcal{N}$=4)	× ($\mathcal{N}$=0)	→	$\mathcal{N}$=4 sugra
($\mathcal{N}$=4)	× (matter)	→	$\mathcal{N}$=4 matter
($\mathcal{N}$=0)	× ($\mathcal{N}$=0)	→	Einstein gravity + axion+ dilaton

similar to Kawai-Lewellen-Tye but works at loop level

UV calculations using C-K duality

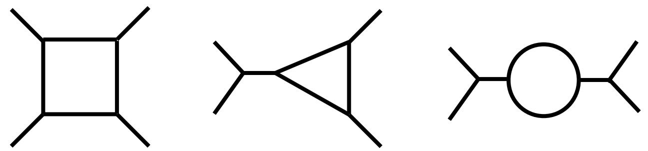
C-K duality unifies 1,2-loop UV behavior

1,2 loops *N*=4,5,6,8 SG are particularly easy to understand

$$\mathcal{M}^{\rm SG} = \sum_i n_i \times ({\rm SYM\ Integral})_i$$

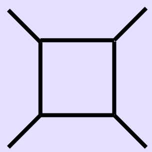
Bern, Boucher-Veronneau, Dixon, HJ

no loop momenta



***N*=4 SYM numerator:**	$stA_4^{\rm tree}$	0	0	*β*-fn vanish
$\times$	$\times$	$\times$	$\times$	
***N*=0,1,2,4 SYM integral:**	*finite*	*div.*	*div.*	*gauge theory renormalizablity*

But *N*=4 matter diverges:
Fischler; Fradkin, Tseytlin



ϕ^4

$stA_4^{\rm tree} \times div$

Bern, Davies, Dennen, Huang
1209.2472 [hep-th]

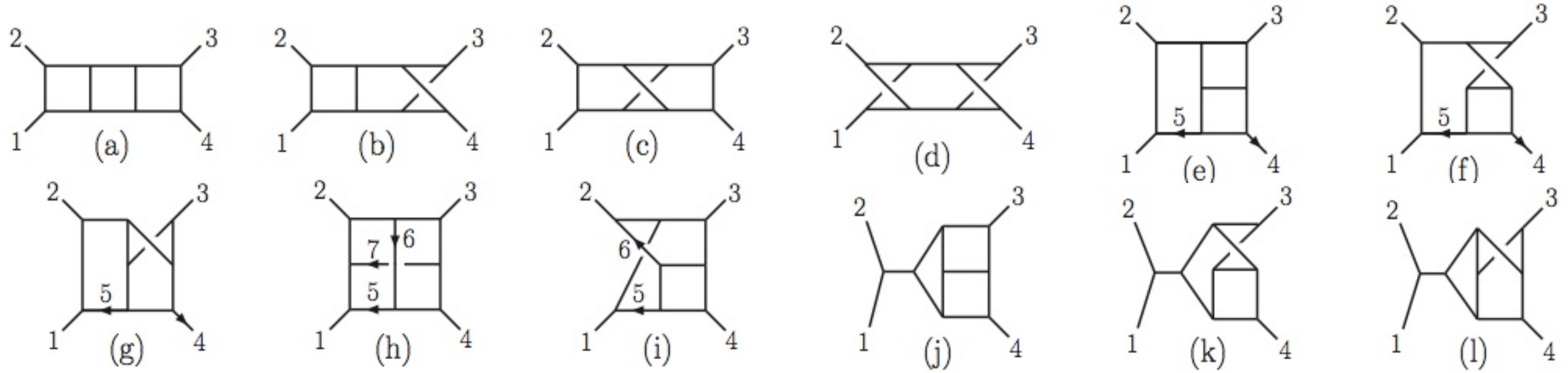
3-loop $\mathcal{N}$=8 SG & $\mathcal{N}$=4 SYM

Color-kinematics dual form:

Bern, Carrasco, HJ

$$N^{(\mathrm{e})} = s(\tau_{45} + \tau_{15}) + \frac{1}{3}(t - s)(s + \tau_{15} - \tau_{25})$$

$$\tau_{ij} = 2k_i \cdot l_j$$



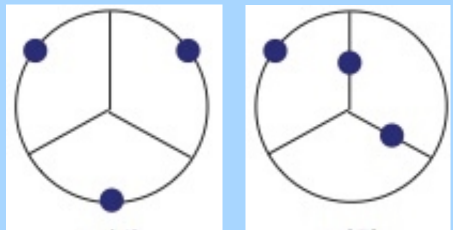
UV divergent in *D*=6:

Bern, Carrasco, Dixon, HJ, Roiban

$$\mathcal{A}^{(3)}\Big|_{\mathrm{pole}} = 2g^8 st A^{\mathrm{tree}}(N_c^3 V^{(\mathrm{A})} + 12N_c(\underline{V^{(\mathrm{A})} + 3V^{(\mathrm{B})}})) \times (u\mathrm{Tr}[T^{a_1}T^{a_2}T^{a_3}T^{a_4}] \ + \ \mathrm{perms})$$

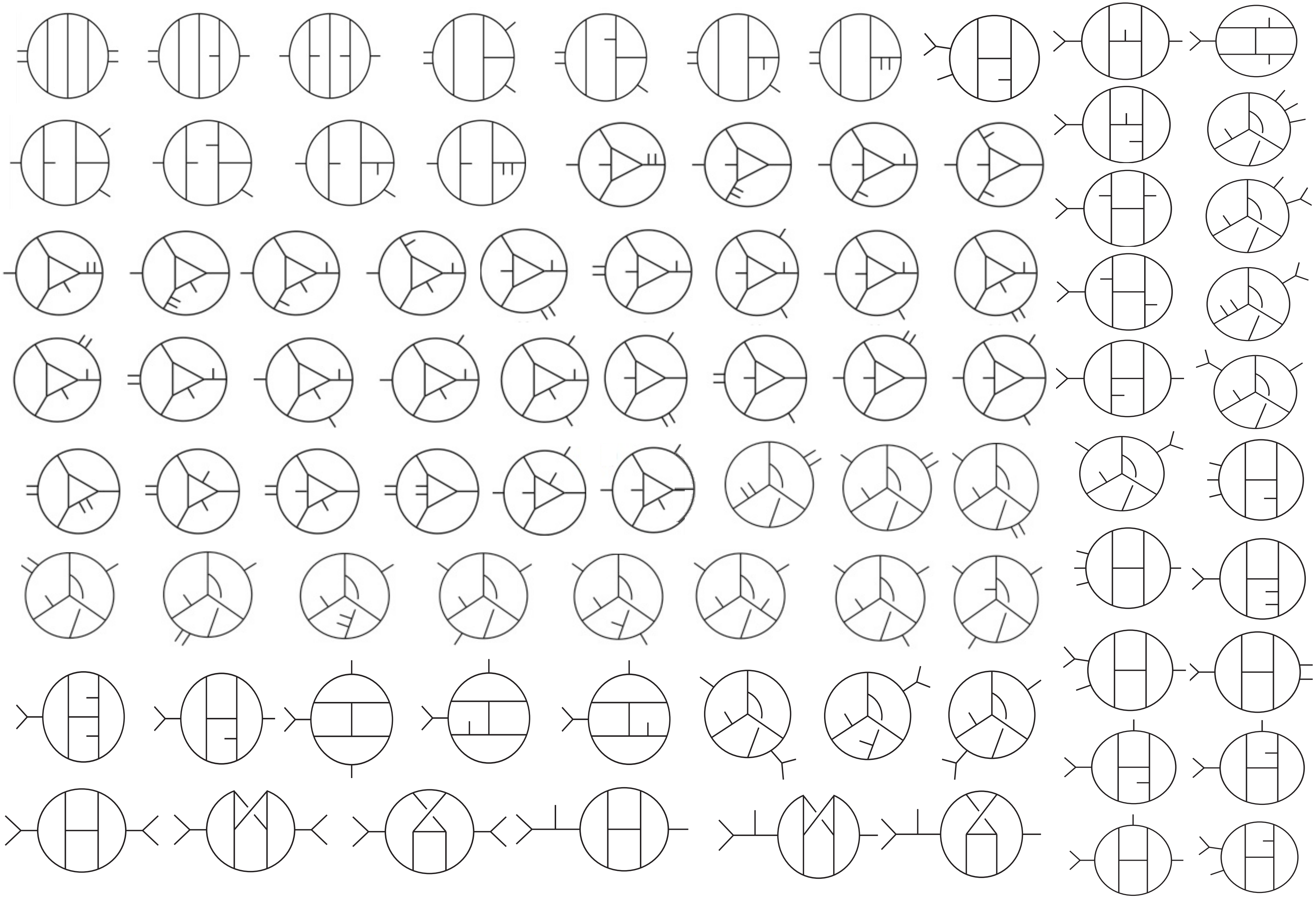
$$\mathcal{M}^{(3)}\Big|_{\mathrm{pole}} = 10\left(\frac{\kappa}{2}\right)^8 (stu)^2 M^{\mathrm{tree}}(\underline{V^{(\mathrm{A})} + 3V^{(\mathrm{B})}})$$

$$V^{(\mathrm{A})} + 3V^{(\mathrm{B})} = \frac{\zeta_3}{6}$$



$V^{(A)}$ $\quad$ $V^{(B)}$

4-loops: 85 integral types



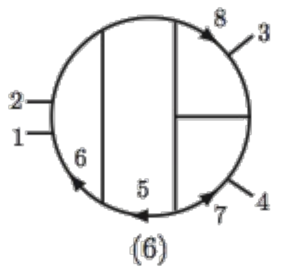
4-loops $\mathcal{N}$=4 SYM and $\mathcal{N}$=8 SG

Bern, Carrasco, Dixon, HJ, Roiban 1201.5366

- 85 diagrams
- Power counting manifest
- $\mathcal{N}$=4 & $\mathcal{N}$=8 diverge in *D*=11/2

$$N_6^{\rm SYM} = \tfrac{1}{2} s_{12}^2 (\tau_{45} - \tau_{35} - s_{12})$$

$$N_6^{\rm SG} = \left[\tfrac{1}{2} s_{12}^2 (\tau_{45} - \tau_{35} - s_{12})\right]^2$$

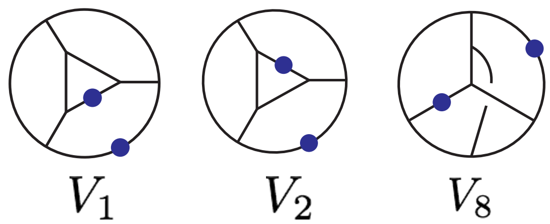


$\tau_{ij} = 2k_i \cdot l_j$

$$\mathcal{A}_4^{(4)}\Big|_{\rm pole} = -6 g^{10} s t A^{\rm tree} N_c^2 \Big(N_c^2 V_1 + 12 (V_1 + 2V_2 + V_8) \Big) \times (u {\rm Tr}_{1234} + {\rm perms})$$

$$\mathcal{M}_4^{(4)}\Big|_{\rm pole} = -\frac{23}{8} \left(\frac{\kappa}{2}\right)^{10} s t u (s^2 + t^2 + u^2)^2 M^{\rm tree} (V_1 + 2V_2 + V_8)$$

up to overall factor, divergence same as for $\mathcal{N}$=4 SYM $1/N_c^2$ *part*



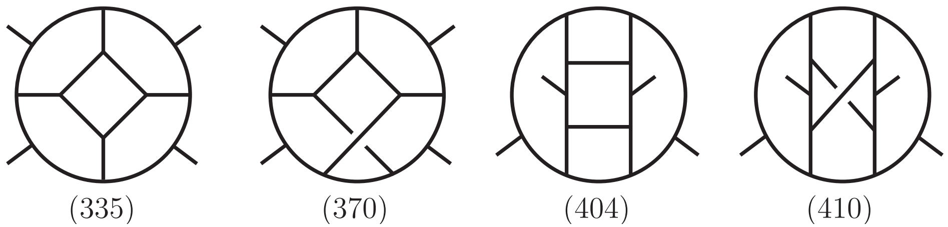
5-loop status

$\mathcal{N}$=4 SYM 5-loop Amplitude

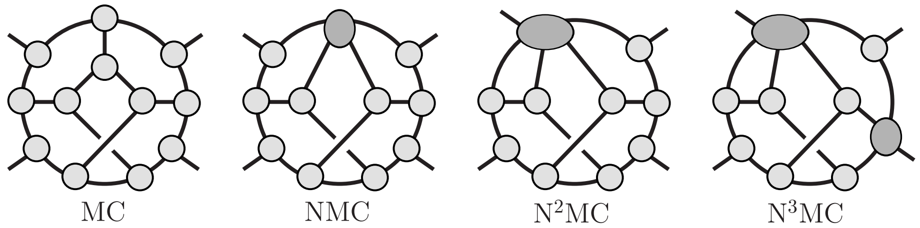
$\mathcal{N}$=4 SYM important stepping stone to $\mathcal{N}$=8 SG

1207.6666 [hep-th]
Bern, Carrasco, HJ, Roiban

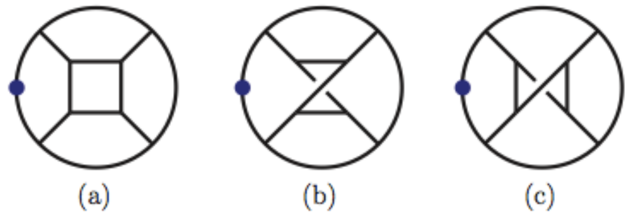
- 416 integral topologies:



- Used maximal cut method
  Bern, Carrasco, HJ, Kosower



- UV divergence in $D=26/5$:



$$\mathcal{A}_4^{(5)}\Big|_{\text{div}} = -\frac{144}{5} g^{12} s t A_4^{\text{tree}} N_c^3 \left(N_c^2 V^{(\text{a})} + 12\underline{(V^{(\text{a})} + 2V^{(\text{b})} + V^{(\text{c})})} \right) \times \text{Tr}[T^{a_1} T^{a_2} T^{a_3} T^{a_4}]$$

UV calc. makes it to Hollywood

The Parking Spot Escalation

with some help from Sheldon Cooper…

$\mathcal{N}$=8 SG 5-loop Status

Construction using only unitarity difficult

- Works for 5-loop *N*=4 SYM
- 5-loop SG too difficult this way
  (ansatz: billions of terms)

$$n^{\rm SYM} \sim 8000 \text{ terms}$$
$$n^{\rm SG} \sim (8000)^2/2$$
$$\sim 30\,000\,000 \text{ terms}$$

Only way: use color-kinematics duality

In principle: need only reorganize 5-loop $\mathcal{N}$=4 SYM Bern, Carrasco, HJ, Roiban

- 416 + 336 = 752 integral topologies
- Minimal ansatz: 1112 free parameters
- 2500 functional Jacobi eqns $\sim$ 20 000 000 linear eqns
- Solution exists: 29 free parameters
- Unfortunately, not all unitarity cuts work, there's some glitch
- Working with enlarged Ansätze … stay tuned for results!

UV div. $\leftrightarrow$ ubiquitous Casimir

5 loops, D=26/5:
SYM $\leftrightarrow$ SG
?

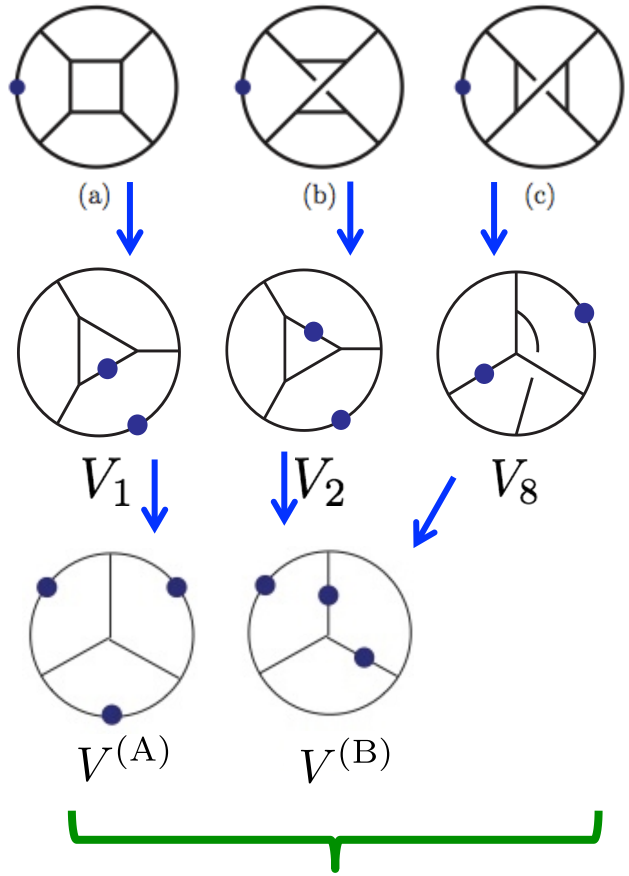
$$V^{(a)} + 2V^{(b)} + V^{(c)}$$

4 loops, D=11/2:
SYM $\leftrightarrow$ SG

$$V_1 + 2V_2 + V_8$$

3 loops, D=6:
SYM $\leftrightarrow$ SG

$$V^{(A)} + 3V^{(B)}$$
$$= V^{(A)} + 2V^{(B)} + V^{(B)}$$



$$\sim (d^{abcd})^2$$

related to diagrams in the quartic Casimir

Pattern → reason to be optimistic About UV behavior at 5 loops !

Summary

- Explicit calculations in $\mathcal{N}=8$ SUGRA up to four loops show that the power counting exactly follows that of $\mathcal{N}=4$ SYM – a finite theory
- 5 loop calculation in *D*=24/5 probes the potential 7-loop *D*=4 counterterm – will provide critical input to the $\mathcal{N}=8$ question !
- Color-Kinematics duality allows for gravity calculations for multiloop multipoint amplitudes – greatly facilitating UV analysis in gravity.
- Numbers in UV divergences of $\mathcal{N}$=8 SUGRA and $1/N_c^2$ $\mathcal{N}$=4 SYM coincide, suggesting a deeper connection between the theories
- Stay tuned for the 5-loop SUGRA result…

THANK YOU!