# Exploring Universal Extra-Dimensions at the LHC

#### **Alexander Belyaev**

NEXT

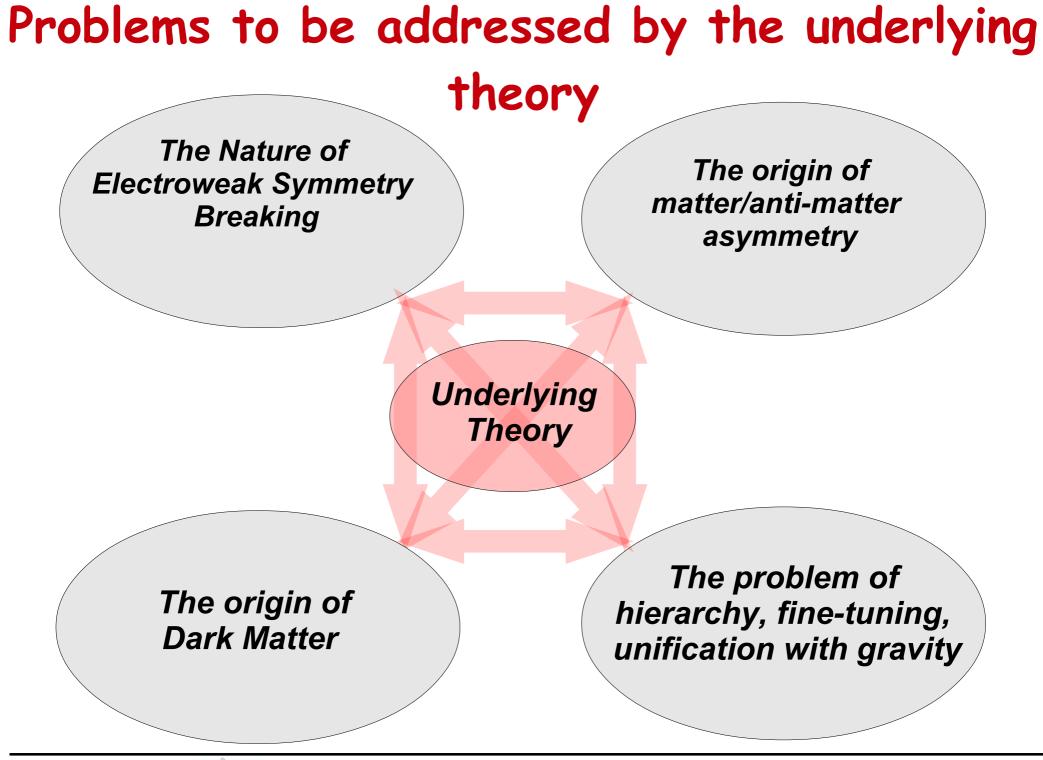
#### **Southampton University & Rutherford Appleton Laboratory**

# SUSY 2013

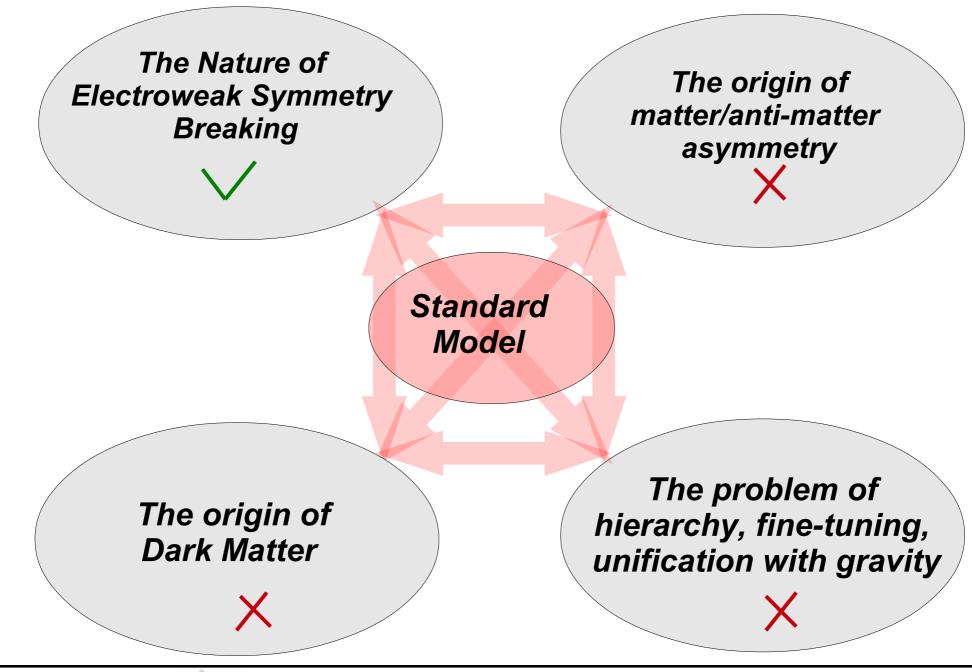
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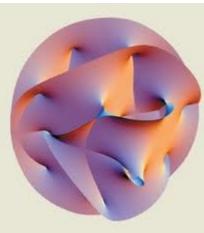


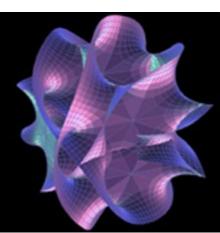
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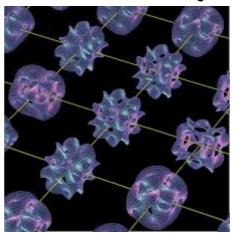


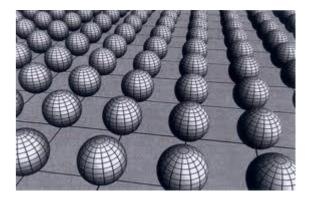
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## What could lie below the 10<sup>-19</sup>m scale? Extra Dimensions! (ED)





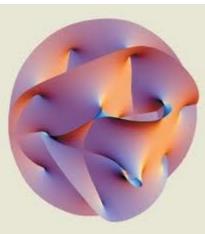


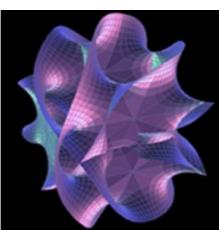


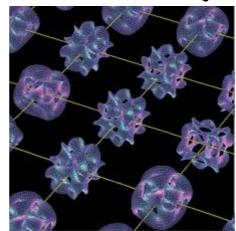


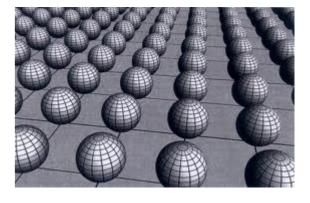


## What could lie below the 10<sup>-19</sup>m scale? Extra Dimensions! (ED)









#### Motivations

- String theory, the best candidate to unify gravity & gauge interactions, is only consistent in 10 D space-time
- Extending symmetries:

Internal symmetries - GUTs, technicolour...; Fermionic spacetime- SUSY Bosonic spacetime - Extra dimensions

• The presence of XD could have an impact on scales  $<< M_{planck}$  (started with ADD)

#### The question is what is the size and the shape of ED ?!



# New perspectives of XD

- The nature of electroweak symmetry breaking
- The origin of fermion mass hierarchies
- The supersymmetry breaking mechanism
- The description of strongly interacting sectors (provide a way to model them)



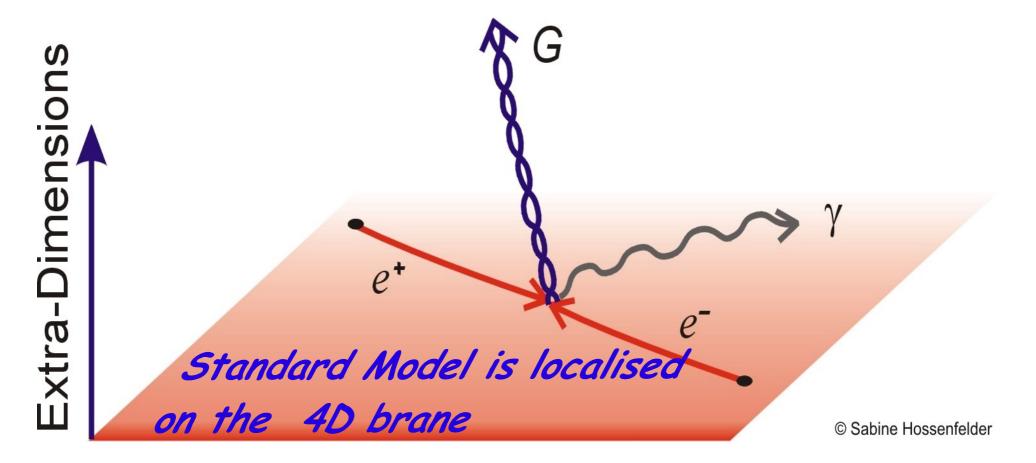


- 1914: Nordstrom tried to unify gravity and electromagnetism in 5D

   (A<sub>µ</sub> -> A<sub>µ</sub>, where M = 0,1,2,3,4)
- 1920's: Kaluza and Klein tried using Einstein's equations in 5D ( $g^{\mu\nu} \rightarrow g^{MN} \sim g^{\mu\nu} g^{\mu4} g^{44}$ )
- 1970's: Development of superstring theory and supergravity required extra dimensions
- 1998: Arkani-Hamed, Dimopoulos, and Dvali propose
   Large Extra Dimensions (ADD) as a solution to the
   Hierarchy /Fine tuning problem of the Standard Model



• The Standard Model has been tested to r ~  $10^{-16}$  mm, Gravity has been tested to r ~ 1 mm only





- The Standard Model has been tested to r ~ 10<sup>-16</sup> mm, Gravity has been tested to r ~ 1 mm only
- 4D -> (4 + n)D

 $V_n = (2\pi R)^n$ 

The effective D = 4 action is

$$\frac{M_{\mathbf{f}}^{2+n}}{2} \int d^4x \int_0^{2\pi R} d^n Z \sqrt{G} R_{4+n} \longrightarrow \frac{1}{2} M_{\mathbf{f}}^{2+n} V_n \int d^4x \sqrt{g} R$$

In case of toroidal compactification of equal radii, R

$$M_P^2 = M_f^{2+n} V_n$$



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$$V_n = (2\pi R)^n \qquad \qquad M_P^2 = M_f^{2+n} V_n$$

$$\Rightarrow R \Rightarrow \text{ the torus} \qquad V(r) = -G_N \frac{m_1 m_2}{r} = -\frac{m_1 m_2}{M_P^2 r}$$
effectively disappear

NE>



- The Standard Model has been tested to  $r \sim 10^{-16}$  mm, Gravity has been tested to  $r \sim 1$  mm only
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In case of toroidal compactification of equal radii, R

$$V_{n} = (2\pi R)^{n}$$

$$M_{P}^{2} = M_{f}^{2+n}V_{n}$$

$$r > R \Rightarrow \text{ the torus}$$

$$V(r) = -G_{N}\frac{m_{1}m_{2}}{r} = -\frac{m_{1}m_{2}}{M_{P}^{2}r}$$

$$r < R \Rightarrow \text{ observer}$$

$$V(r) = -G_{*}\frac{m_{1}m_{2}}{r} = -\frac{m_{1}m_{2}}{M_{f}^{2+n}r^{1+n}}$$

is able to feel the bulk

2



 $(M_f^{2+n}r^{1+n})$ 

- The Standard Model has been tested to  $r \sim 10^{-16}$  mm. Gravity has been tested to  $r \sim 1$  mm only
- 4D -> (4 + n)D

The effective D = 4 action is

 $\frac{M_{\mathbf{f}}^{2+n}}{2} \int d^4x \int_0^{2\pi R} d^n Z \sqrt{G} R_{4+n} \longrightarrow \frac{1}{2} M_{\mathbf{f}}^{2+n} V_n \int d^4x \sqrt{g} R$ 

In case of toroidal compactification of equal radii, R

$$V_n = (2\pi R)^n \qquad \qquad M_P^2 = M_f^{2+n} V_n$$

 $V(r) = -G_N \frac{m_1 m_2}{r} = -\frac{m_1 m_2}{M_P^2 r}$  $r >> R \Rightarrow$  the torus effectively disappear  $V(r) = -G_* \frac{m_1 m_2}{r} =$  $r < R \Rightarrow observer$  $M_f^{2+n}r^{1+n}$ is able to feel the bulk Fundamental quantum gravity scale

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 $m_1 m_2$ 

## The current status of ADD

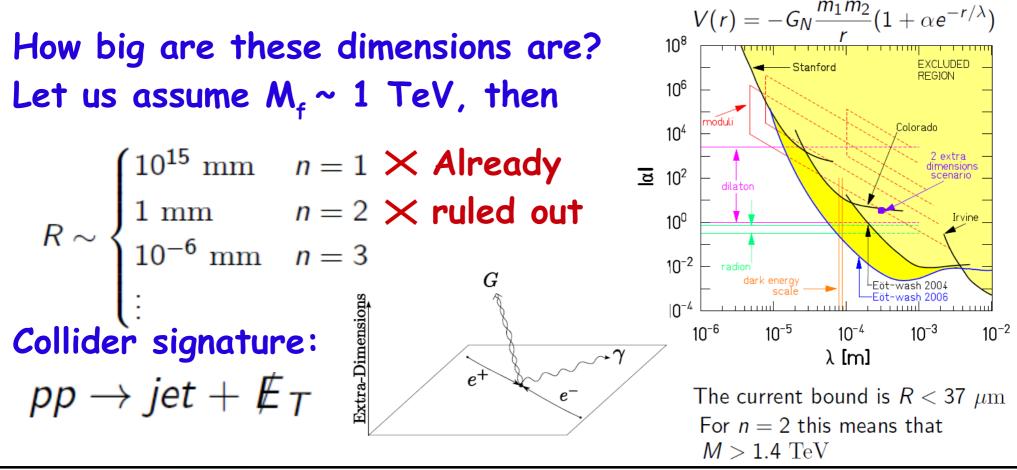
So, 
$$M_P^2 = M_f^{n+2} (2\pi R)^n$$
 and respectively,  
$$R = \frac{1}{2\pi} \frac{1}{M_f} \left(\frac{M_P}{M_f}\right)^{\frac{2}{n}} [\text{GeV}^{-1}] \times 0.197 [\text{ GeV m}]$$



# The current status of ADD

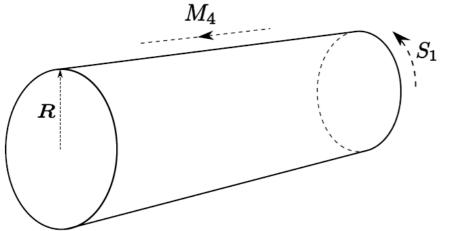
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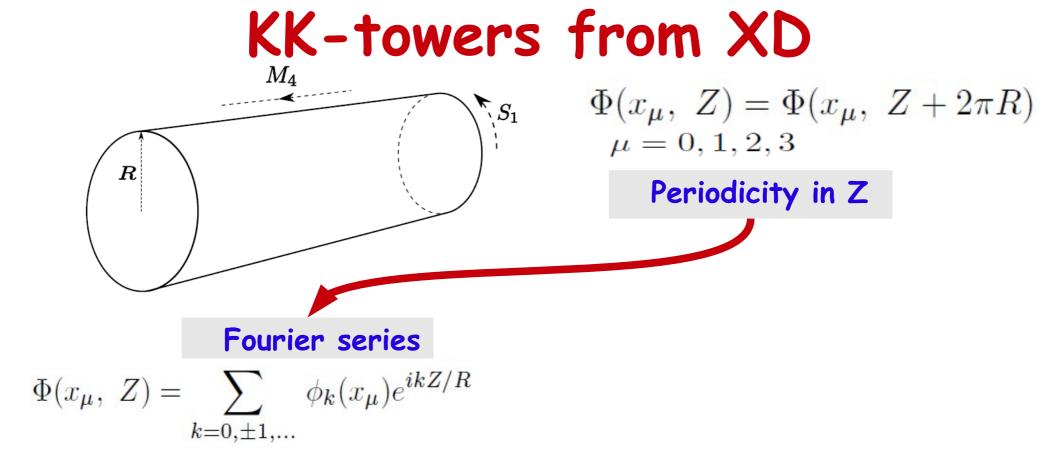
## **KK-towers from XD**



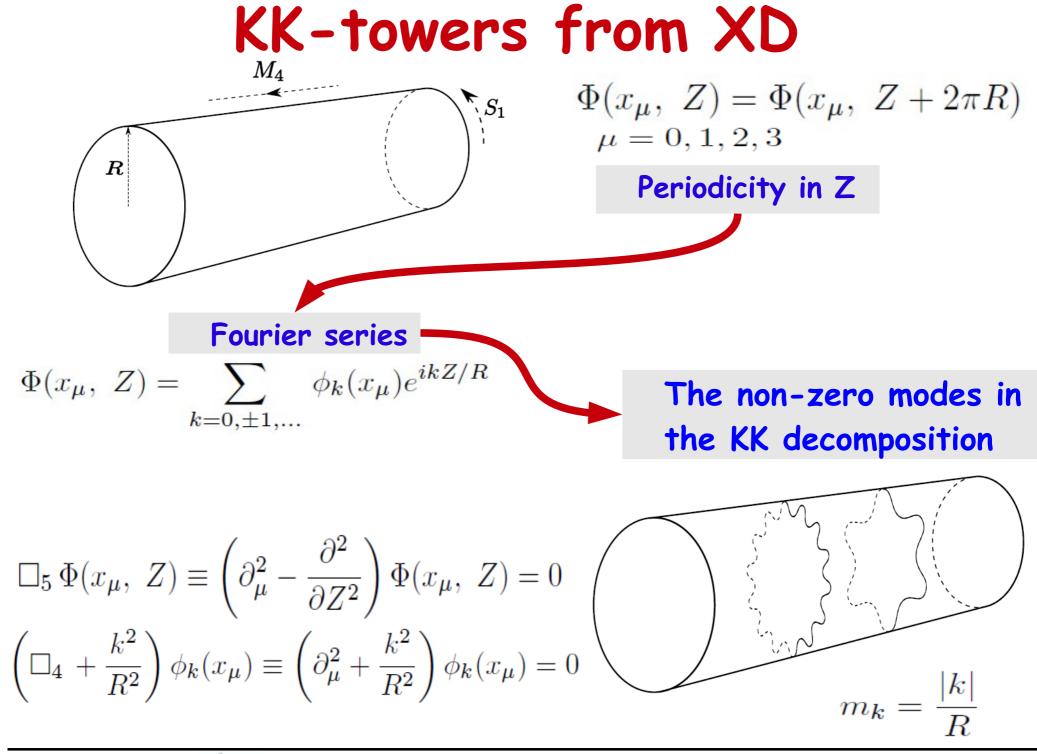
 $\Phi(x_{\mu}, Z) = \Phi(x_{\mu}, Z + 2\pi R)$  $\mu = 0, 1, 2, 3$ 

Periodicity in Z











#### From Brane - to Bulk: Universal Extra Dimensions (UED)

[Appelquist, Cheng, Dobrescu '01]

- all fields propagate in the extra dimensions, so 1/R > 1 TeV to obey experimental data
- for D=5 (minimal UED = MUED) we immediately find that M<sub>f</sub>=10<sup>15</sup> GeV for 1/R = 1TeV
- hierarchy problem is not addressed but MUED has interesting features ...



#### Minimal Universal Extra Dimensions compactifying on the circle

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}}\phi_0(x) + \sqrt{\frac{\pi}{R}}\sum_{n=1}^{\infty} \left[\phi_n^+(x)\cos\frac{ny}{R} + \phi_n^-(x)\sin\frac{ny}{R}\right]$$

$$S = \int d^4x \underbrace{\int_0^{2\pi R} dy \frac{1}{2} \left[ \partial_M \phi \partial^M \phi - m^2 \phi(x, y)^2 \right]}_{\mathcal{L}_5}$$

$$\mathcal{L}_{4} = \frac{1}{2} \left[ \partial_{\mu} \phi_{0} \partial^{\mu} \phi_{0} - m^{2} \phi_{0}^{2} \right] + \sum_{n=1}^{\infty} \frac{1}{2} \left[ \partial_{\mu} \phi_{n}^{\pm} \partial^{\mu} \phi_{n}^{\pm} - \overbrace{\left(m^{2} + \frac{n^{2}}{R^{2}}\right)}^{n} \phi_{n}^{\pm 2} \right]$$

- all fields propagate in the bulk 5D momentum \ conservation
- This leads to the KK-number conservation at this point:  $\pm n_1 \pm n_2 = \pm n_3$



OK

 $m^2$ 

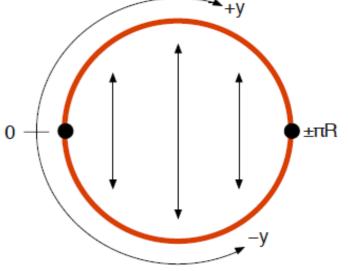
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#### Universal Extra Dimensions (UED) compactifying on the orbifold

 Choose action of Z<sub>2</sub> symmetry on Dirac Fermions to project out <sup>1</sup>/<sub>2</sub> of them and arranges chirality:

$$\psi_{\pm}(y) \mapsto \psi'_{\pm}(-y) = \pm \gamma^5 \psi_{\pm}(y)$$

If we identify  $y \sim -y$  then we require  $\psi'_{\pm}(y) = \psi_{\pm}(y)$ , so  $\psi_{\pm}(y) = \psi_0^{R,L} + \sum_n \left(\psi_n^{R,L} \cos_n + \psi_n^{L,R} \sin_n\right)$ 



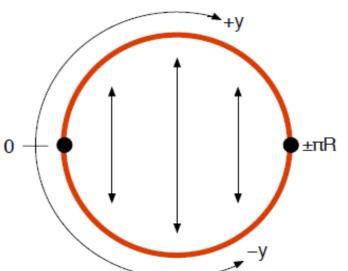


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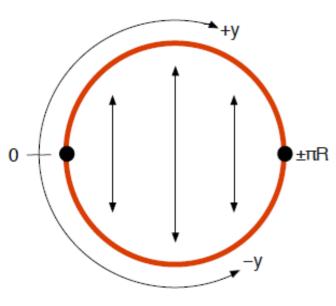
 Translational invariance along the 5<sup>th</sup> D is broken, but KK parity is preserved!
 KK number n broken 2 down to the KK parity, (-1)<sup>n</sup>:

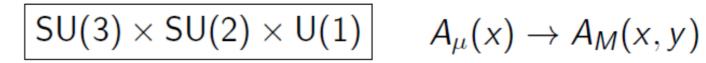
KK excitations must be produced in pairs

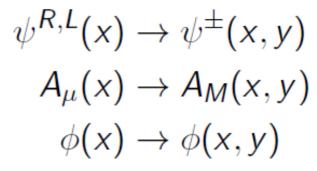
#### • LKP is stable DM candidate!

These vertices are allowed and can be generated at loop-level

### **Minimal Universal Extra Dimensions**





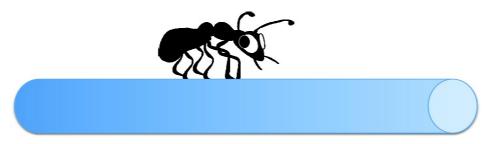


 $\mathsf{S}^1/\mathcal{Z}_2$  orbifold

SM Gauge group

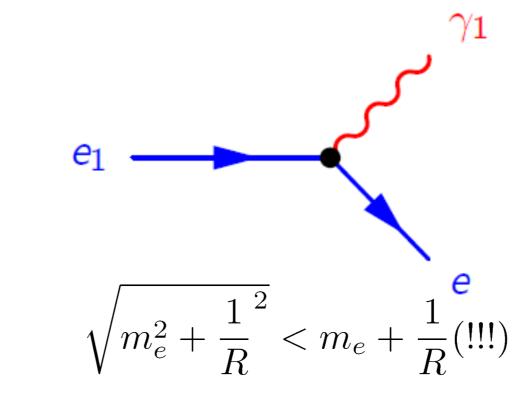
SM field content

brane localised terms are zero at the cutoff scale





### The role of radiative corrections



e.g. the 1<sup>st</sup> KK excitation of the electron is stable at tree-level!

#### Dark Matter would be charged - which is not acceptable



#### MUED at one loop

Cheng, Matchev, Schmaltz 2002

Loop corrections come from 5D Lorentz violating processes. They appear as tree-level mass corrections in 4D.

#### Bulk corrections :

the gauge bosons receive an extra mass which is KK-independent

$$\delta m_n^2 = \alpha_i \ \frac{1}{R^2}$$

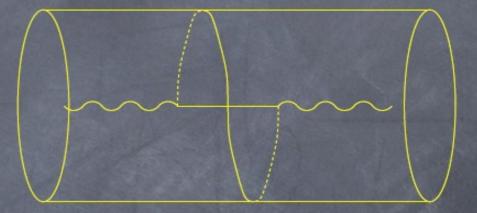
 $\bullet$  Brane corrections :  $p_5$  is not conserved, all particles receive a mass correction

$$\delta m_n = eta_i \, rac{n}{R} \ln rac{\Lambda^2}{\mu^2}$$
 for fermion  
 $\delta m_n^2 = eta_i \, rac{n^2}{R^2} \ln rac{\Lambda^2}{\mu^2}$  for bosons

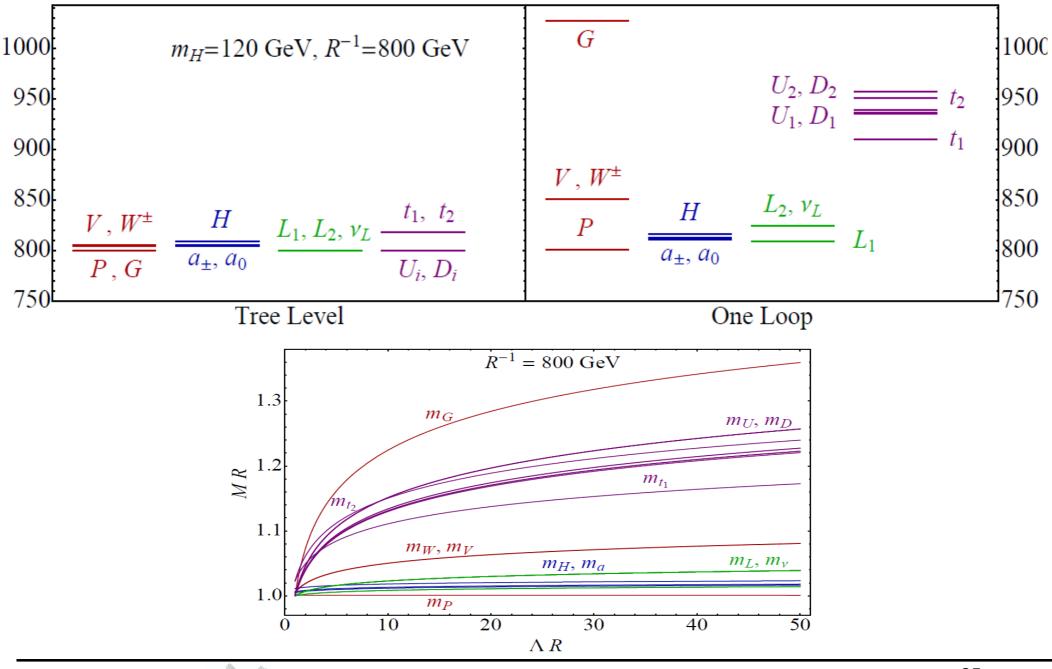
#### **Problem :** Electroweak symmetry breaking was not included

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## MUED spectrum at 1100p vs tree-level



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#### Our setup

We model the corrections to the self-energy by wave-function normalisations. We replace a 5D-Lorentz conserving action

$$\frac{1}{4}F^a_{MN}F^{a\,MN} + \left|D_M\Phi\right|^2$$

by the following

$$-\frac{1}{4}F^{a\,\mu\nu}F^{a}_{\mu\nu} + \frac{1}{2}Z_{v}F^{a}_{\mu5}F^{a\,\mu}_{5} + |D_{\mu}\Phi|^{2} - Z_{\Phi}|D_{5}\Phi|^{2}$$

which is gauge invariant but not Lorentz covariant. In this way, the fields receive a KK mass

$$m_n \;=\; Z\, {n\over R}$$
 for fermions ,  $\;m_n^2 \;=\; Z\, {n^2\over R^2}$  for bosons

We are free to match our normalisations with the previous results  $Z_i = 1 + \beta_i \ln \frac{\Lambda^2}{\mu^2}$ 



#### Model implementation

• In LanHEP : LanHEP is a package that generates the Feynman rules out of a Lagrangian.

We have implemented MUED@1L in Feynman and unitary gauges. We discart the bulk corrections.

• In CalcHEP/CompHEP : CalcHEP calculates cross-sections out of Feynman rules of a theory. The vertices generated by LanHEP are included into CalcHEP. We have taken particular care of the splitting of 4-gluon vertices.

Model is available at High Energy Physcs Model Database (HEPMDB) http://hepmdb.soton.ac.uk/hepmdb:1212.0121



#### **Model Validation**

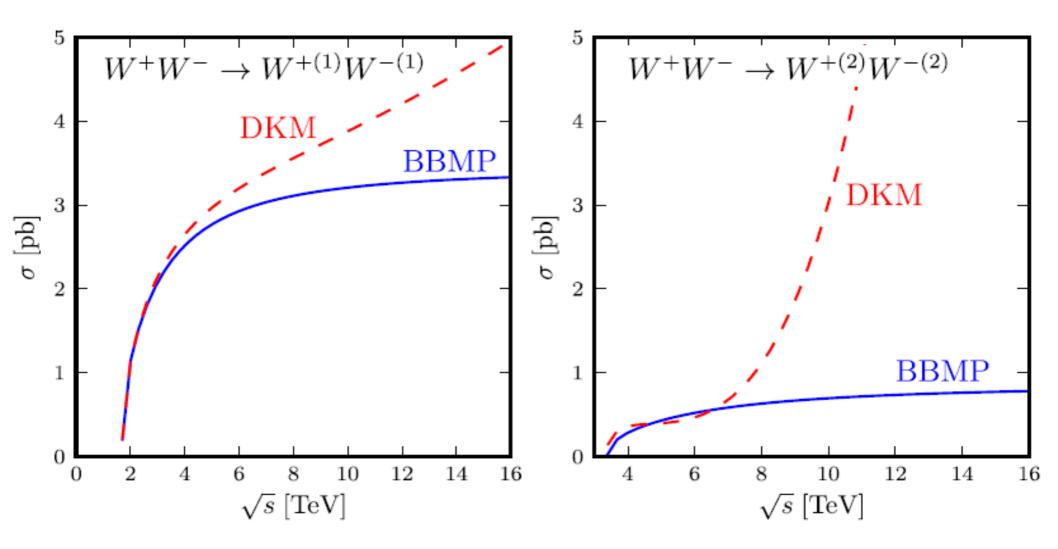
#### Sample of processes with two-gauge bosons for cross-section comparison (in pb) between previous implementation by Datta,Kong, Matchev (DKM) and our implementation (BBMP) arXiv:1212.4858

	Process	DKM $\sigma$ [pb]	BBMP $\sigma$ [pb]
1	$G^{(1)} G^{(1)} \to G G$	$3.952 \times 10^{1}$	$3.952 \times 10^{1}$
2	$G^{(1)} G \to G^{(1)} G$	$7.600 \times 10^{3}$	$7.600 \times 10^{3}$
* 3	$G^{(1)} G^{(1)} \to G^{(1)} G^{(1)}$	$8.619 \times 10^{3}$	$8.600 \times 10^{3}$
* 4	$G^{(1)} Z^{(1)} \to c \bar{c}$	$2.132 \times 10^{-1}$	$2.037 \times 10^{-1}$
* 5	$G^{(1)} \gamma^{(1)} \to b  \bar{b}$	$3.651 \times 10^{-2}$	$3.249 \times 10^{-2}$
* 6	$\gamma^{(1)} \gamma^{(1)} \to t  \bar{t}$	$2.641 \times 10^{-2}$	$2.758 \times 10^{-2}$
* 7	$Z^{(1)} Z^{(1)} \to d  \bar{d}$	$9.098 \times 10^{-2}$	$9.165 \times 10^{-2}$
* 8	$Z^{(1)} Z^{(1)} \to W^+ W^-$	$9.293 \times 10^{0}$	$9.288 \times 10^{0}$
* 9	$W^{+(1)} W^{-(1)} \to Z Z$	$2.744 \times 10^{0}$	$2.761 \times 10^{0}$
10	$W^{+(1)} W^{-(1)} \to Z \gamma$	$1.653 \times 10^{0}$	$1.653 \times 10^{0}$
*11	$W^{+(1)} W^{-(1)} \to W^{+} W^{-}$	$3.152 \times 10^{0}$	$3.081 \times 10^{0}$

 $\sqrt{s}=2$  TeV  $P_T > 100$  GeV KK up to n=2: if KK numbers of the external particles is 5 or less [<2\*(n+1) in general] gauge invariance is ensured



## **Model Validation**



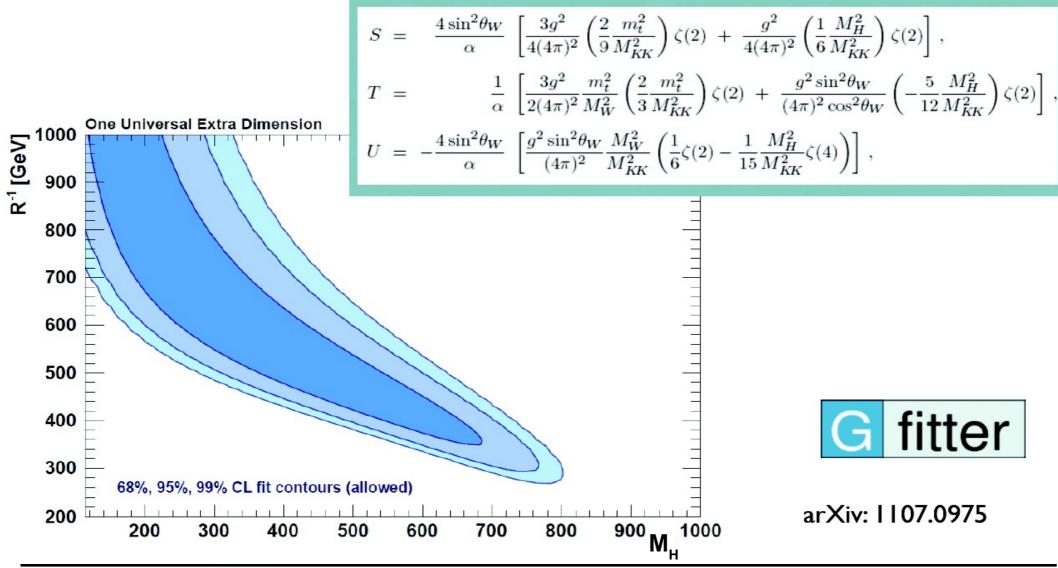
Proper implementation of the Higgs sector lead to the correct High Energy asymptotic which respects Unitarity



### EW precision constraints

The tower of KK particles modify the gauge bosons self-energies, contributing to the S,T, and U electroweak parameters:

T. Appelquist H.-U. Yee 2001 I. Gogoladze and C. Macesanu, 2006



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## FCNC and DM constraints

FCNC

K. Agashe, N.G. Deshpande, G.-H. Wu L. A. J. Buras, A. Poschenrieder, M. Spranger, A. Weiler

KK modes will give contributions to FCNC processes . From  $b \! \rightarrow s \gamma$ 

I/R > 600 GeV

Cosmology (DM)

Belanger, Kakizaki, Pukhov

The evaluation of the LKP relic abundance depends on the spectrum details and on the number of KK levels included in the calculation (eg level 2 resonances, level 2 particles in the final state, etc) Electroweak symmetry breaking effects are also important.

Matsumoto, Senami '05; Kong, Matchev '05 Brunel, Kribs '05; Belanger, Kakizaki, Pukhov '10

WMAP imposes a bound from above to DM scale: if DM were heavier it would lead to the Universe having a measurable positive curvature

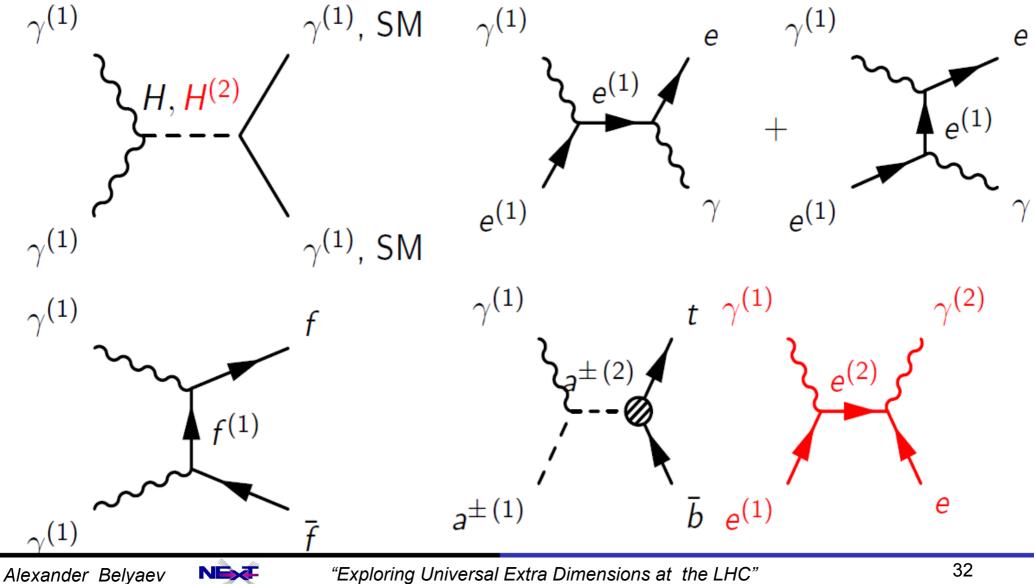
1/R < 1.6 TeV

. . . . . . . . . . . .



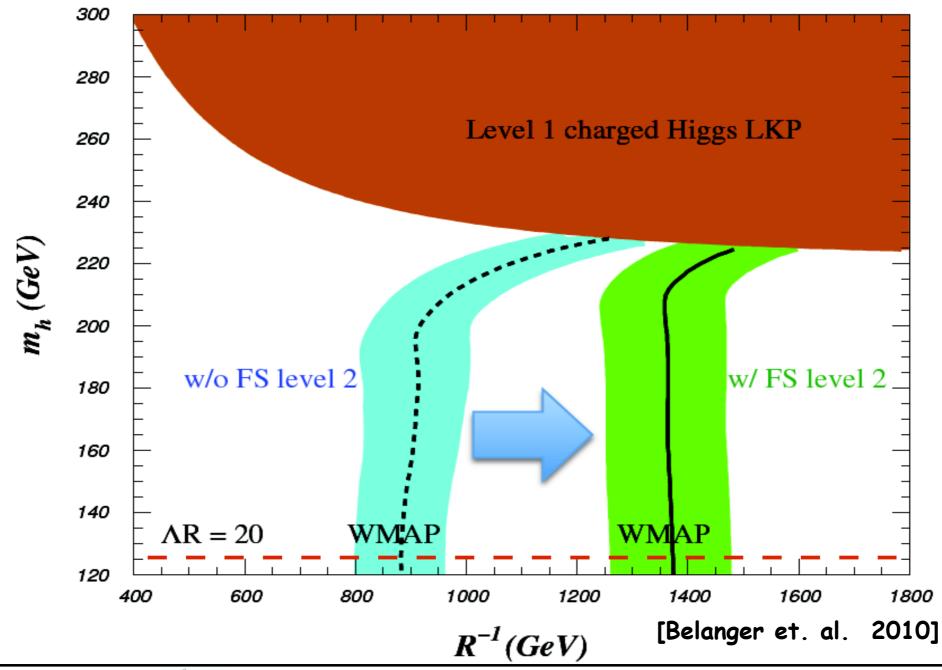
## The role of the 2<sup>nd</sup> level of KK excitation

Processes important for calculating DM relic abundance... Self-annihilation **Co-annihilation** 



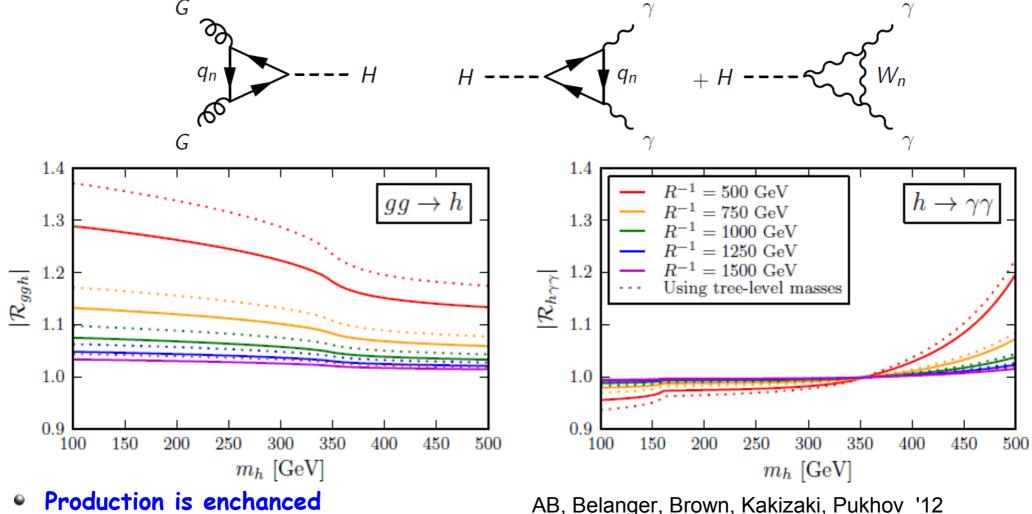
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#### The role of the 2<sup>nd</sup> level of KK excitation





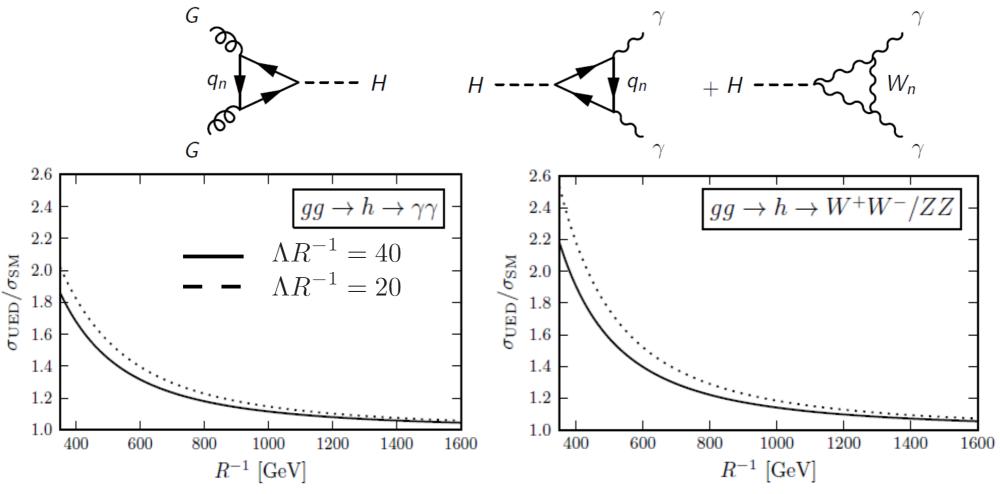
## The role of the Higgs searches in constraining of the mUED model $\gamma$



Decay is slightly suppressed



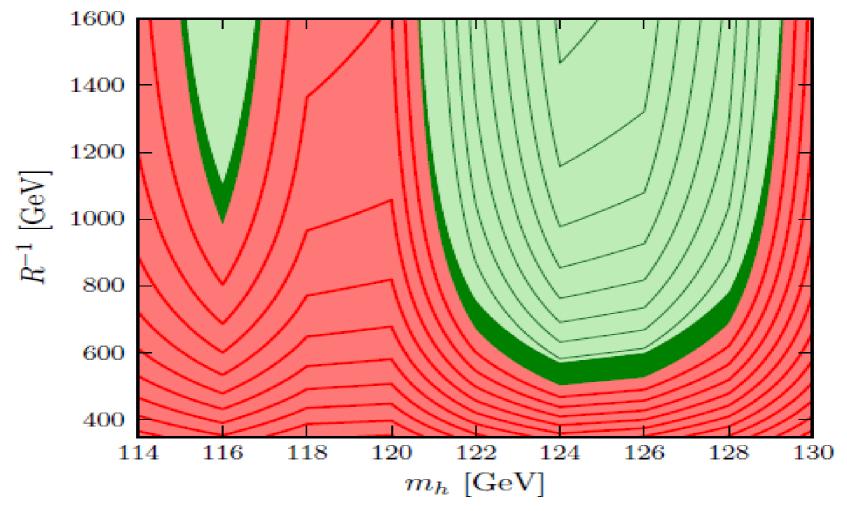
#### Constraints from the Higgs data



- Production is enchanced
  - AB, Belanger, Brown, Kakizaki, Pukhov '12 Decay is slightly suppressed
- Overall, the GG->H-> $\gamma\gamma$  is enhanced ٠



#### Constraints from the Higgs data



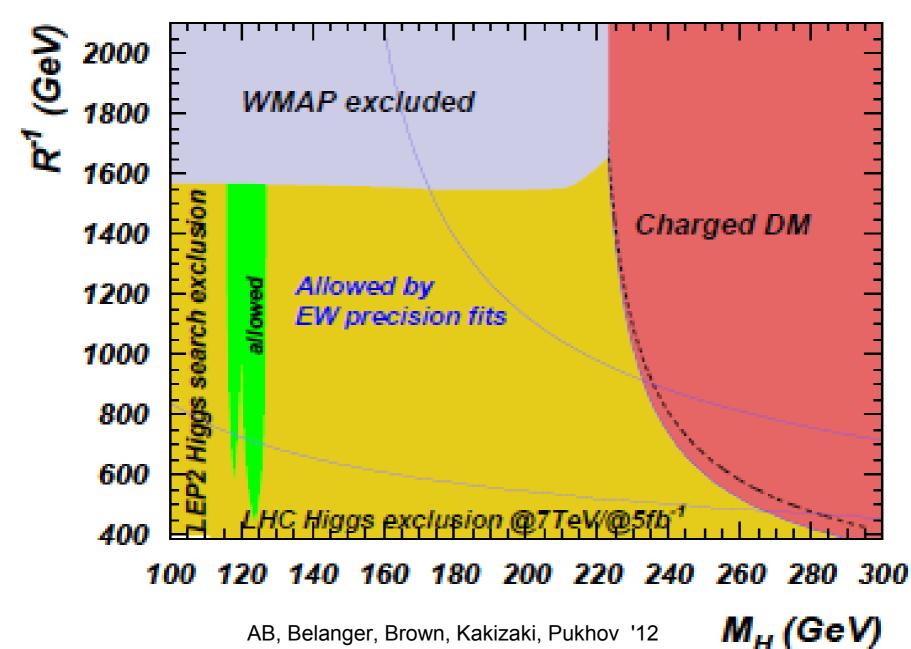
- Same channels (γγ and WW) from CMS/ATLAS are combined
- R<sup>-1</sup><500 is excluded at 95% CL</p>
- overall, the GG->H-> $\gamma\gamma$  is enhanced
- Narrow window around 125 GeV is left

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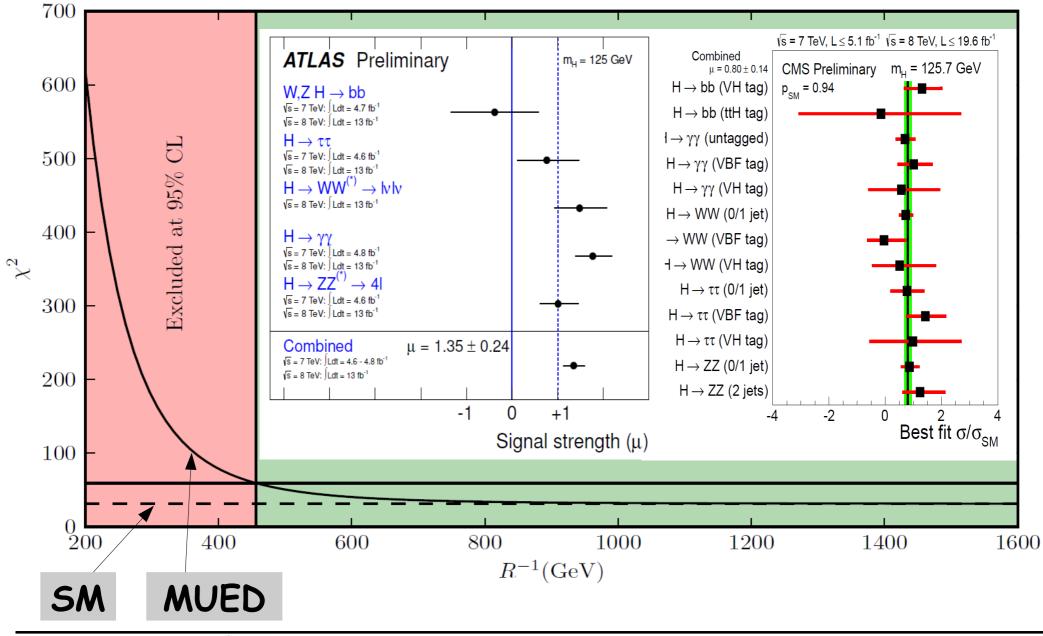
AB, Belanger, Brown, Kakizaki, Pukhov '12

## The Status of MUED (with LHC@7 TeV Higgs data)





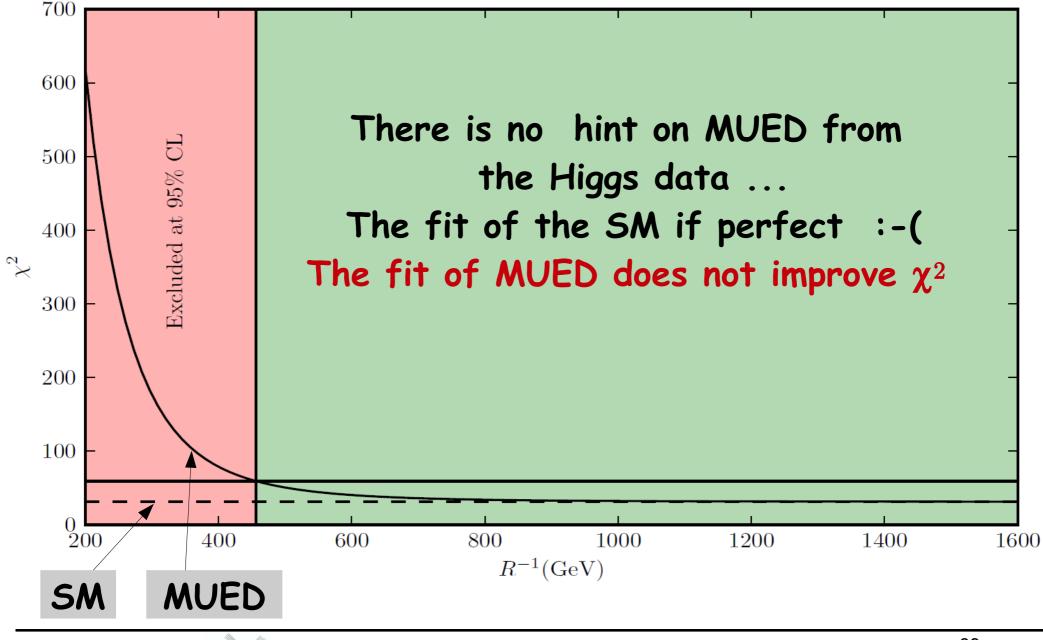
## Data Fit with MUED vs SM



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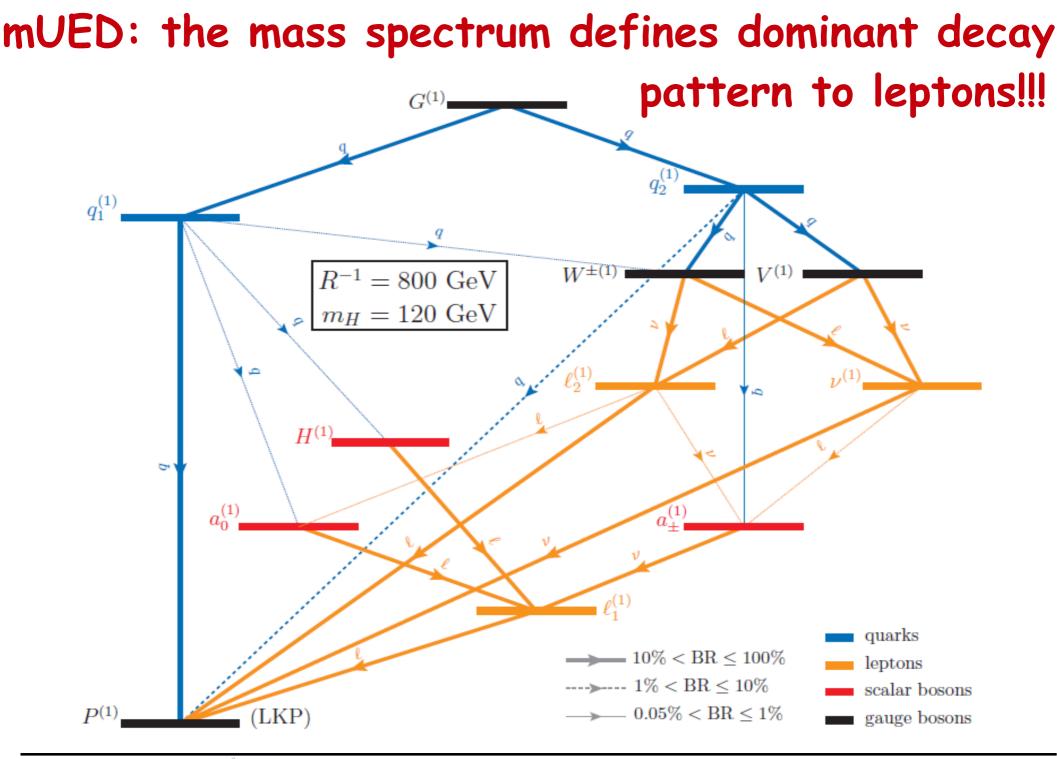
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## Data Fit with MUED vs SM

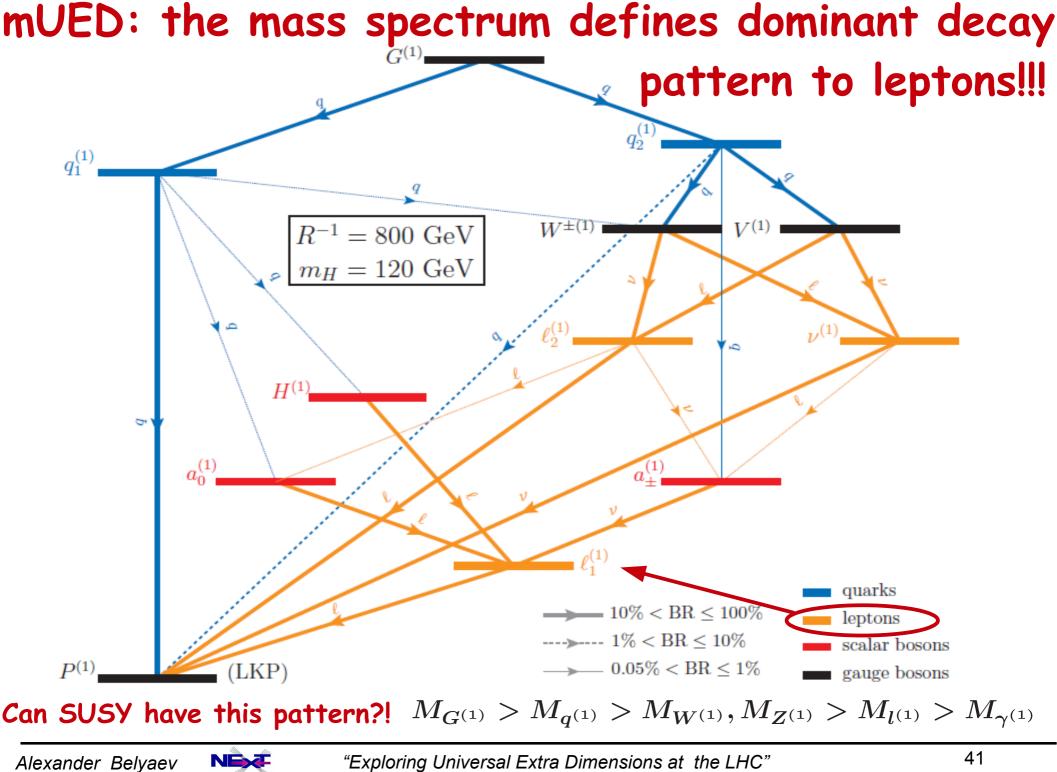


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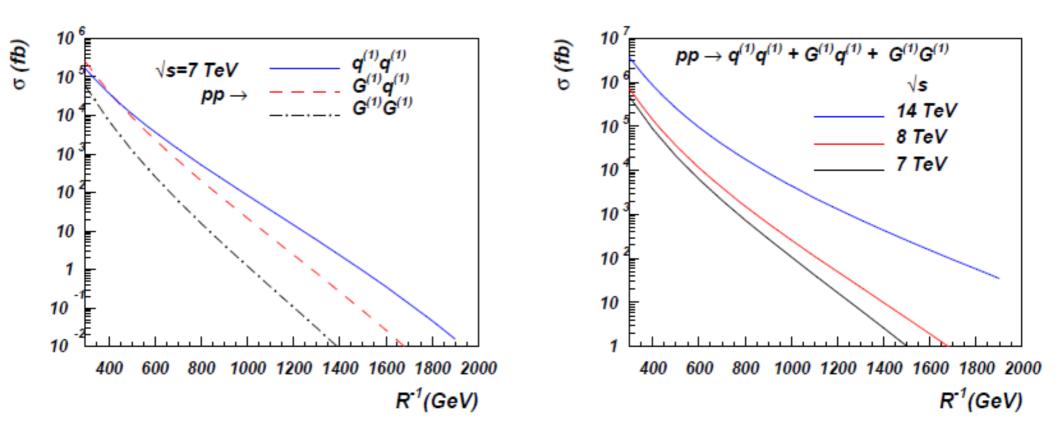


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# mUED collider phenomenology with leptons

AB, Brown, Moreno, Papineau'12

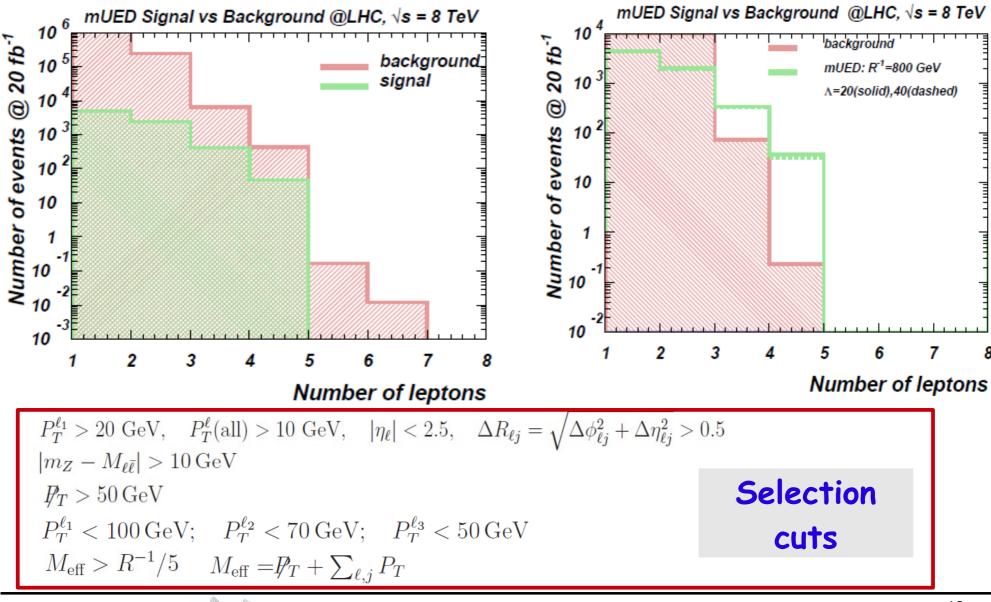


#### $Q^1$ $Q^1$ production rate is the highest



### **mUED collider phenomenology with leptons** Lepton multiplicity: AB, Brown, Moreno, Papineau'12

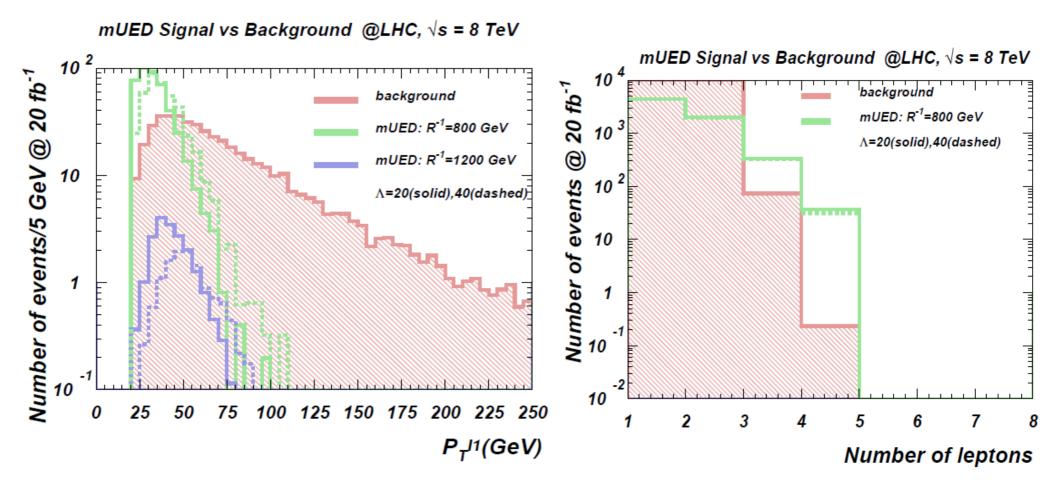
#### Signal vs BG before (left) and after(right) selection cuts



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#### mUED collider phenomenology with leptons AB, Brown, Moreno, Papineau'12 Cut on the maximum $P_{\tau}$ of the lepton is important!

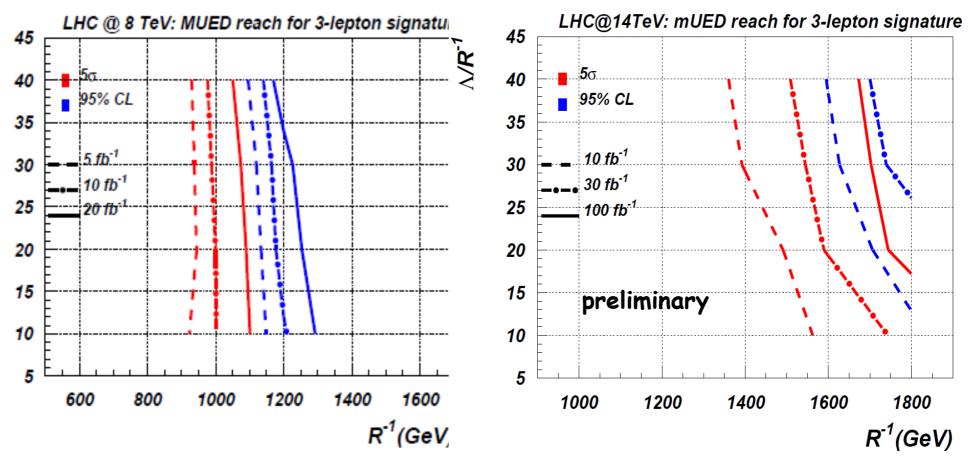


#### 3-lepton signature has the highest significance in comparison with 4-lepton signature



# mUED collider phenomenology with leptons

AB, Brown, Moreno, Papineau'12

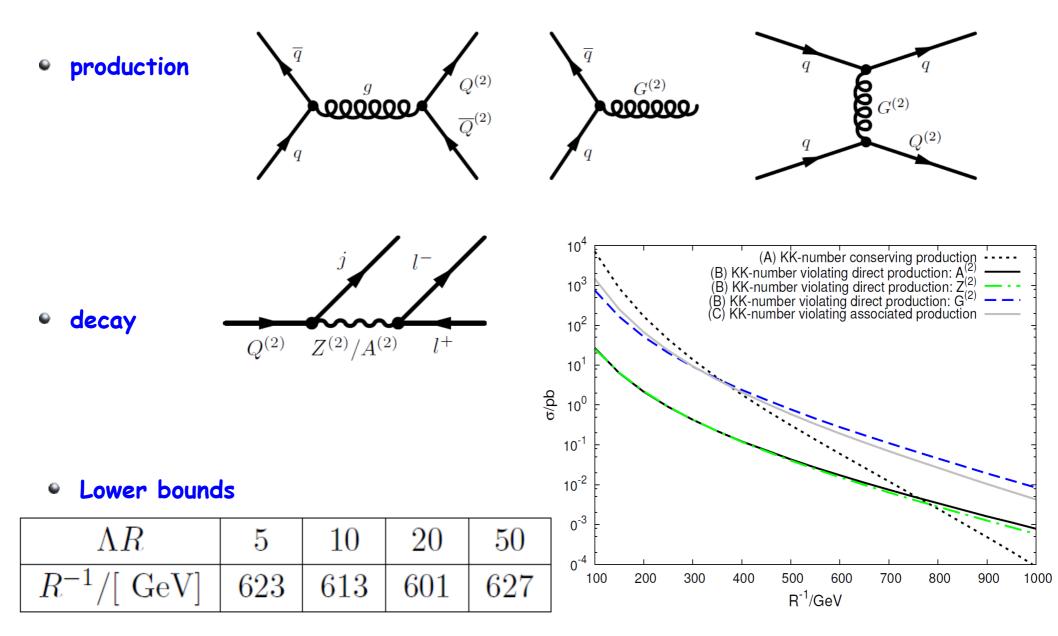


- Small mass gap (as compared to MSSM) much lower missing PT
- Quite a few PHENO papers, but there are no experimental limits!!! the projected limit from this study: R<sup>-1</sup> > 1.2—1.3 TeV
- 3-lepton signature is very promising: LHC@14 will eventually discover or close MUED!



## Constraints from di-lepton searches

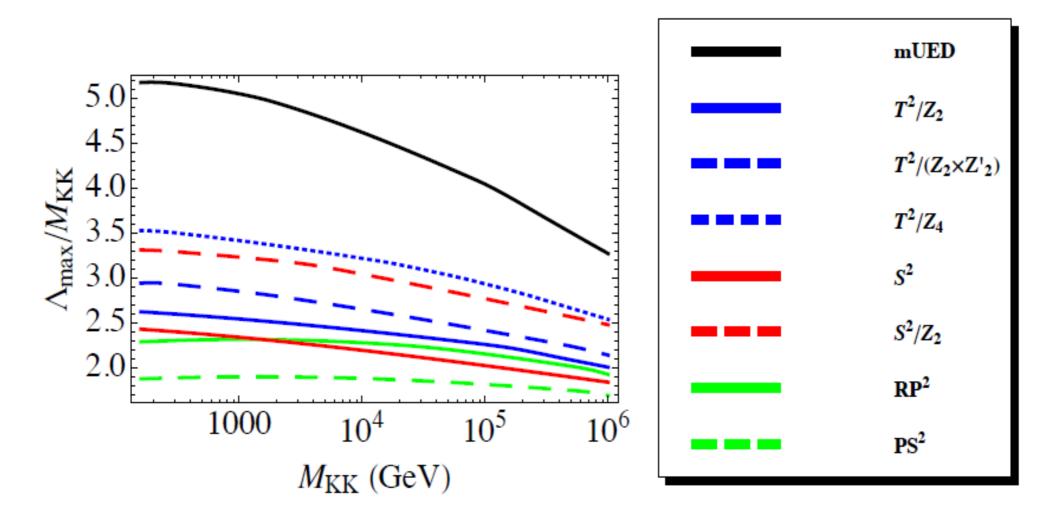
Edelhäuser, Flacke, Kramer, '13





## Vacuum stability bounds

Kakuda, Nishiwaki, Oda, Watanabe, '13

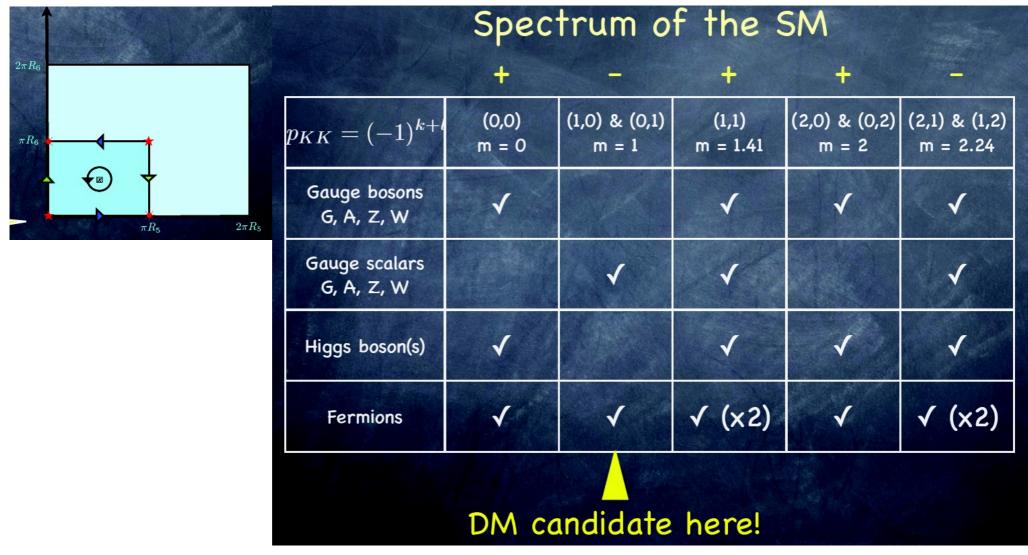


			$T^2/(Z_2 \times Z'_2)$	$T^{2}/Z_{4}$	$S^2$	$S^{2}/Z_{2}$	$RP^2$	$\mathbf{PS}$
$ ilde{\Lambda}_{ ext{max}}$	5.0	2.5	2.9	3.4	2.3	3.2	2.3	1.9



# 6D UED (Dark Matter in a twisted bottle) Arbey, Cacciapaglia, Deandrea, Kubik'12

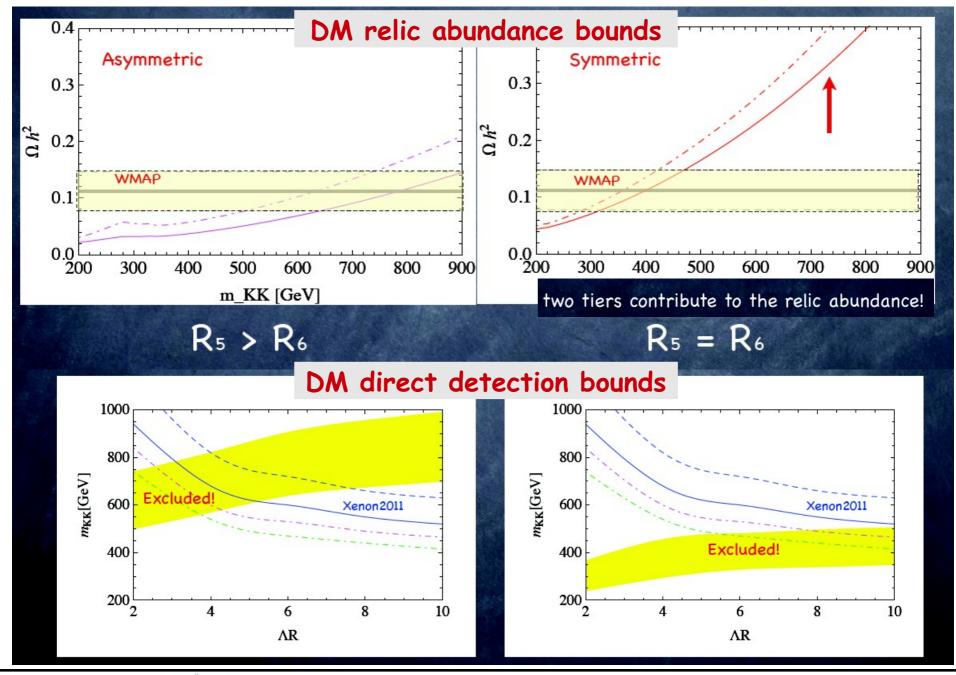






## 6D UED DM bounds

#### Arbey, Cacciapaglia, Deandrea, Kubik'12



Alexander Belyaev

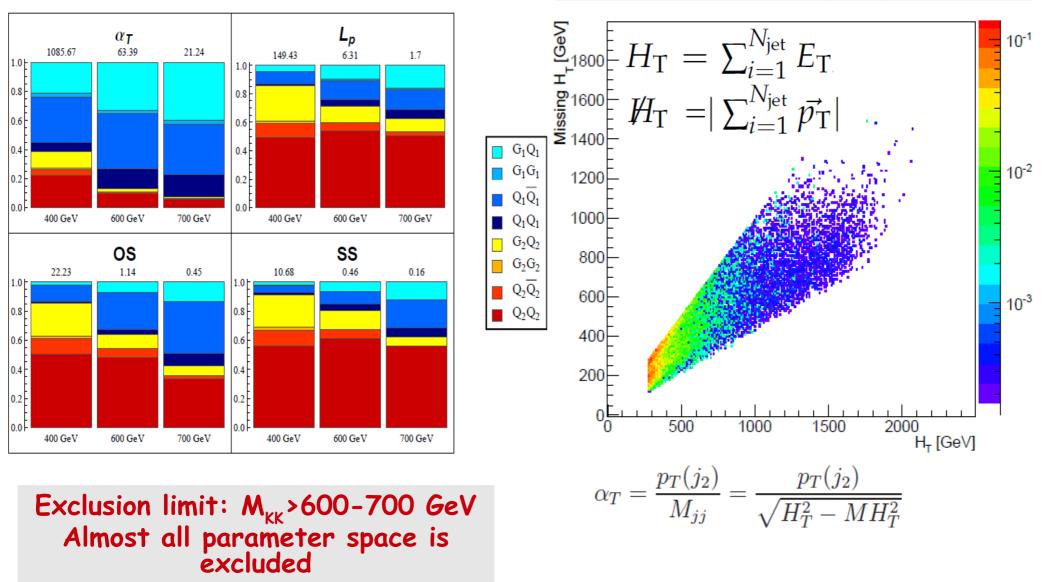


## 6D UED LHC bounds

Cacciapaglia, Deandrea, Ellis, Marrouche, Panizzi '13

#### "composition " of signal signatures

#### MHT-HT analysis plane





# Conclusions

- UED are limited from above by DM relic abundance and from below by the LHC searches
   LHC and DM search experiments provide an important test:
   LHC@14 TeV will discover or exclude the complete parameter space for 5 & 6D UED (no boundary localised terms).
- There are still no dedicated experimental searches for MUED signals which could be in data! It is time to check them!
   3-lepton signal is very promising for MUED at the LHC.
- Consistent MUED with EWSB and loop-corrections is implemented into LanHEP and publicly available at HEPMDB [CalcHEP and UFO(Madgraph5) formats are available].
   It is ready to be used by experimentalists and theorists!

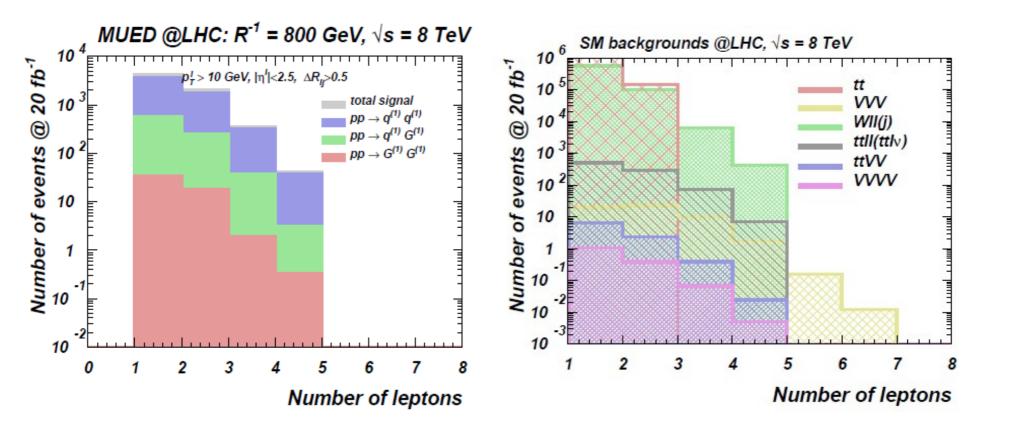






# mUED collider phenomenology with leptons

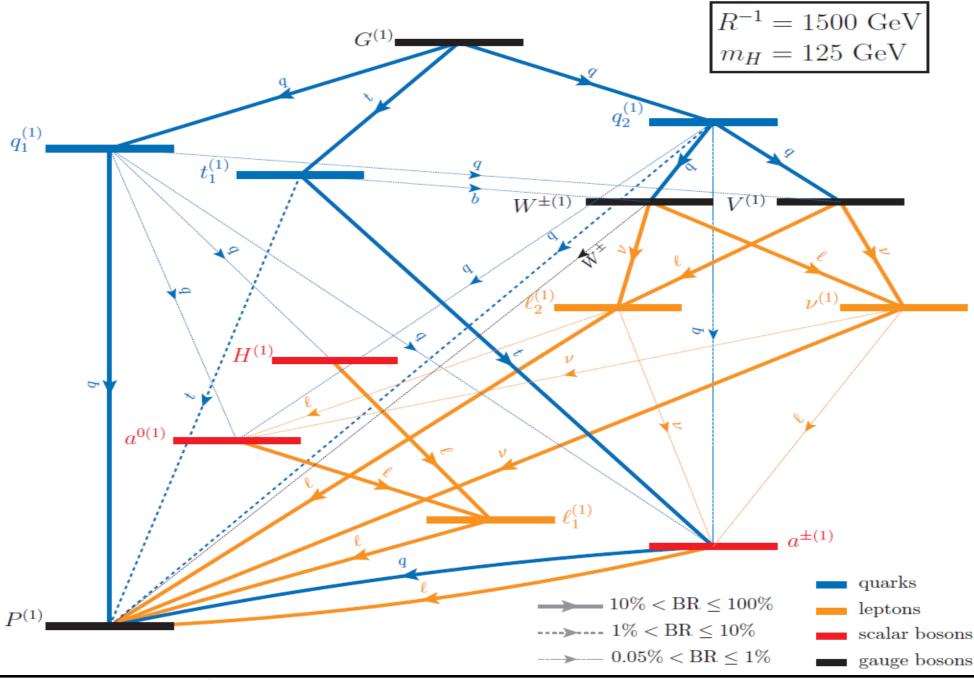
AB, Brown, Moreno, Papineau'12



#### Signal vs BG in lepton multiplicity



# **Backup slides**



NEX

# MUED; Direct DM detection rates

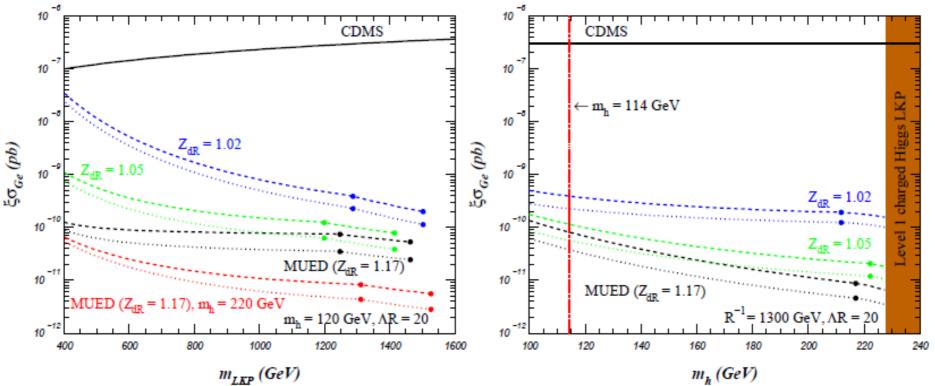


Figure 5: Rescaled LKP-nucleon cross section on  $Ge^{76}$  vs  $m_{LKP}$  for  $m_h = 120$  GeV,  $\Lambda R = 20$  and 2 sets of quark coefficients ( $(\sigma_{\pi N}, \sigma_0) = (56 \text{ MeV}, 35 \text{ MeV})$  (dash) or (47 MeV, 42.9 MeV) (dot) ) and for different values of the mass splitting between the KK singlet d-quarks and the LKP including the MUED case (left panel). The MUED results for  $m_h = 220$  GeV are also shown. In each line the region between the blobs is consistent with the  $3\sigma$  WMAP range. Rescaled LKP-nucleon cross section on  $Ge^{76}$  vs  $m_h$ for  $R^{-1} = 1300$  GeV,  $\Lambda R = 20$  (right). In each line the region left of the blob is consistent with the  $3\sigma$  WMAP range.

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### The spectrum

Because of the loop-corrections, the B and  $W^3$  do not mix with the Weinberg angle

$$\begin{pmatrix} Z_B \frac{n^2}{R^2} + \frac{1}{4}g_1^2 v^2 & -\frac{1}{4}g_1g_2v^2 \\ \\ -\frac{1}{4}g_1g_2v^2 & Z_W \frac{n^2}{R^2} + \frac{1}{4}g_2^2v^2 \end{pmatrix}$$

Consequently, the mass eigenstates are <u>not</u> the KK photon or KK Z-boson. We call them  $P^{(n)}$  and  $Q^{(n)}$ .

There is a tree-level  $H^{(k)}P^{(l)}P^{(m)}$  vertex.

Associated with the KK vectors  $A_{\mu}^{(n)}$ , the Goldstone bosons are combinations of the fifth components  $A_5^{(n)}$  and the Higgses  $\chi^{(n)}$ .

Finally, there are two KK fermions per SM one, and they mix with angles related to the  $Z_i$ .

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## The spectrum

Spin	Name	Particle	Mass
1	Gluon P boson Q boson W boson	$G^{(n)} \ P^{(n)} \ Q^{(n)} \ W^{\pm  (n)}$	$\begin{split} m_{G(n)}^2 &= Z_G \; \frac{n^2}{R^2} \\ m_{P(n)}^2 \\ m_{Q(n)}^2 \\ m_{W(n)}^2 &= Z_W \frac{n^2}{R^2} + M_W^2 \end{split}$
1/2	Neutrinos Charged leptons 1 Charged leptons 2 Up-quarks 1 Up-quarks 2 Down-quarks 1 Down-quarks 2	$\begin{matrix} \nu_{iL}^{(n)} \\ e_1^{(n)}, \mu_1^{(n)}, \tau_1^{(n)} \\ e_2^{(n)}, \mu_2^{(n)}, \tau_2^{(n)} \\ u_1^{(n)}, c_1^{(n)}, t_1^{(n)} \\ u_2^{(n)}, c_2^{(n)}, t_2^{(n)} \\ d_1^{(n)}, s_1^{(n)}, b_1^{(n)} \\ d_2^{(n)}, s_2^{(n)}, b_2^{(n)} \end{matrix}$	$\begin{split} m_{\nu_i(n)} &= Z_{eL} \frac{n}{R} \\ m_{e1(n)},  m_{\mu1(n)},  m_{\tau1(n)} \\ m_{e2(n)},  m_{\mu2(n)},  m_{\tau2(n)} \\ m_{u1(n)},  m_{c1(n)},  m_{t1(n)} \\ m_{u2(n)},  m_{c2(n)},  m_{t2(n)} \\ m_{d1(n)},  m_{s1(n)},  m_{b1(n)} \\ m_{d2(n)},  m_{s2(n)},  m_{b2(n)} \end{split}$
0	Higgs scalar neutral scalar charged scalar	$egin{aligned} h^{(n)} \ a^{(n)}_0 \ a^{(n)}_\pm \ \end{array}$	$ \begin{split} m_{h(n)}^2 &= Z_H \frac{n^2}{R^2} \\ m_{a0(n)}^2 &= Z_H \left[ \frac{n}{R} + \frac{v^2}{4} \left( \frac{g_1^2}{Z_B} + \frac{g_2^2}{Z_W} \right) \right] \\ m_{a(n)}^2 &= \frac{Z_H}{Z_W} \left[ Z_W \frac{n^2}{R^2} + M_W^2 \right] \end{split} $

