One loop blow-up amplitudes and the LVS

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Work with L. Witkowski ...

Overview

- Sequestering and loop corrections in the LVS
- Toward a desequestering calculation for blow-up moduli

Sequestering and the LVS

- The string scale depends on the compactification volume as $m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$
- In the LARGE volume scenario, gravitino mass is given by $m_{3/2} \sim \frac{|W_0|}{\mathcal{V}} M_P$
- If we want a GUT theory, we need a large string scale $\sim 10^{14\div16}$ GeV and thus $\mathcal{V} \sim 10^{4\div8}$, meaning $m_{3/2} \sim 10^{10\div14}$ GeV.
- Original LVS with the SM on a geometric cycle supporting an instanton or gaugino condensate lead to $M_{\mathrm{SOFT}} \sim \frac{m_{3/2}}{\log \mathcal{V}} \rightarrow$ soft masses only as low as 10^9 GeV.

Hence we either

1. Consider intermediate string scales and LHC-accessible SUSY.
2. Have a high string and intermediate SUSY-breaking scale (nb the gauginos would not be light $\rightarrow$ not unified).
3. A GUT and a SUSY solution to the hierarchy problem, if we can suppress soft masses to be $M_P/\mathcal{V}^{3/2}$ or $M_P/\mathcal{V}^2 \rightarrow$ sequestering.

Kähler corrections and Yukawa couplings

Following [Cicoli's] talk:

- Since physical Yukawa couplings on magnetised branes, at least to leading order, should not depend on the overall volume, and $Y_{ijk}^{phys} = \frac{e^{K/2} Y_{ijk}^{hol}}{\sqrt{K_{ii} K_{jj} K_{kk}}}$, $K = -2 \log \mathcal{V}$ at leading order, so matter Kähler metrics at leading order are $\sim \mathcal{V}^{-2/3}$.
- Tree-level Kähler potential has known α' corrections

$$-2 \log(\mathcal{V} + \xi/2) = -2 \log \mathcal{V} - \frac{\xi}{\mathcal{V}} + \ldots$$

- Hence tree-level Kähler metric should have the form [Blumenhagen et al, 0906.3297]

$$K_{ii} = \frac{k}{\mathcal{V}^{2/3}} (1 - \delta \frac{Re(S)^{3/2}}{\mathcal{V}} + \ldots)$$

- Equivalently, if Kähler metric is $e^{K/3}$ the Yukawa couplings would be invariant.
- Furthermore, since soft masses are $m_{\tilde{q}_i}^2 = m_{3/2}^2 + V_0 - F^I \overline{F}^J \partial_I \overline{\partial}_J \log K_{ii}$ the above form leads to vanishing soft masses if $V_0 = 0$ (F-term uplifting).
- This cancels the $(\alpha')^3$ correction to soft to soft terms from $e^{K/2}$ for appropriate δ.
- $\rightarrow$ Kähler modulus dependence of physical Yukawa couplings implies corrections scalar masses.
- We currently do not know at what order this occurs $\rightarrow$ we do not know what the soft masses are in the LVS!

Known knowns and known unknowns

- (We think we) Know the Kähler potential to one-loop:

$$K_0 = -2\log(\mathcal{V} + \xi/2 + c\tau_i \cap \tau_j) + k_a^{0,2}\frac{\tau_a^2}{\mathcal{V}} + \sum \frac{\mathcal{E}(U,\overline{U})}{\text{Re}(S)\tau_i} + \sum \frac{\tilde{\mathcal{E}}(U,\overline{U})}{\tau_i\tau_j} + \dots$$
$$+ \sum_{n=1,m=1}^{\infty} g_s^m \frac{k_a^{m,n}}{\mathcal{V}}\tau_a^n$$

- We know the Kähler metric at tree-level

$$K_{i\bar{i}} = \frac{k}{\mathcal{V}^{2/3}}\left[1 - \delta\frac{\text{Re}(S)^{3/2}}{\mathcal{V}} + \delta^{(1)}\frac{1}{\text{Re}(S)}\left(\frac{\text{Re}(S)}{\mathcal{V}^{2/3}}\right)^{n/2} + \sum_{n,m} g_s^m \epsilon^{m,n}\tau_s^n + \dots\right]$$

- May also be non-perturbative corrections to superpotential and A-terms which can lead to desequestering [Berg,Conlon,Marsh,Witkowski '12].

Unknowns

- Soft scalar masses schematically, $V_0 \sim \frac{m_{3/2}^2}{\tau_s g_s^{3/2} \mathcal{V}}$ without uplifting:

$$m_{\tilde{q}}^2 = \frac{2}{3} V_0 + c \frac{m_{3/2}^2}{g_s^{3/2} \mathcal{V}} (\delta - \xi/3)$$

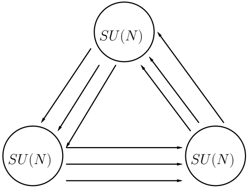
- Gaugino masses depending on anomaly-mediation contribution, uplifting and thus matter Kähler metrics:

$$M_\lambda = \frac{M_P}{\mathcal{V}^{3/2}} \div \frac{M_P}{\mathcal{V}^2}$$

- Tree-level Kähler metrics: [Conlon, Witkowski '11] showed no dependence of the physical Yukawas on blow-up moduli, $\delta = \xi/3$, $\epsilon^{(1)} = 3k_a^{0,2}/\mathcal{V}$ etc (or more generally $e^{\sum k_a^{0,m} \tau_a^m / \mathcal{V}} (1 + \sum_{n,m} g_s^m \epsilon^{m,n} \tau_s^n k/\mathcal{V}^{2/3})^{-3/2} = 1$).
- [Lawrence, Sever '07] attempted to calculate the disk correction $k_a^{1,2} \to$ non-zero and finite as $\mathcal{V} \to \infty$, so can absorb into $k_a^{0,2}$.
- For ultra-local models (e.g. locally $\mathbb{C}/\mathbb{Z}_3$ orbifold) $\delta^{(1)} = 0$, no BHK corrections.
- The actual values of the soft masses in the sequestered scenario are therefore not known. It is necessary to calculate then the additional pieces. Hence we need to calculate blow-up amplitudes at one-loop.

Toroidal orbifolds as prototypes

- Orbifolds $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2/\mathbb{Z}_N$ are useful as prototypes for real compactifications: they contain bulk moduli (up to three Kähler moduli) and blow-ups with isolated singularities.
- Matter fields ϕ_i, ψ_i are projected out of $N = 4$ adjoints: have Chan-Paton matrices λ_i, $i = 1, 2, 3$ for each torus and projection $\lambda = e^{2\pi i b_r/N} \gamma_\theta \lambda \gamma_\theta^{-1}$ for $\sum_r b_r = 0$.
- These are particularly simple to calculate with as we have already seen ...



Corrections to Yukawa couplings

- Want to calculate corrections to Kähler potential and metric involving blow-up modes.
- Attempt to go straight for the goal: calculate Yukawa couplings to see whether there are modulus-dependent corrections

Recall that for ultra-local models, e.g. $\mathbb{Z}_3$ orbifolds, have no corrections at one loop for several reasons:

1. In twisted sector, the only amplitude possible is connects the same stack - so cannot feel distant branes
2. Furthermore, in twisted sector, amplitude vanishes due to tadpole cancellation:

$$\langle \phi_i \psi_j \psi_k \rangle \propto \mathrm{tr}_L(\gamma_\theta \lambda_i \lambda_j \lambda_k) \mathrm{tr}_R(\gamma_\theta) = 0$$

3. In untwisted sector, amplitude vanishes due to effective $N = 4$ supersymmetric states running in the loop: amplitude is proportional to a sum over spin structures

$$\sum_\nu \delta_\nu \prod_{i=1}^{4} \vartheta_\nu \Big(\sum_{j=1}^{3} a_i^j z_j \Big) = 0 \text{ if } \sum_i a_i^j \ \forall j$$

Amplitudes with blow-up moduli

- Want to calculate

$$\langle \phi_i \psi_j \psi_k \prod_{m=1}^{L} \tau_{\theta_m} \rangle$$

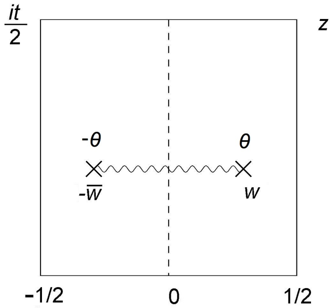
- The vertex operator for blow-up modes τ_{θ_m} contains bosonic twist fields which greatly complicate the analysis:

$$V^{-1,-1}_{\sigma_\theta}(w, \overline{w}) = e^{-\phi} e^{-\tilde{\phi}} e^{ik\cdot X} \prod_{\kappa=1}^{3} e^{i\theta_\kappa H(w)} e^{-i\theta_\kappa \tilde{H}\overline{w}} \sigma_{\theta_\kappa}(w, \overline{w})$$

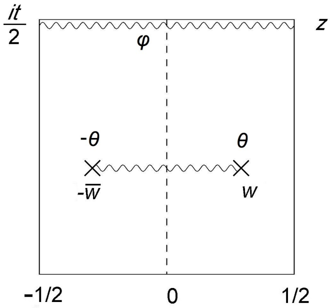
- We are interested in the case where the blow-ups are associated with distant fixed-points under the orbifold group.
- The bosonic twists introduce branch cuts on the worldsheet which are accompanied by chan-paton rotations γ_θ. We must also consider diagrams in twisted sectors which introduce a further twist. Tadpole cancellation then eliminates many diagrams.

Twisted worldsheets

Consider e.g. $\mathbb{Z}_3$, where $\mathrm{tr}(\gamma_\theta) = \mathrm{tr}(\gamma_\theta^2) = 0 = \sum_{\mathrm{perms}(\gamma)} \mathrm{tr}(\gamma_\theta \lambda_1 \lambda_2 \lambda_3)$. Insert matter fields at $\mathrm{Re}(z) = -0.5$:



Vanishes through $\mathrm{tr}_R(\gamma_\theta) = 0$



Vanishes when $\mathrm{tr}_R(\gamma_\theta \gamma_\varphi) = 0$ and always through $\sum_{\mathrm{perms}(\gamma)} \mathrm{tr}_L(\gamma_\varphi \lambda_1 \lambda_2 \lambda_3) = 0$

So we have no twisted-sector amplitudes that can contribute to the Yukawa coupling, and there is no contribution to the matter Kähler metric linear in τ_θ.

Untwisted sectors

- Recall that for the Yukawa couplings it was the Riemann summation part of the amplitude that caused the amplitude to vanish.
- Let us calculate an untwisted amplitude with a twist and antitwist: we need

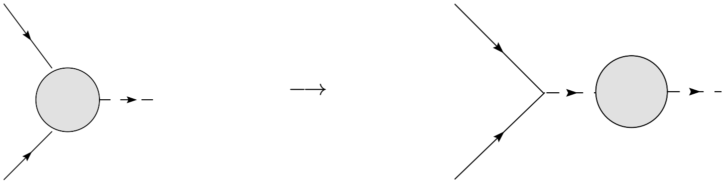
$$\langle V^0_\phi V^{-1/2}_\psi V^{1/2}_\psi V^{0,0}_{\tau_\theta} V^{0,0}_{\tau_{1-\theta}} \rangle$$

- Note that we insert six Picture-Changing Operators (PCOs). These must be in pairs for each complex dimension.
- We want to extract momentum-independent piece; thus we either need a contribution from amplitude with PCOs inserted only on internal directions, or need momentum poles (PCOs inserted on non-compact dimensions lead to $k \cdot \psi$ operators).
- We can evaluate the spin-structure sum for the all-internal case and, just as for normal Yukawa couplings, find zero.

$5 \to 4$

- This is not a surprise! Actually it implies that the only corrections to the Yukawa couplings come from factorising down to bosonic propagators (expression of non-renormalisation theorem):

$$\langle V^0_\phi V^{-1/2}_\psi V^{1/2}_\psi V^{0,0}_{\tau_\theta} V^{0,0}_{\tau_{1-\theta}} \rangle \to \langle V^0_\phi V^0_{\bar{\phi}} V^{0,0}_{\tau_\theta} V^{0,0}_{\tau_{1-\theta}} \rangle$$



- So in the end we are calculating matter Kähler metrics anyway.
- In fact, it is also important to compute the correction to the blow-up Kähler metric:
- For this we require simply

$$\langle V^{0,0}_{\tau_\theta} V^{0,0}_{\tau_{1-\theta}} \rangle$$

  with two internal PCO insertions
- Evaluating the Riemann summation for both of these we do not find that they vanish.

Volume dependence

- Now want to find the volume dependence of the amplitude and ensure that it does not decrease exponentially with the volume.
- Amplitude itself is complicated; volume dependence comes from bosonic correlators.
- Two pairs of PCOs are now inserted internally:

$$\mathcal{A} \propto \langle \sigma_{\theta_i} \sigma_{1-\theta_i} \rangle \langle \partial_n X \partial_n \overline{X} \sigma_{\theta_j^1} \sigma_{1-\theta_j} \rangle \langle \partial_n X \partial_n \overline{X} \sigma_{\theta_k} \sigma_{1-\theta_k} \rangle$$

- Volume factors come from classical solutions:

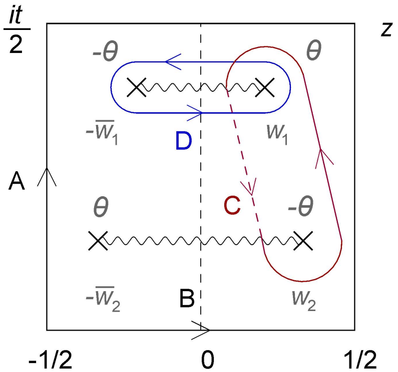
$$\langle \partial_n X \partial_n \overline{X} \sigma_\theta(w_1, \overline{w}_1) \sigma_{1-\theta}(w_2, \overline{w}_2) \rangle =$$
$$\left(\partial_n X_{cl} \partial_n \overline{X}_{cl} f_{qu}(w_i, \overline{w}_i) + \langle \partial_n X \partial_n \overline{X} \sigma_\theta(w_1, \overline{w}_1) \sigma_{1-\theta}(w_2, \overline{w}_2) \rangle_{qu} \right) e^{-S_{cl}}$$

- To find the classical solutions we must construct a basis of cut differentials for the ∂X_{cl} obeying the boundary conditions

$$\oint_\gamma dz \partial X + \oint_\gamma d\bar{z} \bar{\partial} X = v_\gamma$$

- v_γ are given by cosets of the orbifold subject to the condition that the ends of the string attach to separated branes.
- Problem becomes that of finding a set of basis cycles on worldsheet and their associated shifts $\rightarrow$ canonical dissection.

Canonical dissection



$$\partial X(z) \equiv \begin{cases} \partial Z(z) & \mathrm{Re}(z) > 0 \\ \bar\partial Z(-\bar z) & \mathrm{Re}(z) < 0 \end{cases}$$

$$\partial Z(z) = \nu_a (W^{-1})^a_{i'} \omega^{i'}_{1-\theta}(z)$$

$$\bar\partial Z(z) = \nu_a (W^{-1})^a_{i''} \overline{\omega}^{i''}_{\theta}(\bar z)$$

$$\overline{\partial Z}(\bar z) = \Big(\partial Z(z)\Big)^*$$

$$S = \frac{1}{4\pi\alpha'} \int d^2 z \partial X \partial \overline{X} + \bar\partial X \overline{\partial X}$$

$$= \frac{i}{4\pi\alpha'} S_{ab} \nu_a \bar\nu_b$$

From this, we construct S_{ab} in terms of integrals of the ω around the above cycles. Action is substantially simpler than the case for a torus.

Classical part

The classical and quantum amplitudes depend on 8 complex integrals encapsulated in matrix W:

$$W = \begin{pmatrix} W^1_A & W^2_A & -W^1_A & -W^2_A \\ W^1_B & W^2_B & W^1_B & W^2_B \\ W^1_C & W^2_C & -W^1_C & -W^2_C \\ W^1_D & W^2_D & W^1_D & W^2_D \end{pmatrix}$$

$$|W| = -4 \begin{vmatrix} W^1_A & W^2_A \\ W^1_C & W^2_C \end{vmatrix} \begin{vmatrix} W^1_B & W^2_B \\ W^1_D & W^2_D \end{vmatrix}$$

The classical part of the amplitude takes a block-diagonal form (if we swap $B \leftrightarrow C$):

$$S_{ab} = \begin{pmatrix} \mathbf{S^1}_{2\times 2} & \mathbf{0} \\ \mathbf{0} & \mathbf{S^2}_{2\times 2} \end{pmatrix}$$

This determines the scaling with the Kähler moduli.

Expectations and conjectures

- Expect that, since we are looking at a sequestered situation, that the dominant contribution comes from closed string KK modes
- Go to closed-string channel, where $t \to 0$ and annulus becomes a long thin cylinder.
- Dominant contributions from B-cycle only (also $\nu_A = 0$) to go as $\int_{\gamma_B} \omega \sim 1/t$
- Expect other sums to just give constant and exponentially suppressed contribution
- Expect $\partial X_{cl} \propto \nu_B = 2\pi\sqrt{\frac{T_2}{U_2}}(n + mU + y)$
- For matter Kähler metric, amplitude should be $\sim (\partial X_{cl})^4 e^{-S}$; when we go to closed string channel and Poisson-resum we should then find

$$\mathcal{A} \sim \int \frac{dt}{t^3} t^4 \sum_{n_i, m_i} \frac{1}{R^{10} t^5} \prod_i e^{-\frac{c}{R^2 t}|n_i + m_i U_i|^2 + 2\pi i m d y)}$$

$$\sim \frac{1}{R^4}$$

- Would lead to $\delta K_{i\bar{i}} \sim K^0_{i\bar{i}}/\tau_b$.
- This corresponds to the field-theory expectation of exchange of a KK mode in six dimensions.

Conclusions

- Have developed some substantial technology for calculating amplitudes with twist fields on annulus diagrams
- We have some evidence for the corrections to the Kähler metric corrections involving blow-up modes at one-loop
- $\rightarrow$ still working towards blow-up amplitudes ...