Global Embedding of D-branes at singularities

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bctp

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based on collaboration with

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1304.0022, 1206.5237

Motivation

Old Challenge:

Construct an explicit viable string vacuum

satisfying all constraints from particle physics
and cosmological observations.

Status/Approach in type IIB?

... the bottom-up approach to string model building

AIQU: hep-th/0005067

various mechanisms have
been constructed

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Moduli stabilisation

Kähler moduli: perturbative + non-perturbative corrections (e.g. KKLT, LVS)

Complex structure + dilaton: fluxes (e.g. GKP)

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Inflation

open string

closed string

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D-brane SM model

magnetised branes

D-branes at singularities

1002.1790, 1102.1973, 1106.6039

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chirality (BPM), ~~SUSY~~

various mechanisms have been constructed

control of eft, uv sensitivity

Inflation

open string

closed string

re-heating, dark radiation

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Can these mechanisms
be combined leading to
a realistic string vacuum?

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**Focus on combining moduli stabilisation
with D-brane model building**

D-brane SM model

magnetised branes

D-branes at singularities

Content

- Geometric requirements for combining both mechanisms
- Models with(out) flavour branes
- Models with explicit flux stabilisation (scan)

Geometric Requirements

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- Visible sector with D-branes at del Pezzo singularities (2 dPn's mapped on top of each other with O-involution)
- Mechanism for explicit Kähler moduli stabilisation: here LVS (1 additional divisor allowing for non-perturbative effect)
- No intersection between the two sectors (BPM, chirality + moduli stabilisation)
- CYs with $h^{11} \geq 4$ ($h^{11-} \geq 1$), search in available list of CYs (Kreuzer-Skarke list)

1002.1790, 1102.1973, 1106.6039

models with D-branes @ singularities

alternative setups: magnetised branes

Search in Kreuzer-Skarke database

$h^{1,1} = 4 : 1197$ polytopes	Σ	dP_0	dP_1	dP_2	dP_3	dP_4	dP_5	dP_6	dP_7	dP_8
There are 2 dP_n + O-involution	82	9	5	-	-	-	2	10	31	25
The 2 dP_n do not intersect	68	9	2	-	-	-	2	10	27	18
Further rigid divisor	21	3	-	-	-	-	-	4	9	5

$h^{1,1} = 5 : 4990$ polytopes	Σ	dP_0	dP_1	dP_2	dP_3	dP_4	dP_5	dP_6	dP_7	dP_8
There are 2 dP_n & O-involution	386	27	60	21	7	3	13	40	121	94
The 2 dP_n do not intersect	327	27	55	7	3	1	11	39	112	72
Further rigid divisor	168	14	16	-	-	-	5	28	68	37

more Kähler moduli = more computing time

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let’s take one example,
add some D-branes,
check consistency conditions,
and stabilise Kähler moduli

dP0 example: $h^{1,1}=4$, $h^{1,2}=112$

- charge matrix, SR-ideal

z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	D_{eqX}
1	1	1	0	3	3	0	0	9
0	0	0	1	0	1	0	0	2
0	0	0	0	1	1	0	1	3
0	0	0	0	1	0	1	0	2

$$\mathrm{SR} = \{z_4\, z_6,\ z_4\, z_7,\ z_5\, z_7,\ z_5\, z_8,\ z_6\, z_8,\ z_1\, z_2\, z_3\}$$

- basis of divisors

$$\Gamma_b = D_4 + D_5 = D_6 + D_7, \qquad \Gamma_{q_1} = D_4, \qquad \Gamma_{q_2} = D_7, \qquad \Gamma_s = D_8$$

- triple intersection form, volume

$$I_3 = 27\Gamma_b^3 + 9\Gamma_{q_1}^3 + 9\Gamma_{q_2}^3 + 9\Gamma_s^3 \qquad \mathcal{V} = \frac{1}{9}\sqrt{\frac{2}{3}}\left[\tau_b^{3/2} - \sqrt{3}\left(\tau_{q_1}^{3/2} + \tau_{q_2}^{3/2} + \tau_s^{3/2}\right)\right]$$

- 3 dP0's at $z_4=0$, $z_7=0$, $z_8=0$

Orientifold projection

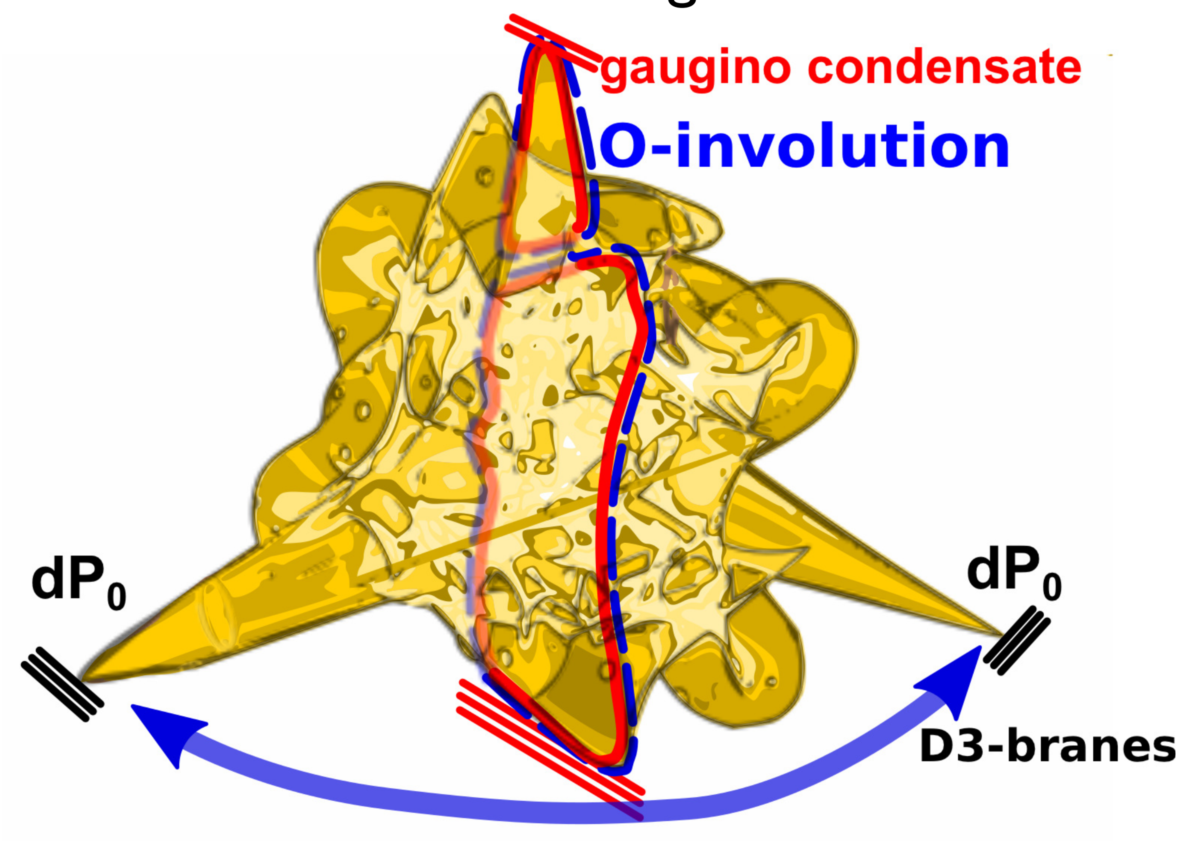
- We take an O-involution exchanging two (shrinking) dP0s:

  $z_4 \leftrightarrow z_7$ and $z_5 \leftrightarrow z_6$ $(h^{1,1-}=1$ and $h^{1,1+}=3)$

- This exchanges the two dP0s: $D_{q1}=D_4$ and $D_{q2}=D_7$

- There are no O3-planes and 2 O7-planes:
  $O7_1$: $z_4z_5 - z_6z_7 = 0 \rightarrow [O7_1]=D_b$. $O7_2$: $z_8 = 0 \rightarrow [O7_2]=D_s$.

- O7-planes do not intersect the shrinking dP0s and each other



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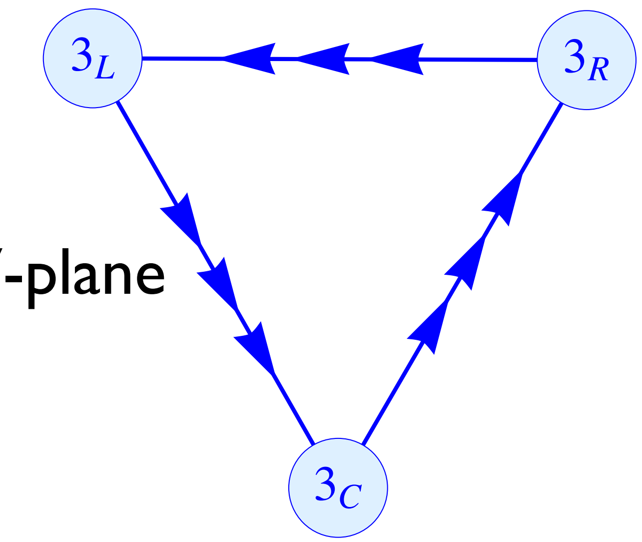
Model without flavour branes



- dP0 trinification model (N=3 D3-branes)
- to cancel D7 tadpole: 4 D7s (+images) on top of each O7-plane
- hidden sector: SO(8)xSO(8)
- FW flux: F_s=-D_s/2 cancelled by choosing B=-D_s/2
  ➛ $\mathcal{F}$s=Fs-B=0 → pure SO(8) SYM on Ds (gaugino condensate)
- FW flux: F_b=-D_b/2 also cancelled by choosing B=-D_b/2 -D_s/2
  ➛ adjoint scalars; can be lifted by flux but SO(8)→SU(4)xU(1)
  (special! no intersection of Γ_s & Γ_b, so cancellation on both possible)
- Non-perturbative superpotential

$$W = W_0 + A_s e^{-a_s T_s} (+A_b e^{-a_b T_b}) \qquad a_s = \frac{\pi}{3} \qquad a_b = \frac{\pi}{2}$$

- D5 tadpole cancelled as $\mathcal{F} = -\mathcal{F}'$.
- Q_{D3} = -60 + 2N_{D3} = -54 (Whitney brane: Q_{D3}=-432, no g.c. on Γ_b)
  freedom to turn on three-form fluxes H_3 & F_3.

Moduli Stabilisation

- complex structure assumed to be stabilised with 3-form fluxes (D3 tadpole allows to turn on fluxes.)
- EFT: $$K = -2\ln\left(\mathcal{V} + \frac{\zeta}{g_s^{3/2}}\right) + \frac{(T_+ + \bar{T}_+ + q_1 V_1)^2}{\mathcal{V}} + \frac{(G + \bar{G} + q_2 V_2)^2}{\mathcal{V}} + \frac{C^i \bar{C}^i}{\mathcal{V}^{2/3}}\,,$$

$$W = W_{\rm local} + W_{\rm bulk} = W_0 + y_{ijk}\, C^i C^j C^k + A_s\, e^{-\frac{\pi}{3}\,T_s} + A_b\, e^{-\frac{\pi}{2}\,T_b}$$

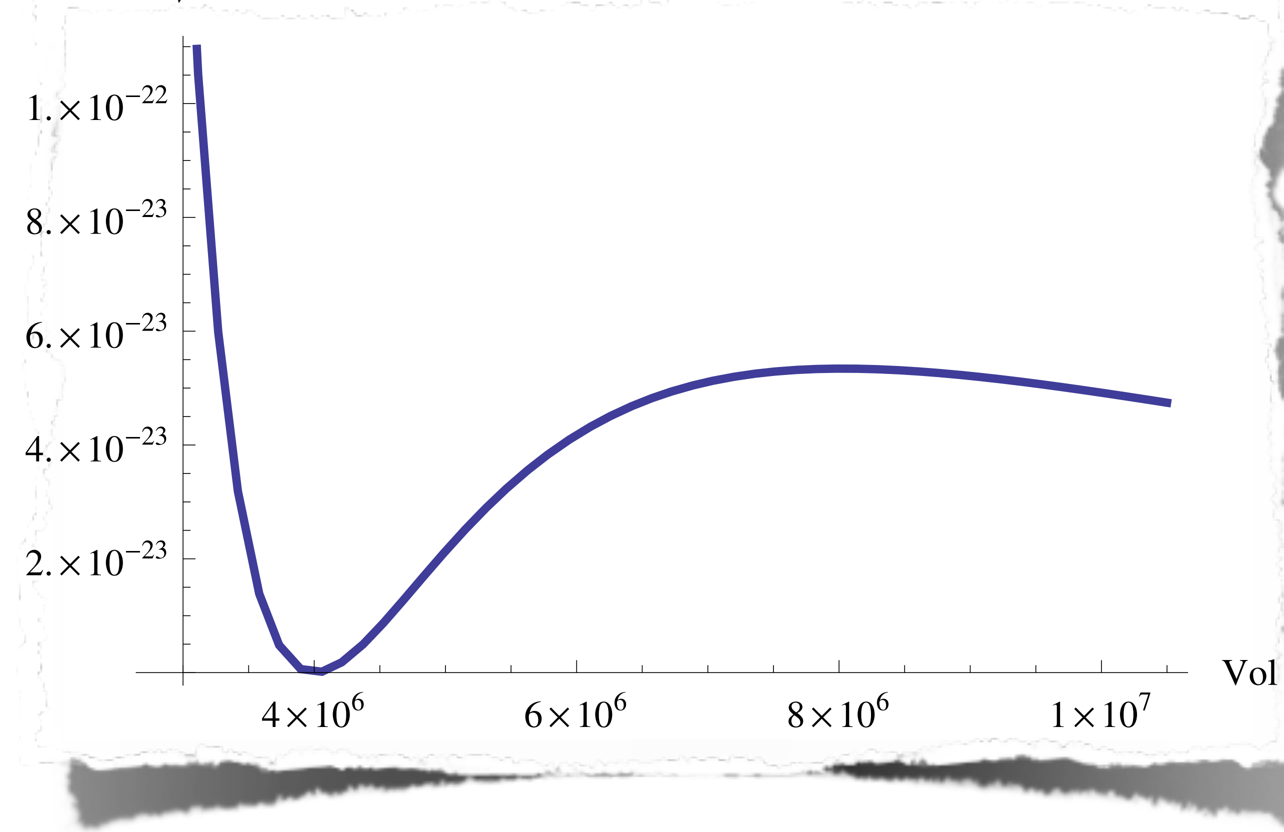
$$\mathcal{V} = \frac{1}{9}\sqrt{\frac{2}{3}}\left(\tau_b^{3/2} - \sqrt{3}\tau_s^{3/2}\right)$$

- singularity stabilisation: D-term minimum at ξ_i=0 and C_i=0 (soft-masses), F-terms sub-leading

$$V_D = \frac{1}{{\rm Re}(f_1)}\left(\sum_i q_{1i} K_i C_i - \xi_1\right)^2 + \frac{1}{{\rm Re}(f_2)}\left(\sum_i q_{2i} K_i C_i - \xi_2\right)^2,$$

$$\xi_1 = -4 q_1 \frac{\tau_+}{\mathcal{V}} \qquad \xi_2 = -4 q_2 \frac{b}{\mathcal{V}}$$

Moduli Stabilisation



- F-term potential

$$V_F \simeq \frac{8}{3}(a_s A_s)^2 \sqrt{\tau_s}\,\frac{e^{-2\,a_s \tau_s}}{\mathcal{V}} - 4\,a_s A_s W_0 \tau_s \frac{e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{3}{4}\frac{\zeta W_0^2}{g_s^{3/2}\mathcal{V}^3}$$

$$\langle \mathcal{V} \rangle \simeq \frac{3W_0\sqrt{\tau_s}}{4a_s A_s}\, e^{a_s\langle \tau_s \rangle} \qquad \langle \tau_s \rangle \simeq \left(\frac{3\zeta}{2}\right)^{2/3}\frac{1}{g_s}$$

$$\zeta \simeq 0.522$$
$$W_0 \simeq 0.2$$
$$g_s \simeq 0.03$$
$$A_s \simeq 1$$

- FW flux on large four-cycle (matter fields), D-term potential

$$V_{\rm tot} = V_D + V_F \simeq \frac{p_1}{\mathcal{V}^{2/3}}\left(\sum_j q_{bj}|\phi_{c,j}|^2 - \frac{p_2}{\mathcal{V}^{2/3}}\right)^2 + \sum_j \frac{W_0^2}{2\mathcal{V}^2}|\phi_{c,j}|^2 + V_F(T)$$

- can account for dS/Minkowski minima...

- KKLT: $\tau_b > \tau_s$ requires $a_b < a_s$ but not realised here...

Gravity/moduli mediated SUSY breaking

0906.3297

- no flavour branes ➔ no redefinitions of moduli
- $F^{TSM}=0$ ➔ sequestered soft-masses
- gravitino mass:

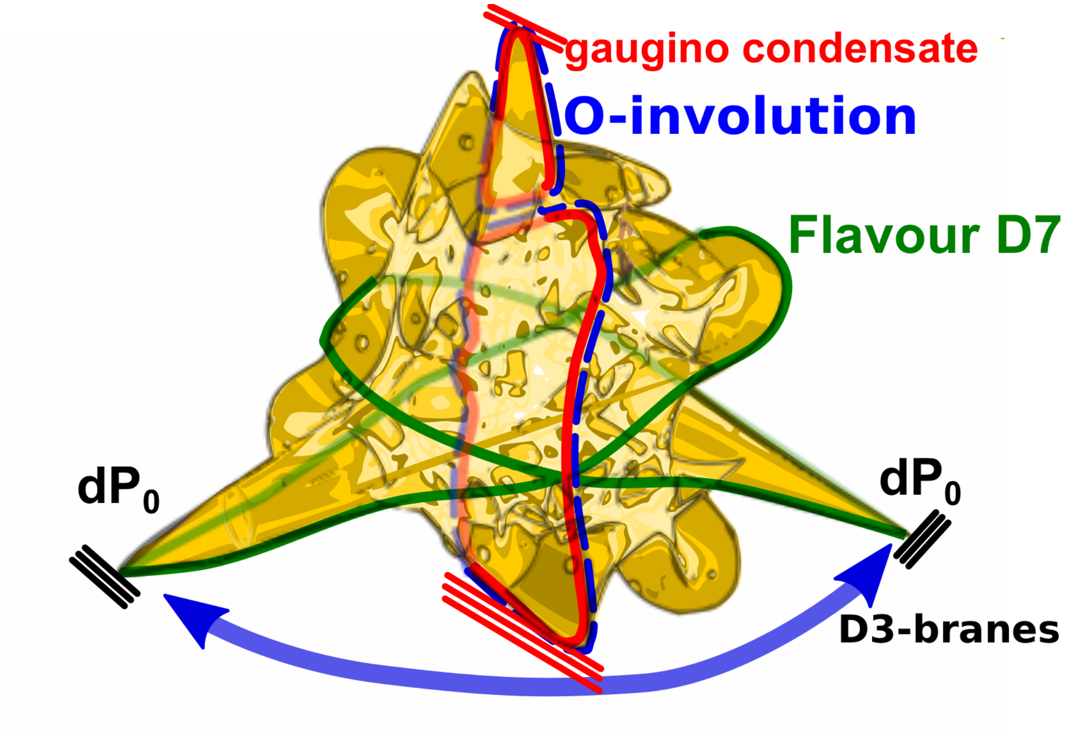
$$m_{3/2} = e^{K/2}|W| \sim \frac{M_P|W_0|}{\mathcal{V}}$$

- remaining soft-masses receive contributions from F^{tb}, F^{s}, gauge kinetic function f=Re(S), after many no-scale cancellations

$M_{\rm gaugino}$	$\frac{m_{3/2}}{\mathcal{V}}$
$m_{\rm scalar}$	$\frac{m_{3/2}}{\sqrt{\mathcal{V}}}$ or $\frac{m_{3/2}}{\mathcal{V}}$
$M_{\rm string}$	$\frac{M_P}{\sqrt{\mathcal{V}}}$
$\mathcal{V}$	10^{6-7}

- Assumption: no D-term contribution [soft scalar masses after gauge breaking will need further study]
- Note: lightest modulus ($m_{Tb} \sim M_P/V^{3/2}$) heavier than TeV soft-masses [➔cosmological moduli problem]
- Pheno: particular slice of CMSSM, resp. Mini Split-SUSY

let’s add flavour D7-branes



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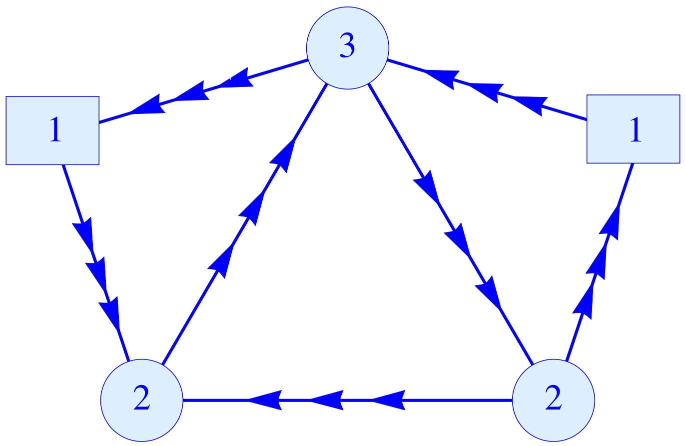
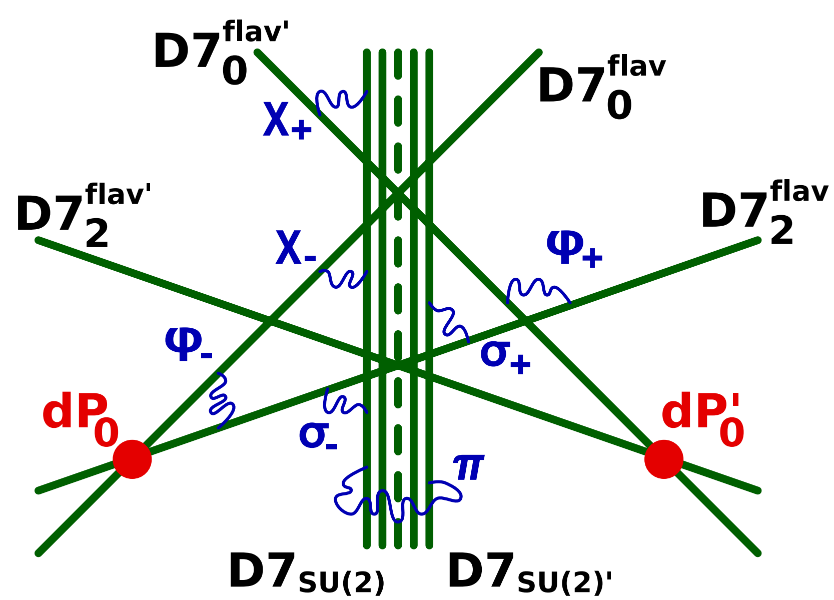
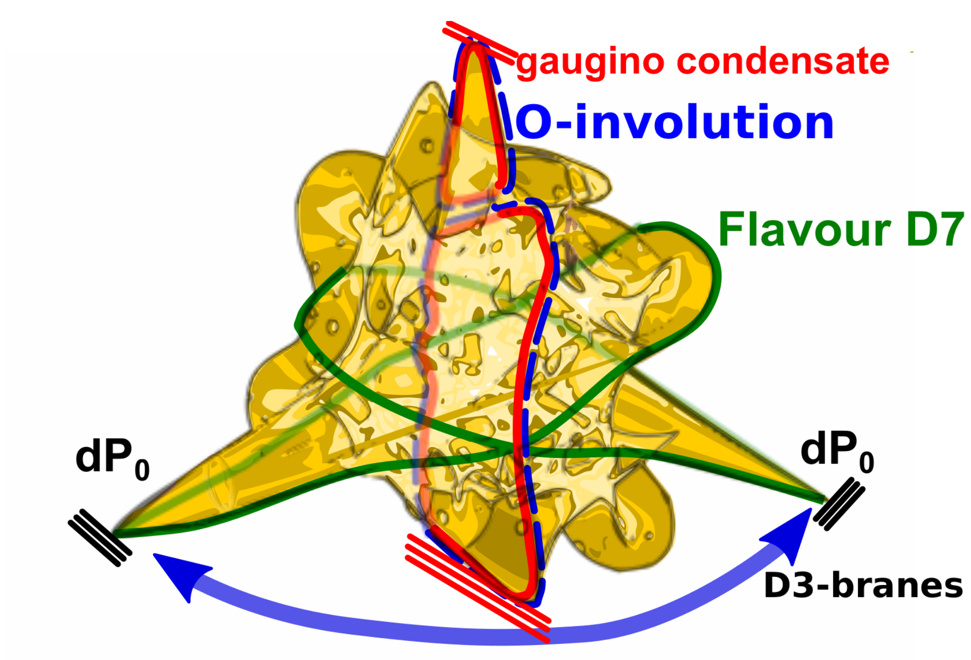
Geometric summary



- Same geometric background and orientifold as before. Can have different D7 brane setup, leading to flavour branes...
- dP0: left-right symmetric model ($n_0=n_2=2$, $m_0=m_2=3$, $m_1=m=0$) as in AIQU hep-th/0005067
- visible sector gauge group: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- hidden sector D7-D7 matter





Moduli stabilisation (with flavour branes)

- D-term potential: quiver D-terms same as before (soft-scalar masses); bulk D-terms some fields receive vev, flat directions at tree-level (need to be fixed by higher order F-term contributions)

- F-term potential for matter fields (leading contribution from soft-masses)

$$V_F^{\rm matter} \simeq \frac{W_0^2}{\mathcal{V}^2\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}\sum_i c_{\rho_i}|\rho_i|^2 \text{ , with: } K_{\rm matter} \simeq \tau_s^{c_\rho}|\rho|^2/\mathcal{V}^{2/3}$$

- Plug in D-term constraints, reasonable assumptions for Kähler matter metric lead to effective matter uplifting contribution

$$V_F^{\rm matter} \simeq p\,\frac{W_0^2}{\mathcal{V}^{8/3}\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}\,, \quad \text{with} \quad p := (2c_\varphi + c_\sigma)\left(\frac{9}{2}\right)^{2/3}\,,$$

- Overall F-term potential

$$V_F \simeq \frac{8}{3}(a_sA_s)^2\sqrt{\tau_s}\,\frac{e^{-2\,a_s\tau_s}}{\mathcal{V}} - 4\,a_sA_sW_0\tau_s\,\frac{e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{3}{4}\,\frac{\zeta W_0^2}{g_s^{3/2}\mathcal{V}^3} + p\,\frac{W_0^2}{\mathcal{V}^{8/3}\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}$$

- After minimising at Large Volume:

$$\Lambda \equiv \langle V\rangle = \frac{W_0^2}{\langle\mathcal{V}\rangle^3}\sqrt{\ln\left(\frac{\langle\mathcal{V}\rangle}{W_0}\right)}\left\{-\frac{3}{4\,a_s^{3/2}} + \frac{p}{9}\,\frac{\langle\mathcal{V}\rangle^{1/3}}{\left[\ln\left(\mathcal{V}/W_0\right)\right]^{5/2}}\left(1-\frac{6}{\ln\left(\mathcal{V}/W_0\right)}\right)\right\}$$

Scales and gauge coupling unification

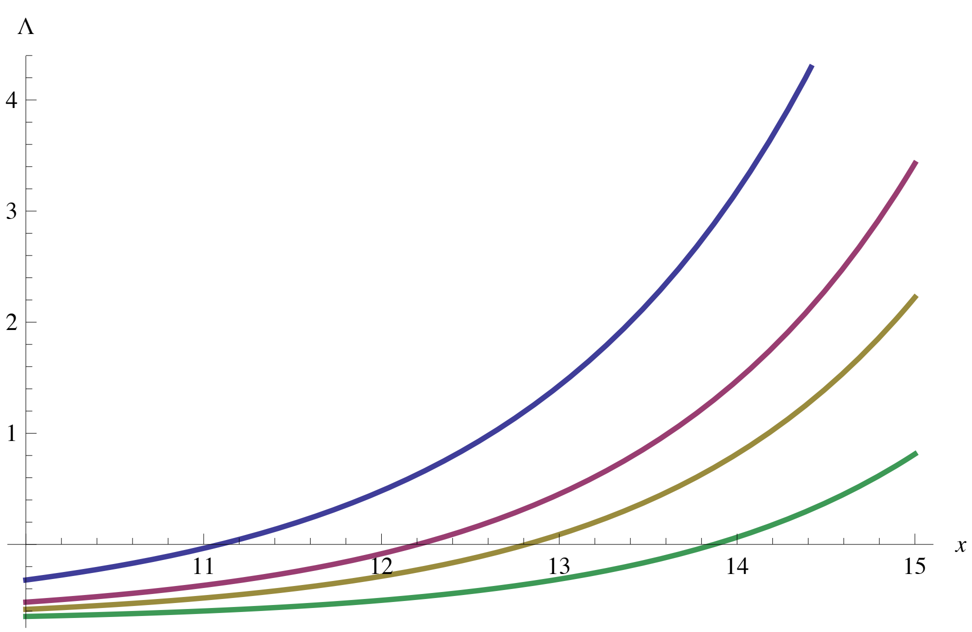
- Soft masses (re-defs due to flavour branes):

$$M_{\rm soft} \simeq \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)} \simeq \frac{W_0\, M_P}{\mathcal{V}\ln\left(\mathcal{V}/W_0\right)}$$

- Required: Soft masses @ TeV, correct gs, and CC=0 by 2 parameters

$$\mathcal{V}/W_0 \simeq 5\cdot 10^{13}, \Lambda = 0 \Rightarrow W_0 \simeq 0.01, \mathcal{V} \simeq 5\cdot 10^{11}$$

$$\zeta \simeq 0.522 \Rightarrow g_s \simeq 0.015 \simeq 1/65, M_s \simeq 10^{12}\mathrm{GeV}$$



p=5.45

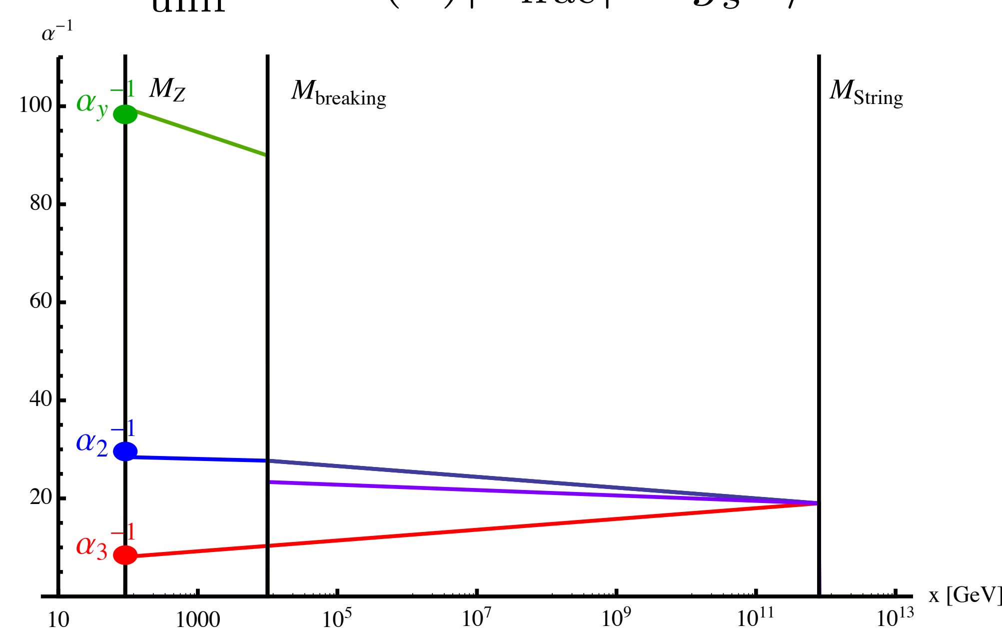
W_0=1

$W_0=10^{-4}$

$W_0=10^{-7}$

$W_0=10^{-14}$

$$\alpha_{\rm unif}^{-1} = \mathrm{Re}(S)|Z_{\rm frac}| = g_s^{-1}/3$$



$$c_\sigma = 1,\ c_\phi = \frac{1}{2} \qquad a_s = \pi/3$$

Conclusion/Outlook

- Successful and explicit combination of moduli stabilisation with D-branes at singularities (with and without flavour branes), satisfying all known consistency conditions
- dS moduli stabilisation with TeV soft-masses (sequestered soft-masses no flavour branes)
- Can the dP0 left-right model pass all low-energy tests?
- Include inflationary sector

Thank you!

Moduli stabilisation (with flavour branes)

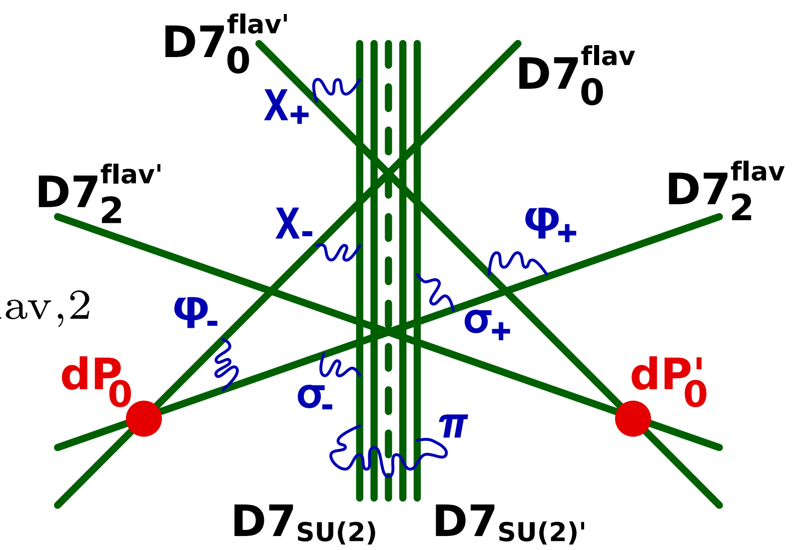
- D-term potential: quiver D-terms same as before (soft-scalar masses); bulk D-terms some fields receive vev, flat directions at tree-level (need to be fixed by higher order F-term contributions)

$$V_D^{\rm bulk} = \frac{1}{{\rm Re}(f_{\rm flav,0})} D^2_{\rm flav,0} + \frac{1}{{\rm Re}(f_{\rm flav,2})} D^2_{\rm flav,2} + \frac{1}{{\rm Re}(f_{D7_{O7_1}})} D^2_{D7_{O7_1}}$$

$$D_{\rm flav,0} = \sum_{i=1}^{12} |\varphi_+^i|^2 + \sum_{i=1}^{9} |\chi_+^i|^2 - \sum_{i=1}^{6} |\varphi_-^i|^2 - \xi_{\rm flav,0}$$

$$D_{\rm flav,2} = \sum_{i=1}^{12} |\varphi_+^i|^2 + \sum_{i=1}^{6} |\varphi_-^i|^2 + \sum_{i=1}^{18} |\sigma_+^i|^2 + \sum_{i=1}^{9} |\sigma_-^i|^2 - \xi_{\rm flav,2}$$

$$D_{D7_{O7_1}} = \sum_{i=1}^{9} |\chi_+^i|^2 + \sum_{i=1}^{18} |\sigma_+^i|^2 + 2\sum_{i=1}^{9} |\pi^i|^2 - \sum_{i=1}^{9} |\sigma_-^i|^2 - \xi_{D7_{O7_1}}$$



$$\xi_{D7_{O7_1}} = \frac{9}{2}\frac{t_b}{\mathcal{V}} = \left(\frac{9}{2}\right)^{2/3} \frac{1}{\mathcal{V}^{2/3}}\,,\ \xi_{\rm flav,0} = \xi_{D7_{O7_1}} + \frac{\sqrt{\tau_q}}{2\mathcal{V}}\,,\ \xi_{\rm flav,2} = 3\,\xi_{\rm flav,0}\,.$$

- F-term potential for matter fields : $K \supset \tau_s^{c_\rho}\,|\rho|^2/\mathcal{V}^{2/3}$

$$V_F^{\rm matter} \simeq \frac{W_0^2}{\mathcal{V}^2\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}\left[c_\varphi\left(\sum_{i=1}^{6}|\varphi_+^i|^2 + \sum_{i=1}^{12}|\varphi_-^i|^2\right) + c_\chi\sum_{i=1}^{9}|\chi_+^i|^2 \right.$$
$$\left. + c_\sigma\left(\sum_{i=1}^{18}|\sigma_+^i|^2 + \sum_{i=1}^{9}|\sigma_-^i|^2\right) + c_\pi\sum_{i=1}^{9}|\pi^i|^2\right]$$

Moduli stabilisation (with flavour branes)

- F-term for matter fields after using D-flatness condition

$$V_F^{\rm matter} \simeq \frac{W_0^2}{\mathcal{V}^2\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}\left[(c_\varphi + c_\chi - c_\sigma)\sum_{i=1}^{9}|\chi_+^i|^2 + 2\,(c_\sigma - c_\varphi)\sum_{i=1}^{9}|\sigma_-^i|^2 + (2c_\varphi + c_\pi - 2c_\sigma)\sum_{i=1}^{9}|\pi^i|^2 + (2c_\varphi + c_\sigma)\,\xi\right]$$

- Minimum for $\chi_+^i = \sigma_-^i = \pi^i = 0$ given

$$K \supset \tau_s^{c_\rho}\,|\rho|^2/\mathcal{V}^{2/3} \quad c_\chi > c_\sigma - c_\varphi > 0\,, \qquad c_\pi > 2\,(c_\sigma - c_\varphi) > 0$$

$$V_F^{\rm matter} \simeq p\,\frac{W_0^2}{\mathcal{V}^{8/3}\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}\,, \quad \text{with} \quad p := (2c_\varphi + c_\sigma)\left(\frac{9}{2}\right)^{2/3}\,,$$

- Overall F-term potential

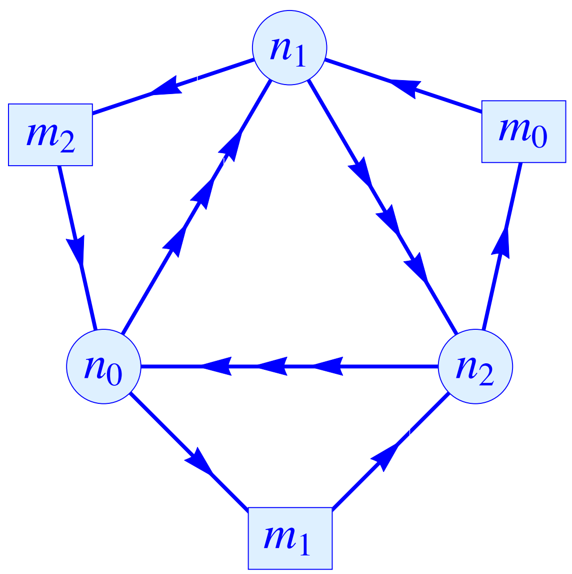
$$V_F \simeq \frac{8}{3}(a_sA_s)^2\sqrt{\tau_s}\,\frac{e^{-2\,a_s\tau_s}}{\mathcal{V}} - 4\,a_sA_sW_0\tau_s\frac{e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{3}{4}\frac{\zeta W_0^2}{g_s^{3/2}\mathcal{V}^3} + p\,\frac{W_0^2}{\mathcal{V}^{8/3}\left[\ln\left(\mathcal{V}/W_0\right)\right]^2}$$

- After minimising at Large Volume:

$$\Lambda \equiv \langle V\rangle = \frac{W_0^2}{\langle\mathcal{V}\rangle^3}\sqrt{\ln\left(\frac{\langle\mathcal{V}\rangle}{W_0}\right)}\left\{-\frac{3}{4\,a_s^{3/2}} + \frac{p}{9}\frac{\langle\mathcal{V}\rangle^{1/3}}{\left[\ln\left(\mathcal{V}/W_0\right)\right]^{5/2}}\left(1 - \frac{6}{\ln\left(\mathcal{V}/W_0\right)}\right)\right\}$$

Restrictions on m_i

- Number of chiral intersections determine local flavour brane charges



$$\langle \Gamma_1, \Gamma_2 \rangle \equiv \int_X \left(-\Gamma_1^{(2-\text{form})} \wedge \Gamma_2^{(4-\text{form})} + \Gamma_2^{(2-\text{form})} \wedge \Gamma_1^{(4-\text{form})} \right)$$

$$\Gamma_{D7_0}^{\text{loc}} = (m + 3(n_1 - n_0))H \left(1 + \frac{1}{2}H\right)$$

$$\Gamma_{D7_1}^{\text{loc}} = -2mH\,(1 + H)$$

$$\Gamma_{D7_2}^{\text{loc}} = (m + 3(n_1 - n_2))H \left(1 + \frac{3}{2}H\right)$$

- $D_{\text{flav}}|_{\text{dP0}} = aH$ is 2-form Poincare dual to the intersection $D_{\text{flav}} \cap D_{\text{dP0}}$

- If D_{flav} is connected and $D_{\text{flav}} \cap D_{\text{dP0}}$ is an effective curve X (described as vanishing locus of hom. eqn. of degree a) $\Rightarrow$ [X]=aH, a > 0.

- The locally induced D7-charge of a flavour brane must be a **positive** multiple of H [note m1=m]

$$0 \leq -m \leq 3(n_1 - \max\{n_0, n_2\})$$

- Not all local models realised globally in this class, e.g.: $n_0=n_1=n_2 \Rightarrow m_i=0$.

Strategy for model building

- gauge theory via favourite technique (e.g. dimers), quiver gauge theory [bi-fundamentals]
- phenomenology: choose your favourite gauge groups (non GUT, e.g. Pati-Salam) to embed SM matter content; break it to the SM, study flavour structure of couplings, proton decay etc.
- bulk effects: couplings, massless hypercharge (simple solution: origin of hypercharge in non-abelian gauge theory)

  [Buican, Malyshev, Morrison, Verlinde, Wijnholt]
- choose your favourite singularity and try (dP0 no mass hierarchy, dP1/dP2 no suff. flavour structure, dP3 flavour structure+diagonal kin. terms)

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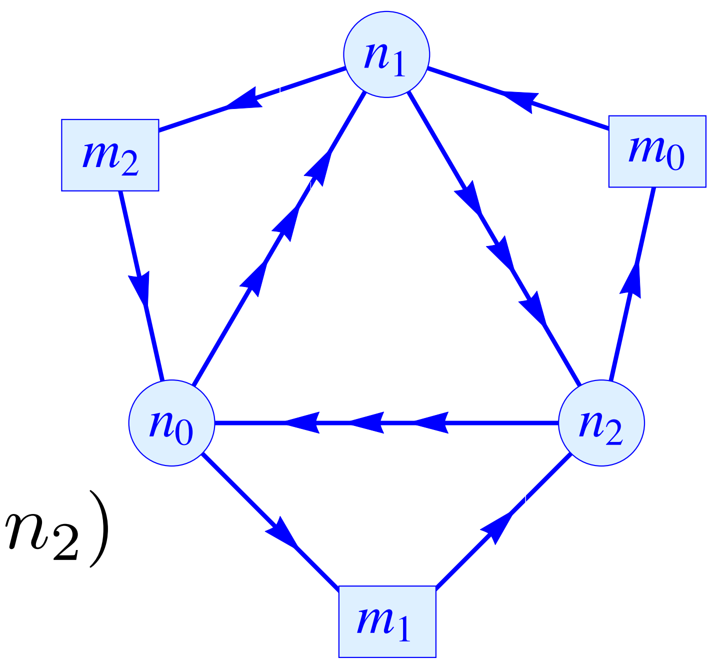
$dP_0 = C^3/Z_3$

- the gauge theory

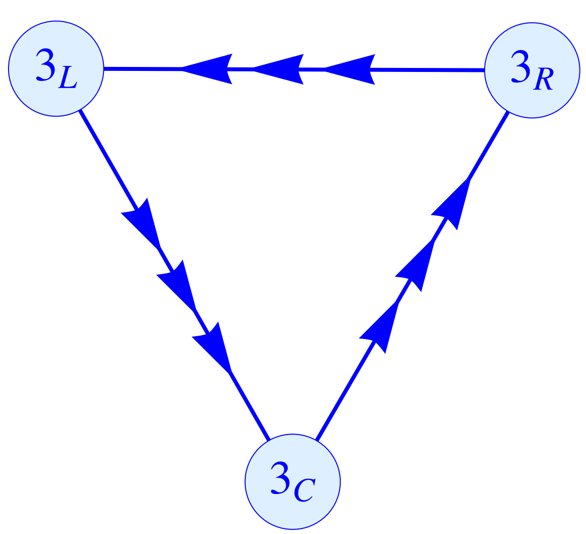
$$W = \epsilon_{ijk} X^i Y^j Z^k + y^{\text{bulk}}_{ijk} X^i Y^j Z^k + W_{D3D7}$$

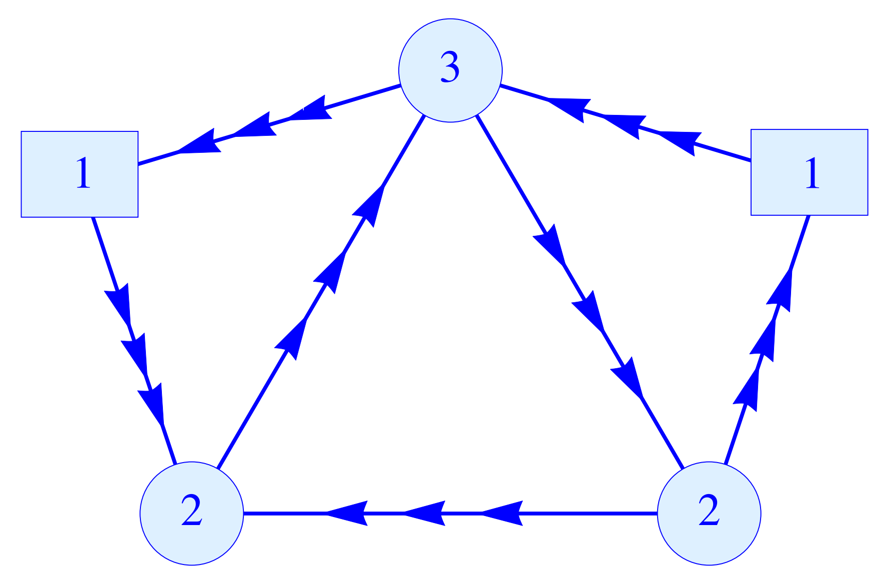


- anomaly cancellation

$$m_0 = m + 3(n_1 - n_0) \quad m_1 = m \quad m_2 = m + 3(n_1 - n_2)$$

- examples: trinification/left-right symmetric models





D-brane charges

- D-brane charges encoded in the Mukai charge vector:

$$\Gamma_{\mathcal{E}} = D \wedge \mathrm{ch}(\mathcal{E}) \wedge \sqrt{\frac{\mathrm{Td}(TD)}{\mathrm{Td}(ND)}}$$

2-form ↔ D7-charge
4-form ↔ D5-charge
6-form ↔ D3-charge

- Fractional branes described by well-chosen collection of sheaves, supported on shrinking cycle (7-branes). For C^3/Z_3:

$$\mathrm{ch}(F_0) = -1 + H - \tfrac{1}{2}\, H \wedge H\,,\ \mathrm{ch}(F_1) = 2 -\ H - \tfrac{1}{2}\, H \wedge H\,,\ \mathrm{ch}(F_2) = -1$$

$D = \mathcal{D}_{dP_0}$ (*H* is hyperplane class for dP0)

- Flavour branes are D7-branes wrapping holomorphic 4-cycles $\mathcal{D}_{\mathrm{flav}}$ that pass through the singularity (i.e. $\mathcal{D}_{\mathrm{flav}} \cdot \mathcal{D}_{dP_0} \neq 0$):

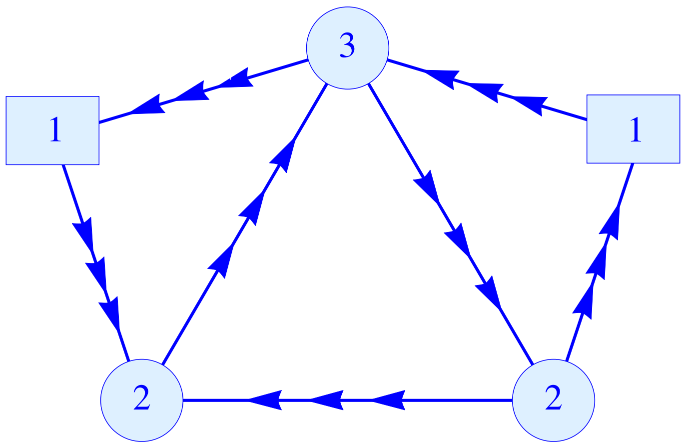
$$\Gamma_{D7}(D_{\mathrm{flav}}, \mathcal{F}) = D_{\mathrm{flav}} \left(1 + \mathcal{F} + \frac{1}{2}\mathcal{F} \wedge \mathcal{F} + \frac{c_2(D_{\mathrm{flav}})}{24}\right)$$

Local models only describe the local charges of $D7_{\mathrm{flav}}$

$$\Gamma^{\mathrm{loc}}_{D7_i} \equiv \Gamma_{\mathcal{D}^i_{\mathrm{flav}}}\Big|_{\mathrm{dP}_0} = a_i H + b_i H \wedge H$$

Diaconescu, Gomis; Douglas, Fiol, Romelsberger; Hanany, Herzog, Vegh

Model with flavour branes



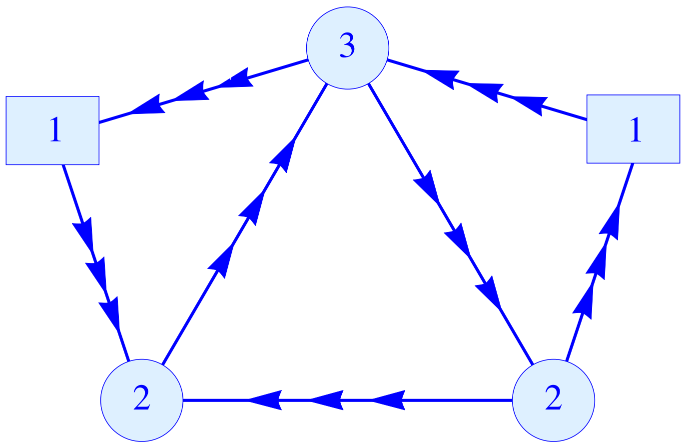
- dP0: left-right symmetric model ($n_0=n_2=2$, $m_0=m_2=3$, $m_1=m=0$)
- gauge group: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- Local D7 charges $\Gamma^{\rm loc}_{D7_0} = 3H(1+\frac{1}{2}H)\,, \qquad \Gamma^{\rm loc}_{D7_2} = 3H(1+\frac{3}{2}H)$
- Which divisor? $D_{\rm flav}|_{Dq1}=3H$

$$\mathcal{D}_{\rm flav} = 3D_1 + \alpha^b \mathcal{D}_b + \alpha^s \mathcal{D}_s + \alpha^{q_2}\mathcal{D}_{q_2}$$

α^s and α^b fixed by $\quad (3D_1 + \mathcal{D}_s + \mathcal{D}_{q_2} + \alpha^b \mathcal{D}_b)\big|_{\mathcal{D}_s,\mathcal{D}_{q2}} = 0$

$$\mathcal{D}^{(0,2)}_{\rm flav} = 3D_1 + \mathcal{D}_s + \mathcal{D}_{q_2} + \alpha^b_{(0,2)}\mathcal{D}_b = (1+\alpha^b_{(0,2)})\mathcal{D}_b - \mathcal{D}_{q_1}$$

Model with flavour branes



- dP0: left-right symmetric model ($n_0=n_2=2$, $m_0=m_2=3$, $m_1=m=0$)

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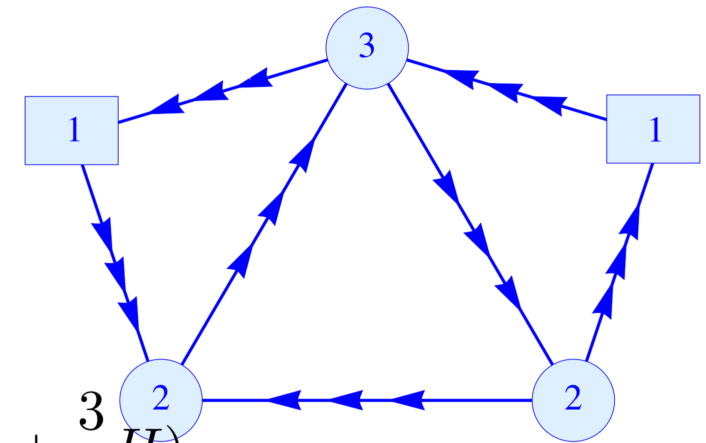
- D7 charge: $$\mathcal{D}^{(0,2)}_{\rm flav} = 3D_1 + \mathcal{D}_s + \mathcal{D}_{q_2} + \alpha^b_{(0,2)}\mathcal{D}_b = (1+\alpha^b_{(0,2)})\mathcal{D}_b - \mathcal{D}_{q_1}$$

- D5 charge: $F_0|_{D_{q1}}$=H/2 and $F_2|_{D_{q1}}$=3H/2. Pullback of D_s and D_{q2} on divisor $D_{\rm flav}$ are trivial. Hence
$$\mathcal{F}^{(0)}_{\rm flav} = \frac{1}{2}D_1 + \beta^b_0\mathcal{D}_b \qquad\qquad \mathcal{F}^{(2)}_{\rm flav} = \frac{3}{2}D_1 + \beta^b_2\mathcal{D}_b$$

- D3 charge:
$$\mathcal{D}_{\rm flav}\cdot\left[\frac{1}{2}\mathcal{F}^2_{\rm flav} + \frac{c_2(\mathcal{D}_{\rm flav})}{24}\right]$$

Model with flavour branes



- Local D7 charges $\Gamma^{\rm loc}_{D7_0} = 3H(1+\frac{1}{2}H)\,, \qquad \Gamma^{\rm loc}_{D7_2} = 3H(1+\frac{3}{2}H)$
- Overall charges from the quiver system

$$\Gamma_{\rm fracD3} = 2\Gamma_{F_0} + 3\Gamma_{F_1} + 2\Gamma_{F_2} = 2\mathcal{D}_{q_1} + 2\mathcal{D}_{q_1}\wedge D_1 - \tfrac{3}{2}d{\rm Vol}^0_X$$
$$\Gamma_{D7^{\rm flav}_0} = (\mathcal{D}_b - \mathcal{D}_{q_1}) + (\mathcal{D}_b - \mathcal{D}_{q_1})\wedge \tfrac{1}{2}D_1 + 5\,d{\rm Vol}^0_X$$
$$\Gamma_{D7^{\rm flav}_2} = (\mathcal{D}_b - \mathcal{D}_{q_1}) + (\mathcal{D}_b - \mathcal{D}_{q_1})\wedge \tfrac{3}{2}D_1 + 7\,d{\rm Vol}^0_X$$
$$\int_X d{\rm Vol}^0_X = 1 \qquad \alpha^b_{(0,2)} = 0 \qquad \beta^b_{(0,2)} = 0$$

- Total charge of quiver system:

$$\Gamma^{z_4=0}_{\rm quiver} = 2\mathcal{D}_b + 2\mathcal{D}_b\wedge D_1 + \left(\tfrac{27}{2} - 3\right) d{\rm Vol}^0_X$$

- To saturate D7 tadpole, still need two D7s on Db (+images)
- D7- and D5-charges cancel globally. Net D3-charge

$$Q^{\rm tot}_{D3} = Q^{z_4=0}_{D3,\,\rm quiver} + Q^{z_7=0}_{D3,\,\rm quiver} + Q^{SU(2)}_{D3} + Q^{SO(8)}_{D3} = -63$$