Predictivity of models with spontaneously broken discrete flavour symmetries

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in collaboration with Mu-Chun Chen, Yuji Omura, Michael Ratz and Christian Staudt.

Chen, MF, Ratz and Staudt, *Phys. Lett.* B **718** (2012), arXiv: `1208.2947 [hep-ph]`.

Chen, MF, Omura, Ratz and Staudt, *Nucl. Phys.* B **873** (2013), arXiv: `1302.5576 [hep-ph]`.

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Neutrino mixing

Experimental success

- Measurements of neutrino mixing angles reach precision phase:

	θ_{12}	θ_{13}	θ_{23}
Best fit values ($\pm 1\sigma$):	$\left(33.6^{+1.1}_{-1.0}\right)^\circ$	$\left(8.93^{+0.46}_{-0.48}\right)^\circ$	$\left(38.4^{+1.4}_{-1.2}\right)^\circ$

Fogli, Lisi, Marrone, Montanino, Palazzo and Rotunno (2012)

$\Updownarrow$

Theory?

- Explanation for the (size of the) mixing.
- Predictions for the yet unknown parameters.

$\Rightarrow$ **Much work to do.**

Outline

Discrete symmetries and flavour model building

Discrete non-abelian symmetry G_F acting on flavour space:

$$L_i \to U_{ij}\,L_j\,, \qquad R_i \to V_{ij}\,R_j\,.$$

e.g. $A_4,\ S_3,\ S_4,\ T',\ \Delta(3\,n^2),\ \dots$

- SM singlet fields charged under G_F: **flavons**.
- G_F spontaneously broken by **flavon** VEVs.

  ⇒ Mass terms generated as effective operators.

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Idea

Angles, phases and masses
completely determined
by flavour symmetry

⟺

Reality

Arbitrariness due to...

...choice of symmetry

...flavon content

...VEV pattern

...more to come

An A_4 example

- 4 irreducible representations: $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$. Altarelli and Feruglio (2006) Ma (2004)
- The leptons:

$$\boxed{(e_\mathrm{R}, \mu_\mathrm{R}, \tau_\mathrm{R}) \sim (\mathbf{1}, \mathbf{1}'', \mathbf{1}'), \quad L \sim \mathbf{3}}$$

- The **flavons** and their VEVs:

$$\begin{array}{ccc} \Phi_\nu \sim \mathbf{3} & \xi_\nu \sim \mathbf{1} & \Phi_e \sim \mathbf{3} \\ \langle\Phi_\nu\rangle = (v, v, v) & \langle\xi_\nu\rangle = w & \langle\Phi_e\rangle = (v', 0, 0) \end{array}$$

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- The superpotential:

$$W_\nu = \frac{\lambda_1}{\Lambda \underset{\text{see-saw scale}}{\Lambda_\nu}} \left\{ [(L\, H_u) \otimes (L\, H_u)]_{\mathbf{3}_\mathrm{s}} \otimes \Phi_\nu \right\}_\mathbf{1} + \frac{\lambda_2}{\Lambda \Lambda_\nu} \, [(L\, H_u) \otimes (L\, H_u)]_\mathbf{1} \, \xi_\nu \,,$$

$$W_e = \frac{h_e}{\underset{\text{cut-off scale}}{\Lambda}} (L \otimes \Phi_e)_\mathbf{1} \, H_d \, e_\mathrm{R} + \frac{h_\mu}{\Lambda} (L \otimes \Phi_e)_{\mathbf{1}'} \, H_d \, \mu_\mathrm{R} + \frac{h_\tau}{\Lambda} (L \otimes \Phi_e)_{\mathbf{1}''} \, H_d \, \tau_\mathrm{R} \,.$$

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$$U_\mathrm{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Harrison, Perkins and Scott (2002)

	θ_{12}	θ_{13}	θ_{23}
TBM prediction:	$\approx 35.3^\circ$	0	45°
Best fit values ($\pm 1\sigma$):	$(33.6^{+1.1}_{-1.0})^\circ$	$(8.93^{+0.46}_{-0.48})^\circ$	$(38.4^{+1.4}_{-1.2})^\circ$

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- Higher–order corrections to the superpotential:

$$W_\nu \to W_\nu + \mathcal{O}\left(\frac{1}{\Lambda^2}\right),$$

$$W_e \to W_e + \mathcal{O}\left(\frac{1}{\Lambda^2}\right).$$

GOOD: more freedom. $\Longleftrightarrow$ **BAD:** loss of predictive power.

Kähler potential corrections

Espinosa and Ibarra (2004); Antusch, King and Malinsky (2008); Antusch, King and Malinsky (2009); Chen, MF, Ratz and Staudt (2012).

- Higher–order terms in the Kähler potential are inevitable:

Canonical terms

$$K = \boxed{{L_i}^\dagger\, L_i + {R_i}^\dagger\, R_i} + \boxed{{L_i}^\dagger\, (\Delta\mathcal{K}_L)_{ij}\, L_j + {R_i}^\dagger\, (\Delta\mathcal{K}_R)_{ij}\, R_j}$$

Correction terms

Contractions with the flavons like

$\frac{1}{\Lambda^2}\,(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}}(L \otimes \Phi_\nu)_{\mathbf{3}}$.

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Correction terms

Contractions with the flavons like $\frac{1}{\Lambda^2}\,(L \otimes \Phi_\nu)^{\dagger}_{\mathbf{3}}(L \otimes \Phi_\nu)_{\mathbf{3}}$.

When $\Phi_\nu \to$ VEV, kinetic terms are re-normalised.

$$\xrightarrow{\textbf{flavons}\to\text{VEV}} \boxed{L_i^{\dagger}\, (\mathbb{1} + \Delta\mathcal{K}_L|_{\text{VEV}})_{ij}\, L_j + R_i^{\dagger}\, (\mathbb{1} + \Delta\mathcal{K}_R|_{\text{VEV}})_{ij}\, R_j}$$

non-diagonal matrices

Canonical normalisation changes the mixing.

Correction terms in the A_4 model

- Corrections linear in flavons can always be forbidden by abelian symmetries.

$$\xcancel{(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}} L}, \qquad \xcancel{(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}} (L \otimes \Phi_e)_{\mathbf{3}}}.$$

- **Quadratic terms cannot be forbidden.**

$$(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}} (L \otimes \Phi_\nu)_{\mathbf{3}} \,.$$

- An A_4 example: Chen, MF, Ratz and Staudt (2012)

$$K \supset \frac{\kappa}{\Lambda^2} (L \otimes \Phi_\nu)^\dagger_{\mathbf{3}_s} (L \otimes \Phi_\nu)_{\mathbf{3}_a} + \text{h.c.}$$

$$\xrightarrow{\Phi_\nu \to \langle \Phi_\nu \rangle = (v,v,v)} \underset{\uparrow}{\kappa} \frac{v^2}{\Lambda^2} \frac{3\sqrt{3}}{2} L^\dagger \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix} L + \text{h.c.}$$

unconstrained

freedom ⇔ arbitrariness

Analytic formulas

Input

Masses
Naive angles and phases

→

Output

Corrected angles and phases

$$K_L = L^\dagger\, L \to \boxed{K_L = L^\dagger\,(\mathbb{1} - 2\,\underset{\text{infinitesimal}}{x}\,\underset{\text{Hermitian}}{P})\,L}$$

- Canonical normalisation: $L \to (\mathbb{1} + x\,P)\,L$.
- Changes mass terms in the superpotential, e.g.

$$\begin{aligned} W_\nu &= \frac{1}{2}\,L^T\,m_\nu\,L \\ &\simeq \frac{1}{2}\,L^T\,\left[m_\nu + x\,\left(P^T\,m_\nu + m_\nu\,P\right)\right]\,L\,. \end{aligned}$$

Analytic formulas

Input

Masses
Naive angles and phases

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- Canonical normalisation: $L \to (\mathbb{1} + x\, P)\, L$.
- Changes mass terms in the superpotential, e.g.

$$W_\nu \simeq \frac{1}{2} L^T \left[m_\nu + x \left(P^T m_\nu + m_\nu P\right)\right] L \,,$$

$$\Rightarrow \frac{\mathrm{d} m_\nu}{\mathrm{d} x} = P^T m_\nu + m_\nu P \,, \quad (\text{for infinitesimal } x)\,.$$

- Differential equations analogous to RGEs.

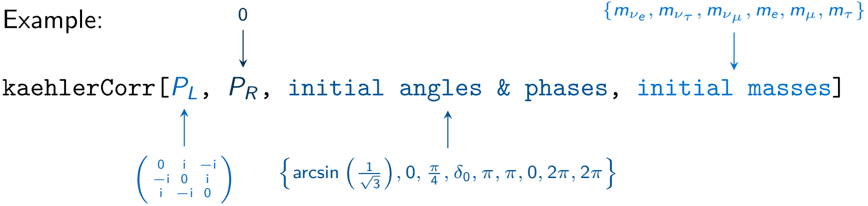
Chen, MF, Omura, Ratz and Staudt (2013)
Antusch, Kersten, Lindner, Ratz and Schmidt (2005)
Antusch, Kersten, Lindner and Ratz (2003)

The MATHEMATICA package

- Contains the pre-computed analytic formulas.
- The package can be found here:

  `http://einrichtungen.ph.tum.de/T30e/codes/KaehlerCorrections/`

- Example:



0

$\{m_{\nu_e}, m_{\nu_\tau}, m_{\nu_\mu}, m_e, m_\mu, m_\tau\}$

`kaehlerCorr[`P_L`, `P_R`, initial angles & phases, initial masses]`

$$\begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

$$\left\{\text{arcsin}\left(\frac{1}{\sqrt{3}}\right), 0, \frac{\pi}{4}, \delta_0, \pi, \pi, 0, 2\pi, 2\pi\right\}$$

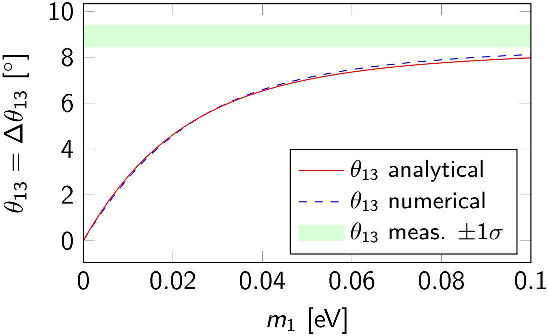
Example I

- Kähler correction of the form:

$$\Delta K = \kappa \frac{v^2}{\Lambda^2} 3\sqrt{3} \cdot L^\dagger \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix} L \, .$$

- This changes θ_{13} by:

$$\Delta\theta_{13} \simeq \kappa \frac{v^2}{\Lambda^2} \; 3\sqrt{6} \; \frac{m_1}{m_1 + m_3}$$



$$\kappa \frac{v^2}{\Lambda^2} = 1 \cdot (0.2)^2$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{13}^2}$$

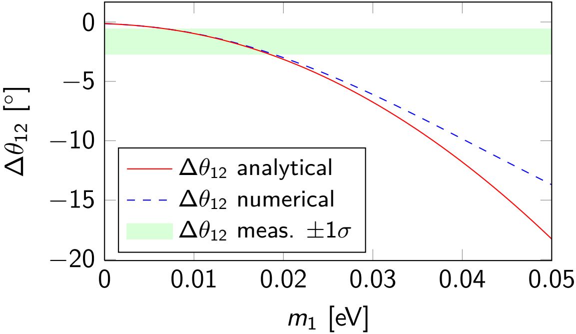
Example II

- Kähler correction of the form:

$$\Delta K = \kappa' \, \frac{v'^2}{\Lambda^2} \cdot L^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} L \, .$$

- This changes θ_{12} by:

$$\Delta\theta_{12} \simeq \kappa' \, \frac{v'^2}{\Lambda^2} \quad \frac{1}{3\sqrt{2}} \, \frac{m_1 + m_2}{m_1 - m_2}$$



$$\kappa' \, \frac{v'^2}{\Lambda^2} = \frac{1}{4} \cdot (0.2)^2$$

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}$$

Conclusion

- **Kähler corrections are inevitable** in models with discrete flavour symmetries.

- The **corrections** to the mixing angles **can be sizable**.
  - ⇒ It might be premature to exclude models only because of their superpotential predictions.

- This introduces a **considerable arbitrariness** into flavour model building.
  - ⇒ Difficult to achieve theoretical precision comparable with experimental precision without knowledge of the Kähler potential.

Thank You!

Bibliography I

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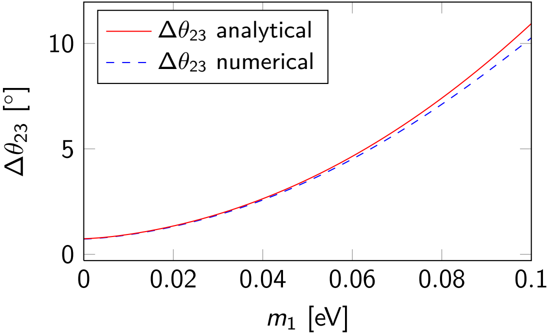
Example III

- Kähler correction of the form:

$$\Delta K = \kappa \frac{v^2}{\Lambda^2} \cdot L^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} L \,.$$

- This changes θ_{23} by:

$$\Delta\theta_{23} \simeq \kappa \frac{v^2}{\Lambda^2} \; \frac{1}{12} \left(3 + \frac{2 m_1}{m_3 - m_1} + \frac{4 m_2}{m_3 - m_2} \right)$$



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