Flavour GUT Models with $\theta_{13}^{\text{PMNS}} = \theta_C/\sqrt{2}$

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30th August 2013

SUSY Conference 2013

Based on arXiv:1305.6612 and arXiv:1306.3984
In collaboration with Stefan Antusch, Christian Gross & Constantin Sluka

Motivation

until 2011 One of flavour model builders' favourites:
Tri-bimaximal lepton mixing

$$\sin^2\theta_{12}^{\text{PMNS}} = \frac{1}{3},\ \sin^2\theta_{23}^{\text{PMNS}} = \frac{1}{2},\ \theta_{13}^{\text{PMNS}} \approx 0^\circ$$

March 2012 Daya Bay: $\theta_{13}^{\text{PMNS}} = 8.8^\circ \pm 1.0^\circ$
Striking resemblence with $\theta_C/\sqrt{2} = 9.2^\circ$

Later 2012 Simple conditions for $\theta_{13}^{\text{PMNS}} \simeq \theta_C/\sqrt{2}$ from GUTs

[Antusch, Gross, V.M., Sluka]

Now Concrete models fulfilling these

Outline

1. Motivation
2. Models with $\theta_{13}^{\mathrm{PMNS}} = \theta_C/\sqrt{2}\ldots$
   - …and Normal Neutrino Mass Hierarchy
   - …and Inverse Neutrino Mass Hierarchy
   - …compared with each other
3. Summary and Conclusions

Common Features

- SUSY + grand unification + flavour symmetry $\rightarrow$ SU(5) $\times A_4$
- Conditions for $\theta_{13}^{\mathrm{PMNS}} \simeq \theta_C/\sqrt{2}$ [Antusch, Gross, V.M., Sluka '12]
  - $\theta_{13}^{\nu} \simeq \theta_{13}^{e} \simeq 0$
  - $\theta_{12}^{e} \simeq \theta_{12}^{d}$
  - $\theta_{12}^{d} \simeq \theta_C$
- Effective operators with H_{24} $\rightarrow$ Discrete SU(5)-breaking ratios

[Antusch, Spinrath '09]

- "Right-handed unitarity triangle" [Antusch, King, Malinsky, Spinrath '10]

$$\theta_{13}^{u} \simeq \theta_{13}^{d} \simeq 0 \Rightarrow \theta_{12}^{d} = 12^\circ$$
$$\Rightarrow \alpha = \delta_{12}^{d} - \delta_{12}^{u}$$

- Spontaneous CP violation
- Alignment ✓ Messengers ✓

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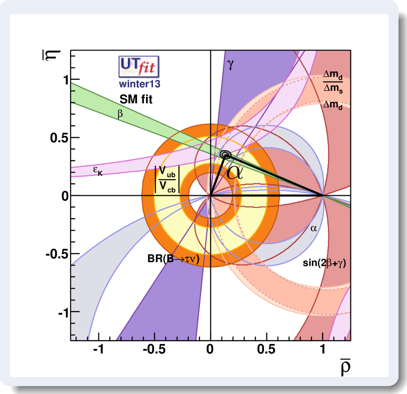
- Conditions for $\theta^{\text{!}}$ …
  - $\theta^{\nu}_{13} \simeq \theta^{e}_{13} \simeq$ (
  - $\theta^{e}_{12} \simeq \theta^{d}_{12}$
  - $\theta^{d}_{12} \simeq \theta_C$
- Effective operat… …reaking ratios

[Antusch, Spinrath '09]

- "Right-handed u… …pinrath '10]



$$\theta^{u}_{13} \simeq \theta^{u}_{13} \simeq 0 \Rightarrow \theta^{u}_{12} = 12^\circ$$

$$\Rightarrow \alpha = \delta^{d}_{12} - \delta^{u}_{12}$$

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Normal Hierarchy Model: Quark & Ch. Lepton Sector

$$W_{Y_d} = [T_1 H_{\overline{45}}]_{45} \, [F H_{24}]_{\overline{45}} \, \phi_2 + [T_2 H_{24}]_{10} \, [F H_{\bar{5}}]_{\overline{10}} \, \phi_{ab} + [T_3 H_{\bar{5}}]_5 \, [F H_{24}]_{\bar{5}} \, \phi_3 + [T_3 H_{24}]_{10} \, [F H_{\bar{5}}]_{\overline{10}} \, \chi \phi_2$$

$$W_{Y_u} = H_5 (T_3^2 + T_2^2 \phi_{ab}^2 + T_1^2 (\phi_2^2)^2 + T_2 T_3 \xi_{23} + T_1 T_2 \xi_{12}^5)$$

$$+$$

flavon:	ϕ_2	ϕ_3	ϕ_{ab}	ξ_{12}	ξ_{23}	$\chi \sim \mathbf{1}'$
VEV:	$\epsilon_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\epsilon_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\epsilon_{ab} \begin{pmatrix} \cos\theta_{ab} \\ -\mathrm{i}\sin\theta_{ab} \\ 0 \end{pmatrix}$	ϵ_{12}	ϵ_{23}	ϵ_χ

$$\Rightarrow$$

$$Y_d = \begin{pmatrix} 0 & \tilde\epsilon_2 & 0 \\ \tilde\epsilon_{ab} c_{ab} & \mathrm{i}\,\tilde\epsilon_{ab} s_{ab} & 0 \\ 0 & \omega^2 \hat\epsilon_\chi & \tilde\epsilon_3 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 6\tilde\epsilon_{ab} c_{ab} & 0 \\ -\frac{1}{2}\tilde\epsilon_2 & \mathrm{i}\,6\tilde\epsilon_{ab} s_{ab} & 6\omega^2 \hat\epsilon_\chi \\ 0 & 0 & -\frac{3}{2}\tilde\epsilon_3 \end{pmatrix}, \quad Y_u = \begin{pmatrix} \epsilon_2^4 & \epsilon_{12}^5 & 0 \\ \epsilon_{12}^5 & \epsilon_{ab}^2 & \epsilon_{23} \\ 0 & \epsilon_{23} & y_t \end{pmatrix}$$

$\omega = \exp \frac{2}{3}\pi \mathrm{i}$

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$$W_{Y_u} = H_5 (T_3^2 + T_2^2 \phi_{ab}^2 + T_1^2 (\phi_2^2)^2 + T_2 T_3 \xi_{23} + T_1 T_2 \xi_{12}^5)$$

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Normal Hierarchy Model: Neutrino Sector

$$W_{Y_\nu} = (H_5 F)(N_1 \phi_{N_1} + N_2 \phi_{N_2})$$
$$W_{M_R} = \xi_M^2 (N_1^2 \phi_{N_1}^2 + N_2^2 \phi_{N_2}^2)$$

+

flavon:	ϕ_{N_1}	ϕ_{N_2}	ξ_M
VEV:	$\epsilon_{N_1} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$	$\epsilon_{N_2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	ϵ_M

$$\Rightarrow \quad Y_\nu = \begin{pmatrix} 0 & \epsilon_{N_2} \\ \epsilon_{N_1} & \epsilon_{N_2} \\ -\epsilon_{N_1} & \epsilon_{N_2} \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{R_1} & 0 \\ 0 & M_{R_2} \end{pmatrix}$$

$$\xrightarrow{\text{see-saw I}} \quad m_\nu = \frac{v_u^2}{2} \begin{pmatrix} A & A & A \\ A & A+B & A-B \\ A & A-B & A+B \end{pmatrix}, \quad \text{with} \quad A = \frac{\epsilon_{N_2}^2}{M_{R_2}}, \; B = \frac{\epsilon_{N_1}^2}{M_{R_1}}$$

$\Rightarrow$ Tribimaximal neutrino mixing, normal hierarchy

Normal Hierarchy Model: Fit to Data

- We fit 12+2 parameters to 18 observables:
  - 9 quark and charged lepton masses [Xing, Zhang, Zhou '07]
  - 3 quark mixing angles + 1 Dirac CP phase [UTfit '13]
  - 3 neutrino mixing angles
  - neutrino mass square differences [NuFIT '13]
- Predictions for δ^{PMNS} and φ^{PMNS}
- MCMC analysis to determine uncertainties
- Taking into account running from M_{GUT} to $m_t(m_t)$
- SUSY threshold corrections at $M_{\text{SUSY}} = 1\text{TeV}$ with $\tan\beta = 40$

Normal Hierarchy Model: Fit and MCMC Analysis

Observable		Value at m_t		Best fit result	Uncertainty
m_u	in MeV	1.22	$^{+0.48}_{-0.40}$	1.22	$^{+0.49}_{-0.40}$
m_c	in GeV	0.59	± 0.08	0.59	± 0.08
m_t	in GeV	162.9	± 2.8	162.89	$^{+2.62}_{-2.36}$
m_d	in MeV	2.76	$^{+1.19}_{-1.14}$	2.73	$^{+0.30}_{-0.70}$
m_s	in MeV	52	± 15	51.66	$^{+5.60}_{-13.68}$
m_b	in GeV	2.75	± 0.09	2.75	± 0.09
m_e	in MeV	0.485	$\pm 1\%$	0.483	± 0.005
m_μ	in MeV	102.46	$\pm 1\%$	102.83	$^{+1.01}_{-0.98}$
m_τ	in MeV	1742	$\pm 1\%$	1741.75	$^{+17.38}_{-17.10}$
$\sin\theta_C$		0.2254	± 0.0007	0.2255	± 0.0007
$\sin\theta_{23}^{\text{CKM}}$		0.0421	± 0.0006	0.0422	± 0.0006
$\sin\theta_{13}^{\text{CKM}}$		0.0036	± 0.0001	0.0036	± 0.0001
δ^{CKM}	in $^\circ$	69.2	± 3.1	65.65	$^{+1.78}_{-0.53}$
$\sin^2\theta_{12}^{\text{PMNS}}$		0.306	± 0.012	0.317	± 0.006
$\sin^2\theta_{23}^{\text{PMNS}}$		0.437	$^{+0.061}_{-0.031}$	0.387	$^{+0.017}_{-0.023}$
$\sin^2\theta_{13}^{\text{PMNS}}$		0.0231	$^{+0.0023}_{-0.0022}$	0.0269	$^{+0.0011}_{-0.0015}$
δ^{PMNS}	in $^\circ$	-		268.79	$^{+1.32}_{-1.72}$
φ_2^{PMNS}	in $^\circ$	-		297.34	$^{+8.66}_{-10.01}$
Δm^2_{sol}	in 10^{-5} eV^2	7.45	$^{+0.19}_{-0.16}$	7.45	$^{+0.18}_{-0.17}$
Δm^2_{atm}	in 10^{-3} eV^2	2.421	$^{+0.022}_{-0.023}$	2.421	$^{+0.022}_{-0.023}$

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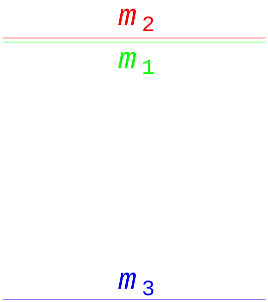
$$\chi^2/\text{d.o.f.} = 2.0$$

Inverse hierarchy:



Problem for Flavour Models

$$\frac{m_1 - m_2}{m_2} \ll 1 \qquad \Rightarrow \text{Finetuning?}$$

Inverse Hierarchy Model: Neutrino Sector

One promising approach [King, Singh '00] :

$$M_R = \hat{M}_R \begin{pmatrix} \varepsilon & 1 \\ 1 & 0 \end{pmatrix}, \; Y_\nu = \begin{pmatrix} a & 0 \\ 0 & b \\ 0 & c \end{pmatrix}$$

$$\xrightarrow{\text{see-saw I}} \quad m_\nu = \begin{pmatrix} 0 & B & C \\ B & 0 & 0 \\ C & 0 & 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & B & C \\ 0 & C & C^2/B \end{pmatrix}$$

$$\text{with } B = b\,\frac{a\,v_u^2}{2\hat{M}_R}, C = c\,\frac{a\,v_u^2}{2\hat{M}_R}, \alpha = -\varepsilon\frac{b}{a}.$$

Inverse Hierarchy Model: Neutrino Sector

$$m_\nu = \begin{pmatrix} 0 & B & C \\ B & 0 & 0 \\ C & 0 & 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & B & C \\ 0 & C & C^2/B \end{pmatrix}$$

$\Rightarrow$ Masses:

$$m_3 = 0$$
$$-\Delta m^2_{\text{atm}} = m_2^2 \approx B^2 + C^2 \,,$$
$$\Delta m^2_{\text{sol}} = m_2^2 - m_1^2 \approx 2\,\alpha\, \frac{(B^2 + C^2)^{3/2}}{|B|} \,,$$

Strong inverse neutrino mass hierarchy
Solar mass splitting $\sim \alpha \sim \varepsilon \Rightarrow$ naturally small

Inverse Hierarchy Model: Neutrino Sector

$$m_\nu = \begin{pmatrix} 0 & B & C \\ B & 0 & 0 \\ C & 0 & 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & B & C \\ 0 & C & C^2/B \end{pmatrix}$$

$\Rightarrow$ Mixing Angles:

$$\tan \theta_{12}^\nu \approx \left| 1 - \frac{\alpha}{2} \frac{\sqrt{B^2 + C^2}}{|B|} \right| \approx \left| 1 + \frac{1}{4} \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} \right|$$

$$\tan \theta_{23}^\nu = \left| \frac{C}{B} \right|$$

$$\theta_{13}^\nu = 0$$

Inverse Hierarchy Model: Neutrino Sector

Implementation:

$$W_{Y_\nu} = (H_5 F)(N_1 \phi_1 + N_2 \phi_{bc})$$
$$W_{M_R} = \xi_M^4 (N_1 N_2 + \phi_{bc}^2 N_1^2)$$

+

flavon:	ϕ_1	ϕ_{bc}	ξ_M
VEV:	$\epsilon_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\epsilon_{bc} \begin{pmatrix} 0 \\ c_{bc} \\ s_{bc} \end{pmatrix}$	ϵ_M

$$\Rightarrow \qquad M_R = \hat{M}_R \begin{pmatrix} \varepsilon & 1 \\ 1 & 0 \end{pmatrix}, \; Y_\nu = \begin{pmatrix} a & 0 \\ 0 & b \\ 0 & c \end{pmatrix}$$

with $a = \epsilon_1$, $b = \epsilon_{bc} \cos\theta_{bc}$, $c = \epsilon_{bc} \sin\theta_{bc}$, $\varepsilon \simeq \epsilon_{bc}^2$

Inverse Hierarchy Model: Quark & Ch. Lepton Sector

$$W_{Y_d} = [T_1 H_{\overline{45}}]_{45}\, [F H_{24}]_{\overline{45}}\, \phi_2 + [T_2 H_{24}]_{10}\, [F H_{\bar{5}}]_{\overline{10}}\, \phi_{ab} + [T_3 H_{\bar{5}}]_{5}\, [F H_{24}]_{\bar{5}}\, \phi_3 + \cancel{[T_3 H_{24}]_{10}\, [F H_{\bar{5}}]_{\overline{10}}\, \chi \phi_2}$$

$$W_{Y_u} = H_5 (T_3^2 + T_2^2 \phi_{ab}^2 + T_1^2 (\phi_2^2)^2 + T_2 T_3 \xi_{23} + T_1 T_2 \xi_{12}^5)$$

$$+$$

flavon:	ϕ_2	ϕ_3	ϕ_{ab}	ξ_{12}	ξ_{23}
VEV:	$\epsilon_2 \begin{pmatrix} 0 \\ -\mathrm{i} \\ 0 \end{pmatrix}$	$\epsilon_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\epsilon_{ab} \begin{pmatrix} \cos\theta_{ab} \\ \sin\theta_{ab} \\ 0 \end{pmatrix}$	ϵ_{12}	ϵ_{23}

$$\Rightarrow$$

$$Y_d = \begin{pmatrix} 0 & \mathrm{i}\,\tilde{\epsilon}_2 & 0 \\ \tilde{\epsilon}_{ab} c_{ab} & \tilde{\epsilon}_{ab} s_{ab} & 0 \\ 0 & 0 & \tilde{\epsilon}_3 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 6\tilde{\epsilon}_{ab} c_{ab} & 0 \\ -\frac{1}{2}\mathrm{i}\,\tilde{\epsilon}_2 & 6\tilde{\epsilon}_{ab} s_{ab} & 0 \\ 0 & 0 & -\frac{3}{2}\tilde{\epsilon}_3 \end{pmatrix}, \quad Y_u = \begin{pmatrix} \epsilon_2^4 & \epsilon_{12}^5 & 0 \\ \epsilon_{12}^5 & \epsilon_{ab}^2 & \epsilon_{23} \\ 0 & \epsilon_{23} & y_t \end{pmatrix}$$

Model with Inverse Hierarchy: Fit and MCMC Analysis

Observable		Value at m_t		Best fit result	Uncertainty
m_u	in MeV	1.22	$^{+0.48}_{-0.40}$	1.22	$^{+0.50}_{-0.39}$
m_c	in GeV	0.59	± 0.08	0.59	$^{+0.07}_{-0.09}$
m_t	in GeV	162.9	± 2.8	162.91	$^{+3.35}_{-2.44}$
m_d	in MeV	2.76	$^{+1.19}_{-1.14}$	2.73	$^{+0.25}_{-0.54}$
m_s	in MeV	52	± 15	50.70	$^{+4.86}_{-9.72}$
m_b	in GeV	2.75	± 0.09	2.75	± 0.09
m_e	in MeV	0.485	$\pm 1\%$	0.483	± 0.005
m_μ	in MeV	102.46	$\pm 1\%$	102.87	$^{+1.04}_{-0.91}$
m_τ	in MeV	1742	$\pm 1\%$	1741.99	$^{+16.84}_{-17.70}$
$\sin\theta_C$		0.2254	± 0.0007	0.2255	± 0.0007
$\sin\theta_{23}^{\mathrm{CKM}}$		0.0421	± 0.0006	0.0421	± 0.0006
$\sin\theta_{13}^{\mathrm{CKM}}$		0.0036	± 0.0001	0.0036	± 0.0001
δ^{CKM}	in $^\circ$	69.2	± 3.1	69.27	$^{+0.91}_{-0.69}$
$\sin^2\theta_{12}^{\mathrm{PMNS}}$		0.306	± 0.012	0.303	± 0.005
$\sin^2\theta_{23}^{\mathrm{PMNS}}$		0.437	$^{+0.061}_{-0.031}$	0.397	$^{+0.023}_{-0.022}$
$\sin^2\theta_{13}^{\mathrm{PMNS}}$		0.0231	$^{+0.0023}_{-0.0022}$	0.0267	$^{+0.0016}_{-0.0015}$
δ^{PMNS}	in $^\circ$	-		180	-
φ^{PMNS}	in $^\circ$	-		180	-
$\Delta m^2_{\mathrm{sol}}$	in 10^{-5} eV^2	7.45	$^{+0.19}_{-0.16}$	7.45	$^{+0.18}_{-0.17}$
$\Delta m^2_{\mathrm{atm}}$	in 10^{-3} eV^2	-2.410	$^{+0.062}_{-0.063}$	-2.410	$^{+0.062}_{-0.064}$

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$\sin\theta_C$		[illegible]	[illegible]	[illegible]	± 0.0007
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δ^{CKM}	in $^\circ$	69.2	± 3.1	69.27	$^{+0.91}_{-0.69}$
$\sin^2\theta_{12}^{PMNS}$		0.306	± 0.012	0.303	± 0.005
$\sin^2\theta_{23}^{PMNS}$		0.437	$^{+0.061}_{-0.031}$	0.397	$^{+0.023}_{-0.022}$
$\sin^2\theta_{13}^{PMNS}$		0.0231	$^{+0.0023}_{-0.0022}$	0.0267	$^{+0.0016}_{-0.0015}$
δ^{PMNS}	in $^\circ$	-		180	-
φ^{PMNS}	in $^\circ$	-		180	-
Δm^2_{sol}	in 10^{-5} eV^2	7.45	$^{+0.19}_{-0.16}$	7.45	$^{+0.18}_{-0.17}$
Δm^2_{atm}	in 10^{-3} eV^2	-2.410	$^{+0.062}_{-0.063}$	-2.410	$^{+0.062}_{-0.064}$

$$\chi^2/\text{d.o.f.} = 1.1$$

Comparison between IH and NH Model

	IH	NH	Data
Δm^2_{atm}	< 0	> 0	—
δ^{PMNS}	180°	$268.79^\circ{}^{+1.32^\circ}_{-1.72^\circ}$	—
$\theta_{12}^{\text{PMNS}}$	$33.38^\circ{}^{+0.30^\circ}_{-0.28^\circ}$	$34.29^\circ{}^{+0.35^\circ}_{-0.39^\circ}$	$33.57^\circ{}^{+0.77^\circ}_{-0.75^\circ}$
$\theta_{23}^{\text{PMNS}}$	$39.06^\circ{}^{+1.33^\circ}_{-1.32^\circ}$	$38.49^\circ{}^{+1.11^\circ}_{-1.26^\circ}$	$41.4^\circ{}^{+3.5^\circ}_{-1.8^\circ}$
$\theta_{13}^{\text{PMNS}}$	$9.41^\circ{}^{+0.28^\circ}_{-0.27^\circ}$	$9.43^\circ{}^{+0.20^\circ}_{-0.25^\circ}$	$8.75^\circ{}^{+0.42^\circ}_{-0.44^\circ}$
$m_{\beta\beta}$	$(1.83^{+0.05}_{-0.06}) \cdot 10^{-2}$ eV	$(2.31^{+0.12}_{-0.09}) \cdot 10^{-3}$ eV	—
δ^{CKM}	$69.27^\circ{}^{+0.91^\circ}_{-0.69^\circ}$	$65.65^\circ{}^{+1.78^\circ}_{-0.53^\circ}$	$69.2^\circ \pm 3.1^\circ$

- Sign of Δm^2_{atm}
- Dirac CP phase δ^{PMNS}
- $\theta_{12}^{\text{PMNS}}$ with future $\sim$60km baseline reactor experiments
- Effective neutrino mass for $0\nu\beta\beta$ experiments $m_{\beta\beta}$
- CKM phase δ^{CKM}

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- Sign of Δm^2_{atm}
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δ^{CKM}	$69.27°{}^{+0.91°}_{-0.69°}$	$65.65°{}^{+1.78°}_{-0.53°}$	$69.2° \pm 3.1°$

- Sign of Δm^2_{atm}
- Dirac CP phase δ^{PMNS}
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- Effective neutrino mass for $0\nu\beta\beta$ experiments $m_{\beta\beta}$
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Comparison between IH and NH Model

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$\theta^{\text{PMNS}}_{12}$	$33.38^\circ{}^{+0.30^\circ}_{-0.28^\circ}$	$34.29^\circ{}^{+0.35^\circ}_{-0.39^\circ}$	$33.57^\circ{}^{+0.77^\circ}_{-0.75^\circ}$
$\theta^{\text{PMNS}}_{23}$	$39.06^\circ{}^{+1.33^\circ}_{-1.32^\circ}$	$38.49^\circ{}^{+1.11^\circ}_{-1.26^\circ}$	$41.4^\circ{}^{+3.5^\circ}_{-1.8^\circ}$
$\theta^{\text{PMNS}}_{13}$	$9.41^\circ{}^{+0.28^\circ}_{-0.27^\circ}$	$9.43^\circ{}^{+0.20^\circ}_{-0.25^\circ}$	$8.75^\circ{}^{+0.42^\circ}_{-0.44^\circ}$
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- Sign of Δm^2_{atm}
- Dirac CP phase δ^{PMNS}
- $\theta^{\text{PMNS}}_{12}$ with future $\sim$60km baseline reactor experiments
- Effective neutrino mass for $0\nu\beta\beta$ experiments $m_{\beta\beta}$
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- Sign of Δm^2_{atm}
- Dirac CP phase δ^{PMNS}
- $\theta_{12}^{\text{PMNS}}$ with future ~60km baseline reactor experiments
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- Sign of Δm^2_{atm}
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- CKM phase δ^{CKM}

Summary

- Proposed SU(5) $\times$ A_4 models that have
  - $\theta_{13}^{\text{PMNS}} \simeq \theta_C/\sqrt{2}$
  - Spontaneous CPV
  - Right-angled unitarity triangle
  - Natural near-degeneracy in IH case
- Good fits with
  - $\chi^2/\text{d.o.f.} = 2.0$ (NH) and 1.1 (IH)
  - Clear predictions for δ^{PMNS}
- To be tested and distinguished by next round of experiments

End of the Talk

Thank you for your attention!

Backup: Alignment Mechanisms

For ϕ_{bc} (almost analogous for ϕ_{ab}):

$$\begin{aligned} W =& S_{bc}[(\phi_{bc})^6 - M^2] + D^\beta(\phi_{bc} \star \phi_{bc})\phi_{bc} \\ &+ D^\gamma[(\phi_{bs}^2)_{\mathbf{1}'}(\phi_{bs}^2)_{\mathbf{1}''} + k(\phi_{bc} \star \phi_{bc})^2] \end{aligned}$$

For ϕ_1, ϕ_2, ϕ_3:

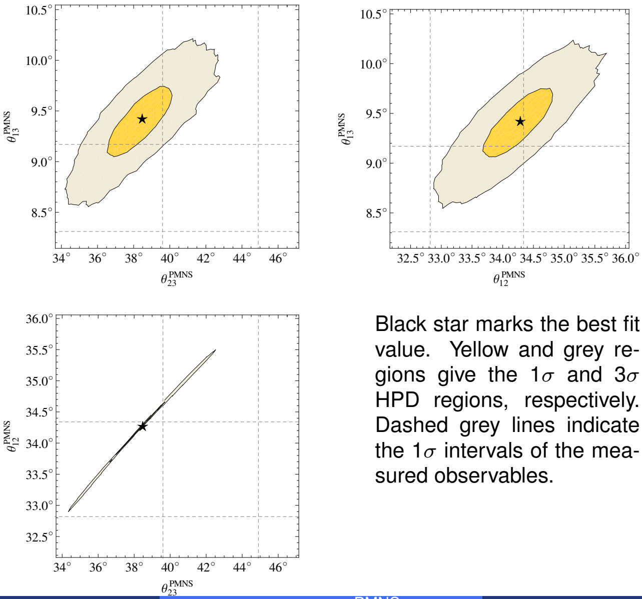
$$W = \sum_i S_i[(\phi_i)^{n_i} - M^2] + O_{i,j}(\phi_i\phi_j) + O'_{i,j}(\phi_i\phi_j)_{\mathbf{1}''}$$

with $n_1 = 2, n_2 = 6, n_3 = 2$

For ϕ_{N_1}, ϕ_{N_2}:

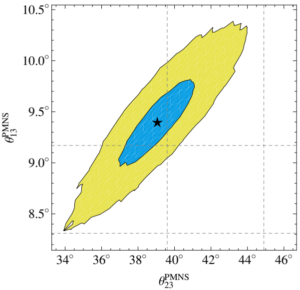
$$\begin{aligned} W =& S_{N_1}[(\phi_{N_1})^6 - M^2] + S_{N_2}[(\phi_{N_2})^6 - M^2] \\ &+ D_{N_1}(\phi_{N_1} \star \phi_{N_1})\phi_{N_1} + O_{N_1,N_2}(\phi_{N_1}\phi_{N_2}) \\ &+ D'_{N_2}(\phi_{N_2}^2)_{\mathbf{1}''} + D''_{N_2}(\phi_{N_2}^2)_{\mathbf{1}'} \end{aligned}$$

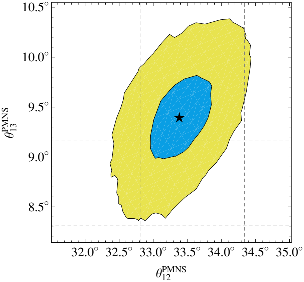
Backup: Correlations (Normal Hierarchy)

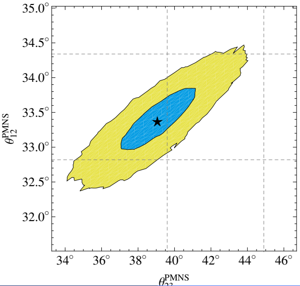


Black star marks the best fit value. Yellow and grey regions give the 1σ and 3σ HPD regions, respectively. Dashed grey lines indicate the 1σ intervals of the measured observables.

Backup: Correlations (Inverse Hierarchy)







Black star marks the best fit value. Blue and golden regions give the 1σ and 3σ HPD regions, respectively. Dashed grey lines indicate the 1σ intervals of the measured observables.