Light field integration in SUGRA theories

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Based on arXiv:1301.6177

SUSY2013, The Abdus Salam ICTP
30 August 2013... 5th Ariel's birthday!

Effective descriptions and symmetries

- Effective theories $\Rightarrow$ neat descriptions.
- Symmetries $\Rightarrow$ robust predictions, and simpler.

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How and under which conditions it is possible to integrate out light fields and obtain a two derivative SUGRA theory?

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The quantum corrections can be neglected if:

- The H are very heavy (hierarchy).
- The two modes are decoupled.

Then

$$S_{Eff}(L) = S(L, H^0(L)) .$$

and the Lagrangian has the structure

$$\mathcal{L}_{eff}(L) = \mathcal{L}\big(L, H^0(L)\big) = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}_i(L) .$$

Higher order operators can be neglected if $\Lambda \sim M_H$ is large or if the corresponding Wilson coefficients, c_i's, are small.

Two derivative truncation

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SUSY case

This procedure is inconsistent with SUSY, if preserved,

$$\delta_\epsilon \phi \sim \epsilon \phi\,, \quad \delta_\epsilon \psi \sim \not{\partial} \phi \epsilon - \frac{1}{2} \epsilon F\,, \quad \delta_\epsilon F \sim \epsilon\, \not{\partial} \psi\,.$$

A parallel truncation in spinor and auxiliary fields is required!

Generic two-derivative Lagrangian, H and L superfields

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(L, H, \overline{L}, \overline{H}) + \int d^2\theta W(L, H) + h.c.\,,$$

then, using $\int d^2\bar{\theta} = -\frac{1}{4}\overline{\mathcal{D}}^2$, $\mathcal{D}$ the SUSY covariant derivative

$$\partial_H \mathcal{L}(L, H) = \partial_H W - \frac{1}{4}\overline{\mathcal{D}}^2 \left(\partial_H K(L, H, \overline{L}, \overline{H})\right) = 0\,.$$

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SUSY two-derivative effective description

[Brizi et al. '09]

In general

$$H = H(L, \bar{L}, \mathcal{D}L, \overline{\mathcal{D}L})\,,$$

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A reliable truncation imply negligible $\mathcal{D}$'s!

Heavy H fields

[N. Arkani-Hamed et al. '98,Choi et al. '09, DG & Serone '09, Brizi et al. '09]

With mass $m_H \sim \partial_H^2 W$ the solution can be written as

$$H(L) = H^o(L) + \Delta H\,, \quad \Delta H \sim \mathcal{O}(\mathcal{D}^2 L/m_H)\,,$$

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Decoupled sectors

Decoupling, no hierarchy, is realized for

$$W = W_H(H) + W_L(L)\,, \quad K = K_H(H,\overline{H}) + K_L(L,\overline{L})\,.$$

However, ΔH is not small in general, only for ϕ_H^o slowly varying

$$\partial_H W\big|_{H^o\,\text{slow}} = 0 \to \partial_H \mathcal{L}(L,H) = \partial_H W - \frac{1}{4}\overline{\mathcal{D}}^2\left(\partial_H K_H(H,\overline{H})\right) \approx 0\,.$$

Rigid Supersymmetry two-derivative description

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Slowly varying solutions to $\partial_H W = 0$ match the ones of the full equation of motion and preserve SUSY!

Supergravity

Generic two-derivative Lagrangian, $G = K + \ln|W|^2$, [Ferrara et al. '82, Kugo et al. '83]

$$\mathcal{L} = -3\int d^2\theta d^2\bar{\theta} e^{-G/3}\Phi\bar{\Phi} + \int d^2\theta\Phi^3 + h.c. + \cdots ,$$

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Some decoupling, no hierarchy, is realized for [Achucarro et al. '08, DG & Serone '09, DG '11]

$$G = G_H(H, \overline{H}) + G_L(L, \overline{L}),$$

such that the e.o.m. reads

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Then slow varying solutions for

$$\partial_H G = 0 \quad \text{if slow varying} \Rightarrow \mathcal{D}H = 0$$

match the ones of the original equation of motion and preserve SUSY.

A two-derivative SUGRA description is obtained!

Hidden sector with its own gauge interactions

$$G = G_H(H, \overline{H}, V_H^r) + G_L(L, \overline{L}, V_L^a)\,,$$
$$\mathcal{L}_{gau-kin} = \frac{1}{4}\int d\theta^2 (f_{rs}\mathcal{W}^r\cdot\mathcal{W}^s + f_{ab}\mathcal{W}^a\cdot\mathcal{W}^b) + h.c.$$

with $\mathcal{W}_\alpha \sim \overline{\mathcal{D}}^2\left(e^{-V}\mathcal{D}_\alpha e^V\right)$, $f_{rs} = f(H) + \tilde{f}_H(H, L)$, $f_{ab} = f_L(L) + \tilde{f}_L(H, L)$.

Then the full hidden sector is integrated out through

$$\partial_H G\,\mathcal{D}^2\left(e^{-G/3}\bar{\Phi}\right) + \mathcal{O}(\mathcal{D}H, \mathcal{D}V_H^r) + \mathcal{O}(\partial_H\tilde{f}_L\mathcal{D}V_L^a) = 0\,,$$
$$(\partial_{V_H^r} G)e^{-G/3}\bar{\Phi}\Phi + \mathcal{O}(\mathcal{D}H, \mathcal{D}V_H^r) + \mathcal{O}(\partial_H\tilde{f}_H\mathcal{D}L) = 0\,,$$

Gauge symmetries

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with $\mathcal{W}_\alpha \sim \overline{\mathcal{D}}^2 \left(e^{-V}\mathcal{D}_\alpha e^V\right)$, $f_{rs} = f(H)$, $f_{ab} = f_L(L)$.

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$$\partial_H G\, \mathcal{D}^2 \left(e^{-G/3}\bar{\Phi}\right) + \mathcal{O}(\mathcal{D}H, \mathcal{D}V_H^r) = 0\,,$$
$$(\partial_{V_H^r} G) e^{-G/3}\bar{\Phi}\Phi + \mathcal{O}(\mathcal{D}H, \mathcal{D}V_H^r) = 0\,,$$

with approximated SUSY slow varying solutions

$$\partial_H G = 0\,, \quad and \quad \partial_{V_H^r} G = 0\,.$$

Comments & remarks

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- Gauge fixing of the conformal symmetries in superspace [Cheung '11]

$$\Phi \equiv e^{Z/3}(1+\theta^2 U)\,, \quad \text{with} \;\; Z = \langle G\rangle + \langle \partial_H G\rangle H + \langle \partial_L G\rangle L\,.$$

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- The form of G guarantees no restriction on the H mass if these are stabilized at nearly SUSY points.

Conclusions

A reliable effective two-derivative SUGRA theory is possible even when the integrated fields are light if: The theory has a factorizable G,i.e.,

$$G = G_H(H, \overline{H}) + G_L(L, \overline{L}) + \epsilon G_{mix}(H, L, \overline{H}, \overline{L}) \,,$$

e.g., in Large volume scenarios, but restricting to slow varying H field configurations, solving

$$\partial_H G = 0 \,.$$

The integrated fields can be charged under a hidden gauge sector and the hidden gauge sector are also integrated by the superspace promotion of the D-flatness condition.