Light field integration in SUGRA theories

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SUSY2013, The Abdus Salam ICTP
30 August 2013... 5th Ariel’s birthday!
Effective descriptions and symmetries

- Effective theories $\Rightarrow$ neat descriptions.
- Symmetries $\Rightarrow$ robust predictions, and simpler.

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Like in string moduli stabilization scenarios.
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How and under which conditions it is possible to integrate out light fields and obtain a two derivative SUGRA theory?
Integration of fields

In a theory with two modes, the dynamics for the $L$ are dictated by $S_{\text{Eff}}(L)$,

$$e^{iS_{\text{Eff}}(L)} = \int {\mathcal{D}H} e^{iS(L,H)}$$
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with $\delta H$ fluctuations around the classical solution $\left. \frac{\delta S}{\delta H} \right|_{H=H^0} = 0$. 

The quantum corrections can be neglected if:

- The $H$ are very heavy (hierarchy).
- The two modes are decoupled.

Then $S_{\text{Eff}}(L) = S(L,H^0)$.

and the Lagrangian has the structure 

$$
L_{\text{eff}}(L) = L(L,H^0) = \sum c_i \Lambda d_i - 4 O_i(L).
$$

Higher order operators can be neglected if $\Lambda \sim M_H$ is large or if the corresponding Wilson coefficients, $c_i$'s, are small.
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# Two derivative truncation

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SUSY case

This procedure is inconsistent with SUSY, if preserved,

\[ \delta_\epsilon \phi \sim \epsilon \phi, \quad \delta_\epsilon \psi \sim \partial \phi \epsilon - \frac{1}{2} \epsilon F, \quad \delta_\epsilon F \sim \epsilon \partial \psi. \]

A parallel truncation in spinor and auxiliary fields is required!
Supersymmetry

Generic two-derivative Lagrangian, $H$ and $L$ superfields

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(L, H, \bar{L}, \bar{H}) + \int d^2\theta W(L, H) + h.c. ,$$

then, using $\int d^2\bar{\theta} = -\frac{1}{4}D^2$, $D$ the SUSY covariant derivative

$$\partial_H \mathcal{L}(L, H) = \partial_H W - \frac{1}{4}D^2 \left( \partial_H K(L, H, \bar{L}, \bar{H}) \right) = 0 .$$
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SUSY two-derivative effective description [Brizi et al. ’09]

In general

$$H = H(L, \bar{L}, \mathcal{D}L, \overline{\mathcal{D}L})$$ ,

namely, the resulting theory

- cannot be cast to a two derivative SUSY.
- in particular, SUSY is broken.
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**A reliable truncation imply negligible $D$’s!**
Rigid Supersymmetry two-derivative description

Heavy $H$ fields

With mass $m_H \sim \partial_H^2 W$ the solution can be written as

$$H(L) = H^o(L) + \Delta H, \quad \Delta H \sim O(\mathcal{D}^2 L/m_H),$$

$H^o(L)$ solution to $\partial_H W = 0$. Similar for SUGRA!
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Decoupled sectors

Decoupling, no hierarchy, is realized for

$$W = W_H(H) + W_L(L), \quad K = K_H(H, \bar{H}) + K_L(L, \bar{L}).$$

However, $\Delta H$ is not small in general, only for $\phi^0_H$ slowly varying

$$\partial_H W \big|_{H^0 \text{ slow}} = 0 \rightarrow \partial_H \mathcal{L}(L, H) = \partial_H W - \frac{1}{4} \bar{D}^2 (\partial_H K_H(H, \bar{H})) \approx 0.$$
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Slowly varying solutions to $\partial_H W = 0$ match the ones of the full equation of motion and preserve SUSY!
Supergravity

Generic two-derivative Lagrangian, $G = K + \ln |W|^2$, [Ferrara et al. ’82, Kugo et al. ’83]

$$\mathcal{L} = -3 \int d^2 \theta d^2 \bar{\theta} e^{-G/3} \Phi \bar{\Phi} + \int d^2 \theta \Phi^3 + h.c. + \cdots,$$

$\Phi$ compensator superfield and the $\cdots$ stand for gravity multiplet terms.
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Some decoupling, no hierarchy, is realized for \[\text{[Achucarro et al. '08, DG & Serone '09, DG '11]}\]

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G = G_H(H, \bar{H}) + G_L(L, \bar{L}) ,
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such that the e.o.m. reads

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\partial_H G_H \mathcal{D}^2 \left(e^{-G/3} \Phi\right) + \mathcal{O}(\mathcal{D}H) = 0 .
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such that the e.o.m. reads

\[
\partial_H G_H D^2 \left( e^{-G/3} \bar{\Phi} \right) + O(DH) = 0 .
\]

Then slow varying solutions for

\[
\partial_H G = 0 \quad \text{if slow varying} \Rightarrow D_H = 0
\]

match the ones of the original equation of motion and preserve SUSY.

**A two-derivative SUGRA description is obtained!**
Gauge symmetries

Hidden sector with its own gauge interactions

\[ G = G_H(H, \bar{H}, \mathcal{V}_H^r) + G_L(L, \bar{L}, \mathcal{V}_L^a), \]

\[ \mathcal{L}_{gau-kin} = \frac{1}{4} \int d^2(\theta^2) (f_{rs} \mathcal{W}^r \cdot \mathcal{W}^s + f_{ab} \mathcal{W}^a \cdot \mathcal{W}^b) + h.c. \]

with \( \mathcal{W}_\alpha \sim \overline{D}^2 (e^{-V} D_\alpha e^V) \), \( f_{rs} = f(H) + \tilde{f}_H(H, L) \), \( f_{ab} = f_L(L) + \tilde{f}_L(H, L) \).

Then the full hidden sector is integrated out through

\[ \partial_H G \bar{D}^2 \left( e^{-G/3} \Phi \right) + \mathcal{O}(D_H, D \mathcal{V}_H^r) + \mathcal{O}(\partial_H \tilde{f}_L D \mathcal{V}_L^a) = 0, \]

\[ (\partial_{\mathcal{V}_H^r} G) e^{-G/3} \Phi \Phi + \mathcal{O}(D_H, D \mathcal{V}_H^r) + \mathcal{O}(\partial_H \tilde{f}_H D \mathcal{L}) = 0, \]
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with \( \mathcal{W}_\alpha \sim \bar{D}^2 \left( e^{-V} D_\alpha e^V \right) \), \( f_{rs} = f(H) \), \( f_{ab} = f_L(L) \).

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\[ \partial_H G \bar{D}^2 \left( e^{-G/3} \bar{\Phi} \right) + \mathcal{O}(\mathcal{D}H, \mathcal{D}V'_H) = 0, \]

\[ (\partial_{V'_H} G) e^{-G/3} \bar{\Phi} \Phi + \mathcal{O}(\mathcal{D}H, \mathcal{D}V'_H) = 0, \]

with approximated SUSY slow varying solutions

\[ \partial_H G = 0, \quad \text{and} \quad \partial_{V'_H} G = 0. \]
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Comments & remarks

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- Gauge fixing of the conformal symmetries in superspace [Cheung '11]

$$
\Phi \equiv e^{Z/3}(1 + \theta^2 U), \quad \text{with} \quad Z = \langle G \rangle + \langle \partial_H G \rangle H + \langle \partial_L G \rangle L.
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The analysis can be generalized by adding a small mix

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G_{\text{mix}}(L, H) \sim \epsilon, \quad \frac{\partial f_L}{\partial H} \sim \epsilon, \quad \frac{\partial f_H}{\partial L} \sim \epsilon.
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Then if \( \partial_\mu \phi \sim \epsilon \), the corrections are small.
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The form of $G$ guarantees no restriction on the H mass if these are stabilized at nearly SUSY points.
A reliable effective two-derivative SUGRA theory is possible even when the integrated fields are light if: The theory has a factorizable $G$, i.e.,

$$G = G_H(H, \bar{H}) + G_L(L, \bar{L}) + \epsilon G_{mix}(H, L, \bar{H}, \bar{L}),$$

e.g., in Large volume scenarios, but restricting to slow varying $H$ field configurations, solving

$$\partial_H G = 0.$$ 

The integrated fields can be charged under a hidden gauge sector and the hidden gauge sector are also integrated by the superspace promotion of the D-flatness condition.